African Journal of Advanced Pure and Applied Sciences (AJAPAS)
Online ISSN: 2957-644X
Volume 4, Issue 2, 2025
Page No: 420-434
Website: https://aaasjournals.com/index.php/ajapas/index

ISI 2025: 1.126 SJIFactor 2024: 6.752 معامل التأثير العربي: 1.62

Advanced Signal Decomposition Techniques for Vibration Diagnostics in Mechanical Systems: A Fourier-Based Approach

Adel Khalleefah Hamad Darmeesh *
Faculty Member, Department of Materials and Metallurgy Engineering, Ajdabiya University, Libya

تقنيات تحليل الإشارة المتقدمة لتشخيص الاهتزازات في الأنظمة الميكانيكية: نهج قائم على فورييه

عادل خليفة حمد درميش *
عضو هيئة التدريس، قسم هندسة المواد والمعادن، جامعة أجدابيا، ليبيا

*Corresponding author: adelsh123431@gmail.com, adelsh_dermesh@uoa.edu.ly

Received: May 06, 2025	Accepted: June 21, 2025	Published: June 25, 2025

Abstract
Vibration analysis plays a critical role in condition monitoring and fault diagnosis of mechanical systems. This paper presents an in-depth investigation into the application of Fourier-based signal decomposition techniques for diagnosing vibrational anomalies in mechanical components. By transforming time-domain vibration signals into the frequency domain using Fourier series expansion and its computational counterpart, the Fast Fourier Transform (FFT) engineers can identify dominant harmonic components that indicate system behavior under dynamic loading conditions. The study outlines the theoretical framework, numerical implementation, and practical applications of Fourier methods in detecting imbalances, misalignments, and structural defects. Additionally, this work explores the integration of these classical tools with modern signal processing strategies to enhance diagnostic accuracy and enable predictive maintenance.

Keywords: Signal Decomposition, Vibration, Diagnostics, A Fourier Series, Approach.

الملخص
يلعب تحليل الاهتزازات دورًا حاسمًا في مراقبة حالة الأنظمة الميكانيكية وتشخيص أعطالها. تقدم هذه الورقة بحثًا متعمقًا في تطبيق تقنيات تحليل الإشارات القائمة على فورييه لتشخيص الشذوذ الاهتزازي في المكونات الميكانيكية. من خلال تحويل إشارات الاهتزاز في المجال الزمني إلى المجال الترددي باستخدام توسيع سلسلة فورييه ونظيرتها الحسابية، يمكن لمهندسي تحويل فورييه السريع (FFT) تحديد المكونات التوافقية السائدة التي تشير إلى سلوك النظام في ظل ظروف التحميل الديناميكي. توضح الدراسة الإطار النظري والتطبيق العددي والتطبيقات العملية لطرق فورييه في الكشف عن الاختلالات، وعدم المحاذاة، والعيوب الهيكلية. بالإضافة إلى ذلك، يستكشف هذا العمل دمج هذه الأدوات التقليدية مع استراتيجيات معالجة الإشارات الحديثة لتعزيز دقة التشخيص وتمكين الصيانة التنبؤية.

الكلمات المفتاحية: تحليل الإشارات، الاهتزاز، التشخيص، سلسلة فورييه، النهج.

1. Introduction

Mechanical systems are inherently subject to dynamic forces that result in vibrations [1]. These oscillatory motions, while sometimes benign, can lead to performance degradation, accelerated wear, or catastrophic failure if not properly monitored and controlled [2]. Vibration diagnostics have therefore become integral to maintenance planning and system reliability, especially in rotating machinery such as turbines, compressors, and gearboxes [3], [4], [5], [6]. Signal decomposition techniques provide a powerful means of interpreting complex vibrational data. Among these, the Fourier series [7], [8], [9], [10], which represents periodic functions as sums of sinusoids, has proven particularly effective due to its ability to isolate frequency-specific contributions within a signal [11], [12], [13], [14]. This approach allows engineers to detect subtle changes in spectral content that magnify emerging faults or deviations from normal operational behavior [15], [16], [17], [18]. This paper aims to demonstrate how Fourier-based decomposition serves as a cornerstone in modern vibration diagnostics, enabling precise identification of system characteristics and facilitating targeted interventions.

