African Journal of Advanced Pure and Applied Sciences (AJAPAS)
Online ISSN: 2957-644X
Volume 4, Issue 1, January - March 2025
Page No: 1-7
Website: https://aaasjournals.com/index.php/ajapas/index
ISI 2024: 0.877 SJIFactor 2024: 6.752 معامل التأثير العربي: 1.6

Harmonic Solutions to Solve Complex Algebraic Equations

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الحلول التوافقية لحل المعادلات الجبرية المعقدة

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Received: October 10, 2024	Accepted: December 21, 2024	Published: January 05, 2025

Abstract
Complex algebraic equations are a fundamental topic in mathematics, requiring the use of multiple techniques to solve them. These technologies include Simplifying, multiplying by reciprocals, and many other different methods, which is used to analyze equations and extract the required variables. Through this study, which aims to enhance students' understanding of the basic concepts in algebra, including algebraic operations such as addition, subtraction, multiplication, and division, and how to apply them in solving equations, in addition to developing students' level in solving complex algebraic equations using...? Combinatorial solutions and developing exploration and criticism skills through analyzing mathematical problems. Through a methodology that relied on several different methodologies, including the descriptive methodology in describing and defining complex algebraic equations and combinatorial solutions, and the quantitative methodology in collecting data on combinatorial solutions and the factors influencing the solution of complex algebraic equations and the nature of the compromise solutions that are chosen, in addition to the scientific methodology in analyzing combinatorial methods and the influencing factors. There is a comparison methodology to compare combinatorial methods in terms of accuracy, ease, and pass ability The results indicated that combinatorial solutions are largely effective in enhancing students' understanding of basic concepts and developing their skills. Exploratory and critical mathematical problems. The flexibility, ease and accuracy of combinatorial methods range from 90 to 95%.

Keywords: Complex Algebraic Equations, Combinatorial Solutions, Influencing Factors, Accuracy and Flexibility.

الملخص
المعادلات الجبرية المعقدة هي موضوع أساسي في الرياضيات، وتتطلب استخدام تقنيات متعددة لحلها. وتشمل هذه التقنيات التبسيط والضرب بالمقلوب والعديد من الطرق المختلفة الأخرى التي تستخدم لتحليل المعادلات واستخراج المتغيرات المطلوبة. من خلال هذه الدراسة التي تهدف إلى تعزيز فهم الطلاب للمفاهيم الأساسية في الجبر، بما في ذلك العمليات الجبرية مثل الجمع والطرح والضرب والقسمة، وكيفية تطبيقها في حل المعادلات، بالإضافة إلى تطوير مستوى الطلاب في حلها المعادلات الجبرية المعقدة باستخدام...؟ الحلول التوافقية وتنمية مهارات الاستكشاف والنقد من خلال تحليل المشكلات الرياضية. من خلال منهجية اعتمدت على عدة منهجيات مختلفة، منها المنهجية الوصفية في وصف وتعريف المعادلات الجبرية المعقدة والحلول التوافقية، والمنهجية الكمية في جمع البيانات عن الحلول التوافقية والعوامل المؤثرة في حل المعادلات الجبرية المعقدة وطبيعة المعادلات الجبرية المعقدة. ويتم اختيار الحلول التوفيقية، بالإضافة إلى المنهجية العلمية في تحليل الأساليب التوافقية والعوامل المؤثرة. توجد منهجية مقارنة لمقارنة الطرق التوافقية من حيث الدقة والسهولة والقدرة على النجاح. أشارت النتائج إلى أن الحلول التوافقية فعالة إلى حد كبير في تعزيز فهم الطلاب للمفاهيم الأساسية وتنمية مهاراتهم. المسائل الرياضية الاستكشافية والنقدية. وتتراوح مرونة وسهولة ودقة الطرق التوافقية من 90 إلى 95%.

الكلمات المفتاحية: المعادلات الجبرية المعقدة، الحلول التوافقية، العوامل المؤثرة، الدقة والمرونة.

1.Introduction

They are algebraic equations that can be defined as interest Mathematical expression that expresses equality between two algebraic expressions. These two quantities may include variables, numbers, and coefficients, in addition to the well-known arithmetic operations of addition, multiplication, subtraction, and division. Which represents the basics of algebra which is one of the branches of mathematics? Algebraic equations are considered the cornerstone of many mathematical, scientific, and engineering fields and applications, and even after important social applications. With the development of science, especially programming sciences, engineering sciences and physics, algebraic equations have developed and become more complex, and these complexities have become among the most important challenges that algebraic equations witness in the years. Finally, to overcome these challenges, studies and research began to develop in the same field to search for methods and techniques that would solve The complexities of algebraic equations and their simplification. One of the most important of these techniques is the combinatorial methods for solving algebraic equations [1]

Complex algebraic equations are algebraic equations that include random variables sometimes and ambiguous variables at other times. The relationship between these variables can also be a linear relationship, that is, a first-degree relationship or a multi-degree relationship. These relationships may be ambiguous relationships or no relationships, and they cannot be formulated in a known mathematical relationship. All of that It ultimately leads to complexity in these equations, and to solve these equations, it was necessary to develop techniques and strategies that contribute to solving these equations and simplifying them. The most important of these techniques are combinatorial solution techniques, which can be defined as Methods and strategies related to arranging and organizing the different elements within the groups. Each of these groups may contain smaller groups or may be part of larger groups, but it must be taken into account that the arrangement and organization must be subject to some restrictions and conditions to ensure the success of the solution and the accuracy of the results. Consensual solution identifications A More accurate It is a branch of mathematics that is concerned with studying problems related to numbers, their counting, and the relationship between those numbers, whether this relationship is represented by the number of ways in which the numbers and elements can be arranged in a certain way or the number of choices for the ways in which a subset of a larger group can be chosen [2].

The importance of combinatorial methods is that they contribute to simplifying complex algebraic expressions, making them easier to understand and analyze. Through techniques such as collecting similar terms and removing parentheses, it also helps isolate variables and can transform equations into more manageable forms. It also provides

2. Theoretical background and basic concepts

This section will present the theoretical background and basic concepts of algebraic equations and combinatorial methods. By presenting this background and basic concepts, an insightful point of view can be formed about the study procedures, their importance, and drawing their conclusions. It also gives a conscious and flexible understanding of the nature of algebraic equations and the use of combinatorial methods in solving them. In addition to providing a historical overview of the development of algebraic equations and combinatorial methods**.**

2.1. Algebraic equations

An algebraic equation is a mathematical expression consisting of two algebraic expressions joined by an equal sign (=). An algebraic equation is also known as a polynomial equation, as both sides contain multiple terms. It can be algebraic equations Univariate (contains one variable) or Multivariate (Containing more than one variable) Current equations may be first-degree, second-degree, or multi-degree equations. Figure (1) shows the components of the algebraic equation [3]

Results and discussion

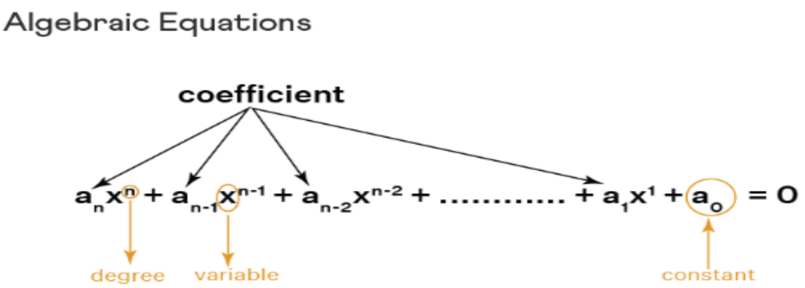

Figure 1: The components of an algebraic equation.

2.1.1. Historical Overview

- The roots of algebraic equations began since time immemorial. The ancient Egyptian and Babylonian civilizations used simple equations to calculate the areas and volumes of geometric shapes. Then the Greeks and Pomegranates added another geometric dimension to the algebraic equations, as they linked the equations to geometric proofs and the basis of the theoretical foundations of some algebraic concepts such as the theorems of Euclid and Pythagoras.

➢ Then came the Islamic era, and Muslim scholars played a decisive role in developing the science of algebra, introducing new algebraic concepts, and creating symbols to facilitate mathematical operations. Abu Abdullah Muhammad Ibn Musa Al-Khwarizmi is considered one of the most important pioneers of algebra, as he wrote the book Al-Mukhtasar fi Arithmetic Algebra, which is considered the first systematic book in the science of algebra.

➢ Then, in modern times, since the sixteenth century, major developments began in solving algebraic equations, especially of the second, third, and fourth degrees, where scholars such as Ghawes contributed to developing the theory of equations and establishing modern algebra, and scholars such as Descartes contributed to developing algebra and introducing modern symbols.