2. Theoretical Background: Fourier Series in Vibration Analysis

2.1 Definition and Mathematical Formulation

A continuous, real-valued periodic function f(t) with period T can be expressed via the Fourier series as:

$$f(t) = a_0 + \sum_{n-1}^{\infty} \left[a_n \cos\left(\frac{2\pi nt}{T}\right) + b_n \sin\left(\frac{2\pi nt}{T}\right)\right]$$

Where the coefficients a_0, a_n, and b_n are given by:

$$a_0 = \frac{1}{T}\int_0^T f(t)dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos\left(\frac{2\pi nt}{T}\right)dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$$

These coefficients represent the amplitude of each harmonic component present in the signal, enabling the reconstruction of the original waveform in terms of its frequency content [19], [20], [21], [22].

2.2 Frequency Domain Interpretation

The transformation of time-domain vibration signals into the frequency domain through Fourier analysis reveals the distribution of energy across various frequencies. Each term in the series corresponds to a specific harmonic frequency fn= Tn, allowing engineers to associate spectral peaks with physical phenomena such as rotational speed harmonics, gear meshing frequencies, or bearing defect frequencies. Resonant frequencies, those at which the system amplifies vibrational energy, can also be identified, providing crucial insights for avoiding operating conditions that could induce instability or failure [23], [24], [25], [26].

3. Application of Fourier Analysis in Vibration Diagnostics

3.1 Detection of Mechanical Faults

In rotating machinery, abnormal vibrations often manifest as distinct frequency components superimposed on the baseline spectrum. For example:
Unbalance: Typically appears as a strong peak at the fundamental rotational frequency.
Misalignment: Generates harmonics at twice the rotational frequency or higher.
Generates harmonics at twice the rotational frequency (2× RPM) or higher. Angular or parallel misalignment between coupled shafts causes periodic stresses that produce vibration energy at harmonic multiples of the running speed.
Gear Tooth Defects: Induce sidebands around the gear mesh frequency.
Induce sidebands around the gear mesh frequency, which is calculated as the product of the number of gear teeth and the rotational speed. Damage such as cracks, wear, or broken teeth results in amplitude modulation of the mesh frequency, creating sidebands spaced at the gear's rotational frequency [27].
Bearing Faults: Exhibit characteristic defect frequencies based on geometry and rotational speed.
Exhibit characteristic defect frequencies determined by the bearing geometry (number of rolling elements, pitch diameter, contact angle) and rotational speed. Common fault locations include the inner race, outer race, rollers, and cage, each producing distinct frequencies identifiable through spectral analysis. By performing a Fourier decomposition, these features can be isolated and analyzed quantitatively, enabling early detection of potential failures. The Fast Fourier Transform (FFT) is commonly used for this purpose in industrial applications due to its computational efficiency and ability to convert time-domain signals into interpretable frequency spectra [28].
By performing a Fourier decomposition, these features can be isolated and analyzed quantitatively, enabling early detection of potential failures.

3.2 Resonance Identification and Damping Design

Resonance occurs when the excitation frequency coincides with one of the natural frequencies of the system, leading to amplified oscillations. Fourier analysis aids in identifying these critical frequencies by revealing sharp peaks in the amplitude spectrum. Once identified, appropriate damping mechanisms such as tuned mass dampers or viscoelastic supports can be designed to suppress unwanted oscillations and ensure safe operation.

3.3 Predictive Maintenance Strategy

Integrating Fourier-based vibration analysis into predictive maintenance programs allows for continuous monitoring of machine health. Deviations from established baseline spectra can be flagged automatically, prompting maintenance before a failure occurs. This approach reduces unplanned downtime and extends equipment life [30].

4. Numerical Implementation Using Fast Fourier Transform (FFT)

While the analytical Fourier series provides the theoretical foundation, practical implementation relies heavily on the Fast Fourier Transform (FFT) algorithm a computationally efficient method for computing discrete Fourier transforms.

Given a sampled vibration signal $x[n]$ of length N, the DFT is defined as:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}, k = 0,1,\ldots,N-1$$

where:

- $X[k]$: Frequency-domain representation
- $x[n]$: Time-domain signal
- j : Imaginary unit ($j^2 = -1$)

The FFT reduces the computational complexity from $O(N^2)$ to $O(N\log\ N)$, making real-time analysis feasible.