- In the modern era, with the emergence of computer science and programming, there has been a revolution in algebraic equations and their solution, especially difficult and complex algebraic equations. Any computer can now solve millions of algebraic equations in record time [4]

2.1.2. Types of Algebraic Equations

Algebraic equations are classified based on the degree of the equation, which is defined as the highest exponent of the variable in the equation. Common types include: Algebraic equations can be classified according to the degree of the equation, as the common types are first-degree equations [5], which are called linear equations, and third-degree equations, and the degree of the equation is determined by x power, as shown in Figure (2).

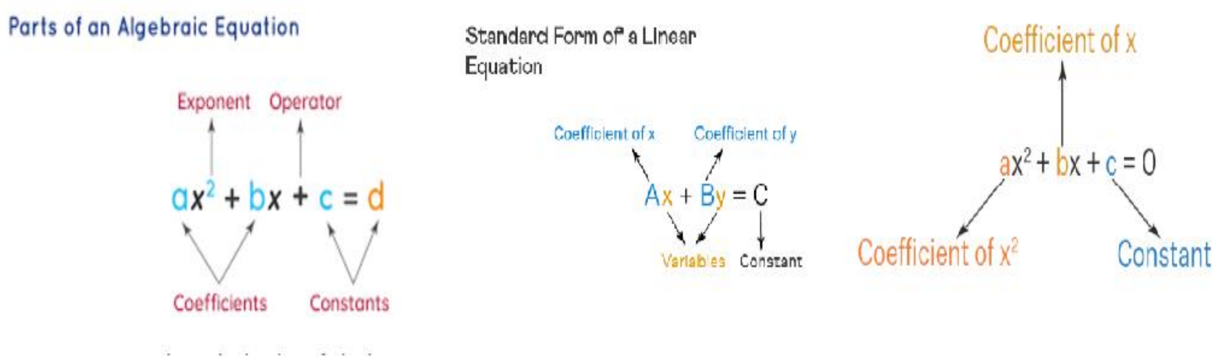

Figure 2: The types of algebraic equations.

2.1.3. Principles and Properties of Algebraic Operations

The basic principles of algebra include the rules that govern mathematical operations such as addition, subtraction, multiplication, and division. It is important to keep the equation balanced, which means that the same operation must be performed on both sides to keep the equation valid as shown in the figure.

Property	Operations			
	Addition	Subtraction	Multiplication	Division
Closure	a + b ∈ R	a - b ∈ R	a × b ∈ R	a ÷ b ∈ R
Commutative	a + b = b + a	a - b ≠ b - a	a × b = b × a	a ÷ b ≠ b ÷ a
Associative	a+(b+c) = (a+b)+c	a-(b-c) = (a-b)-c	a×(b×c) = (a×b)×c	a÷(b÷c) = (a÷b)÷c
Identity	a+0 = 0+a = a	--	a×1/a=1=1/a×a	--
Distributive	a×(b+c) = ab + ac	a×(b-c) = ab - ac	--	--

Figure 3: The basic operations of algebraic equations and their properties.

2.2. Compatible solutions

It is a group of methods that include deletion, substitution, isolation, analysis, and inverse multiplication. The goal of these methods is to simplify and solve complex algebraic equations [6]

2.2.1. Elimination method:

This method is used to remove one of the variables in a system consisting of 30 linear, which makes the coefficient of one of the variables equal in absolute value and reflected in sign. By subtracting or adding the two equations, the equations can be solved. The following is an example that shows the elimination method:

$x + 2y = 7$

$3x - y = 1$

the solution:

We multiply the first equation by 3: $3x + 6y = 21$

We add the two new equations: $(3x + 6y) + (3x - y) = 21 + 1$ $6x + 5y = 22$

Now we have an equation with one variable (x). We can solve it for x, and then substitute it into one of the original equations to find y **[7]**

2.2.2. Isolation method:

It is a method that depends on isolating one variable from one of the two equations and then replacing the resulting value in the other equation. The following is an example that shows the solution method.

$2x + y = 5$

$x - 3y = 1$

the solution:

We isolate x in the first equation: $x = 5 - 2y$

We substitute the value of x into the second equation: $(5 - 2y) - 3y = 1$

We solve the resulting equation to find the value of y, then substitute the value of y into one of the original equations to find the value of x [8]

2.2.3. Substitution method:

A method somewhat similar to the isolation method, but in this method we isolate an expression that contains two variables and then replace it completely in the other equation. The following is an example of using this method:

$x + y = 7$

$2x - y = 1$

the solution:

We isolate y in the first equation: $y = 7 - x$

We substitute the value of y into the second equation: $2x - (7 - x) = 1$

We solve the resulting equation to find the value of x, then substitute the value of x into one of the original equations to find the value of y.

2.2.4. Reciprocal multiplication method:

This method is used to remove fractions from equations by multiplying all terms of the equation by the least common denominator. The following is an example of solving this equation:

$(1/2)x + 3 = (2/3)x - 1$

the solution:

We multiply all terms by 6 (the least common denominator): $3x + 18 = 4x - 6$

Now we can solve the resulting equation to find the value of x.

2.2.5. Analysis method:

It is a method used to analyze quadratic equations or those with higher degrees into several factors and then analyze each factor separately. Below is an example of this method.

$$x^2 - 5x + 6 = 0$$

the solution:

We factorize the equation: $(x - 2)(x - 3) = 0$

So, either $x - 2 = 0$ or $x - 3 = 0$

Therefore, the solutions are $x = 2$ or $x = 3$

3.metthodology

The methodology here is to choose a complex equation and solve it using five different combinatorial methods, clarify the method by which the solution was achieved, then compare the solutions and the methods by which the solution was used, and provide a critical vision of this mathematical problem related to solving the complex equation **[10]** This methodology aims to train students in critical methods for solving mathematical problems. Developing their level of solving algebraic equations and how to choose the appropriate method to solve the equation x2+y2=z2 in five combinatorial ways, the following methods can be used:

1. Compensation method

We can rearrange the equation to find z:

$$Z^2=x^2+y^2$$

Using this equation, we can substitute different values for x and y to get values for z. For example:

Z	X	Y
5	3	4
5	4	3
10	4	6

3.1. Factorization method

The equation can be factored using the difference-of-squares formula, but in this case, we can use the Pythagorean equations. If we consider:

x=a,y=b,z=c

The equation means that (a,b,c)(a,b,c) represents a right-angled triangle. We can use known values like (3,4,5)(3,4,5) or (5,12,13)(5,12,13).

3.2. Chart method

The equation can be plotted in a coordinate system. The circuit representing the equation x2+y2=z2 is drawn. By changing the value of z, it is possible to observe how the circle changes, which helps in finding the points that satisfy the equation.

x\y	1	2	3	4	5	6	7	8	9	10
1	-	-	-	-	-	-	-	-	-	-
2	-	-	-	-	-	-	-	-	-	-
3	-	-	-	-	-	-	-	-	-	-
4	-	-	-	**5**	-	-	-	-	-	-
5	-	-	-	-	-	-	-	-	-	-
6	-	-	-	-	-	-	-	-	-	-
8	-	-	-	-	-	-	-	-	-	-

3.3. Deletion method

If we have two equations like:

x2+y2==z2 and z=k

Where k is a constant, we can substitute Z into the first equation:

x2+y2==k2

when k= 25 x= 3 y=4

3.4. Matrix method

Matrices can be used to solve a system of equations. If we consider that we have a set of values for xx, yy and zz, we can represent them in a matrix and find the solutions using matrix operations.

1. Convert the equation into a matrix form

We begin by converting the equation into a system of equations. We can rewrite the equation as follows:

x2+y2−z2=0

Let's have three variables: x,y,z

We can represent the equation in matrix form. Consider that we have a set of possible values for x, y, and z:

$$[1 \quad 1 \quad -1]\begin{bmatrix} x2 \\ y2 \\ z2 \end{bmatrix}=[0]$$

create an expanded matrix:

[1 1− 1 |0]

=32+42=9+16=25 X=3 y= 4 z=5

4. Results and discussion

In this section, we will discuss the results of solving the previous problems using five methods, along with the arrangement of the five methods for solving the equation x2+y2=z2 in terms of accuracy and flexibility.

Table 1: Arrangement of the five methods accuracy and flexibility and Solution speed.

	Method				
	Newton's Method	**Deletion Method**	**Replacement Method**	**The Graphing Method**	**The Matrix Method**
Accuracy	**Very High.**	**High.**	**Moderate**	**High**	**Moderate**
Flexibility	**Moderate**	**High.**	**Weak**	**High**	**Moderate**
Solution Speed	**Moderate**	**Moderate**	**Moderate**	**Moderate**	**Moderate**