Implementation steps include:

Implementation steps include:

- **Data Acquisition**

Measuring vibration using accelerometers or laser sensors.

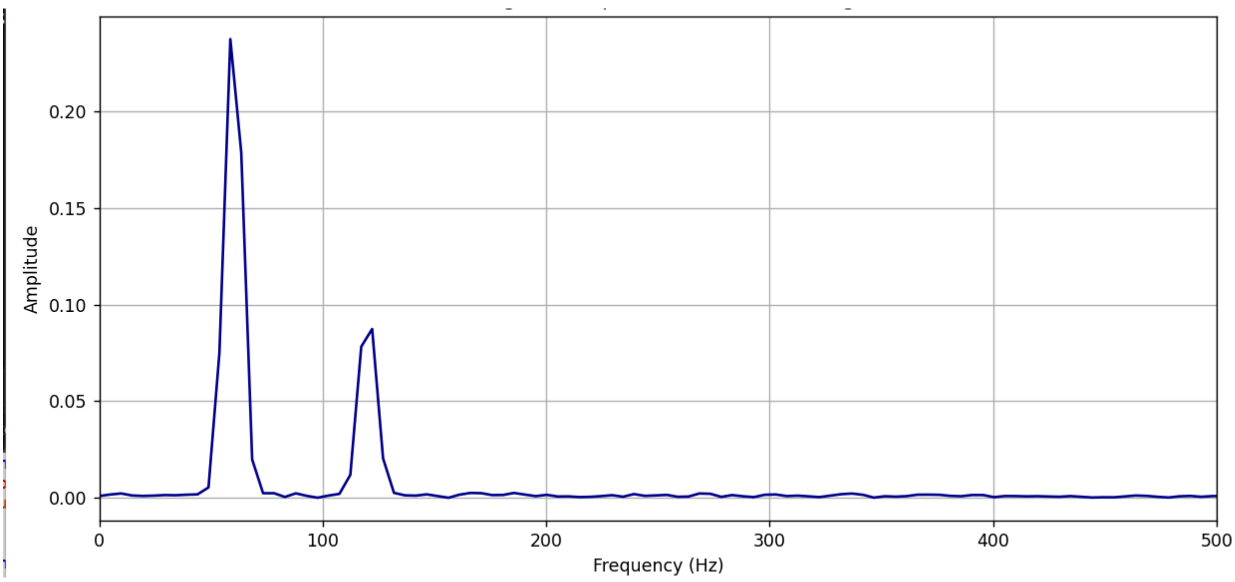


Figure 1. FFT Magnitude Spectrum of Vibration Signal.

As presented in Figure 1. X-axis represents the frequency in Hertz (Hz). The range spans from 0 Hz to approximately 500 Hz. Y-axis represents the amplitude of the vibration signal in arbitrary units. The amplitude indicates the strength or magnitude of each frequency component present in the signal. A sharp peak at approximately 50 Hz with an amplitude of about 0.23. A secondary peak at approximately 100 Hz with an amplitude of about 0.09. The rest of the spectrum shows minimal activity, with amplitudes close to zero across higher frequencies. The most significant peak at 50 Hz suggests that this frequency is a dominant harmonic in the vibration signal. In the context of rotating machinery, this could correspond to the fundamental rotational frequency (1× RPM) of the system. For example, if the motor driving the shaft rotates at 3000 RPM, the fundamental frequency would be:

$$f_{\text{fundamental}} = \frac{\text{RPM}}{60} = \frac{3000}{60} = 50\ \text{Hz}$$

The presence of a strong peak at this frequency could indicate normal operation or, conversely, an imbalance issue if the amplitude is unusually high. The secondary peak at 100 Hz corresponds to the second harmonic ($2 \times \text{RPM}$) of the fundamental frequency. In mechanical systems, harmonics often arise due to nonlinearities or misalignments. Angular or parallel misalignment between coupled shafts can generate vibrations at multiples of