Very high=5 high =4 moderate = 3 weak =1

1. Newton's method: Newton's method is considered one of the most accurate and flexible methods, as it rely on repetition to improve guesses, which leads to rapid convergence towards the correct solution. It is also a method, but in terms of flexibility, it is moderate, requires knowledge of difficulties, and may be a sensitive choice. But it is very effective in solving nonlinear equations
2. Deletion method: The deletion method comes in second place in terms of accuracy, as it allows the removal of variables in a direct way, which facilitates access to solutions and the flexibility is very high.
3. Replacement method: The replacement method comes in third place. It is good and has moderate flexibility, but it may become complicated in some large systems.
4. The graphing method: It comes in fourth place. It is moderately accurate and highly flexible. It can be used to understand the relationships between variables visually, but sometimes it is not as accurate as other methods.
5. The matrix method: The expense method is moderately accurate and moderately flexible, and it can be complex in implementation and requires knowledge **[11]**

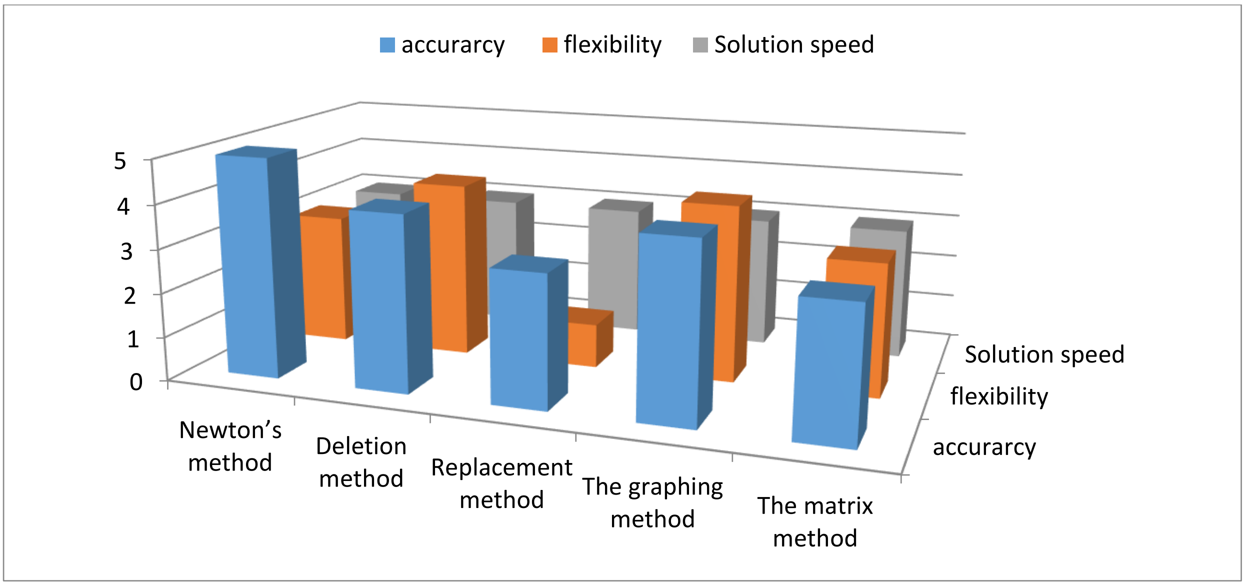

Figure 4: Arrangement of the five methods accuracy and flexibility and Solution speed.

The figure shows that Newton's method comes in first place in terms of accuracy, followed by the graphical method, then the deletion method, then the matrix method, and the substitution method comes in last place. As for flexibility, the deletion method comes in first place, followed by the graphical method in second place, then the matrix method in the third stage, Newton's method in fourth place, and the substitution method in the fifth and final stage. As for the speed of solution, the matrix, graphical, and substitution methods come in first place, the deletion method in second place, and Newton in last place **[12]**

5. Conclusions (Summary)

The study concludes the following:

1. Method Selection: Choosing a combinatorial solution method depends on application requirements such as flexibility, speed, and accuracy **[13]**

1) Flexibility: Elimination and graphing methods are preferred.
2) Accuracy: Newton’s method and elimination method are best.
3) Speed: Expense, graphing, or elimination methods are suitable.

2.Newton's Method: Highly accurate and effective for nonlinear equations due to its iterative nature, leading to rapid convergence. It is moderately flexible but requires careful handling of complexities.

3. deletation Method: Offers high accuracy and flexibility by directly removing variables to simplify solutions.

4. Replacement Method: Provides moderate flexibility but can become complicated in large systems **[14]**

5. Graphing Method: Moderately accurate with high flexibility, suitable for visualizing variable relationships but less precise than other methods.

6. matrix Method: Moderately accurate and flexible, but complex in implementation and requires advanced knowledge.

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