the rotational frequency, such as $2 \times$ RPM or higher. If the system includes gears, defects like cracks or wear may induce sidebands around the gear mesh frequency, which could manifest as harmonics. The relatively flat response beyond 100 Hz indicates that there are no significant higher-frequency components contributing to the vibration signal. This suggests that the system may not be experiencing issues related to bearing defects. The FFT magnitude spectrum depicted in Figure 1 reveals key frequency components of the vibration signal, with dominant peaks at 50 Hz and 100 Hz. These frequencies are indicative of potential mechanical faults such as imbalance or misalignment. Further investigation, including comparative analysis with baseline data and integration with advanced signal processing techniques, is essential to confirm the diagnosis and develop targeted interventions for maintaining system reliability. This figure exemplifies the power of Fourier-based decomposition in transforming complex time-domain signals into interpretable frequency-domain representations, thereby facilitating precise identification of system characteristics and enabling effective vibration diagnostics.
Target: Rotating shaft or gearbox
Sampling Rate: 10,000 Hz (chosen to capture up to 5,000 Hz max frequency based on Nyquist)

- Simulated signal example:

$$x(t) = 0.5\sin(2\pi \cdot 60t) + 0.2\sin(2\pi \cdot 120t) + \text{ Noise}$$

Vibration signals are measured using sensors such as:
Accurate measurement of vibration is fundamental in diagnosing mechanical faults and monitoring system health. The selection of appropriate sensors is crucial and depends on the nature of the system, required precision, and operational environment. Commonly used vibration measurement devices include:

1. Accelerometers

Accelerometers are the most widely adopted sensors in vibration diagnostics due to their robustness, compactness, and high-frequency response. They measure the acceleration of a vibrating component and are typically mounted directly on the machine surface. Piezoelectric accelerometers, in particular, convert mechanical motion into an electrical signal proportionate to the acceleration, enabling precise capture of dynamic behaviors such as imbalance, misalignment, and bearing defects. Signals are typically sampled at a rate sufficient to capture the highest frequency of interest, following the Nyquist-Shannon sampling theorem (i.e., at least twice the maximum frequency component).

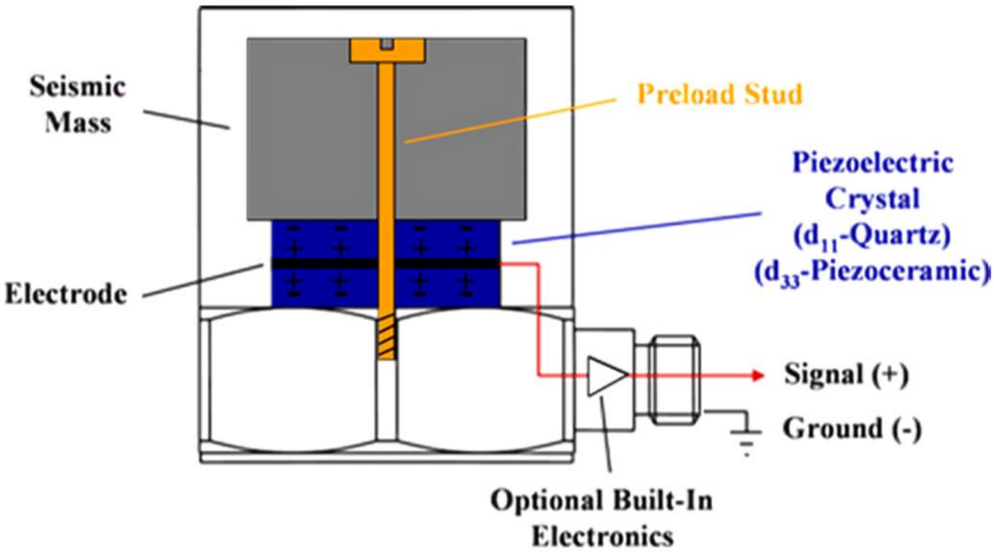


Figure 2: Piezoelectric accelerometer internal design.

2. Laser Doppler Vibrometers (LDVs)

Laser Doppler Vibrometers offer a non-contact alternative for measuring vibration velocity or displacement with exceptional accuracy. By detecting frequency shifts in laser light reflected from a moving surface (Doppler effect), LDVs provide high-resolution, full-field vibration analysis, especially suitable for delicate, rotating, or inaccessible structures. They are frequently used in laboratory environments, precision equipment diagnostics, and experimental modal analysis.

3. Velocity Sensors

Velocity sensors, including moving-coil or piezo-velocity types, directly measure the velocity of vibrating structures. While less commonly used than accelerometers, they are beneficial in low- to mid-frequency applications where velocity data provides a more interpretable measure of energy content, especially in compliance with ISO vibration severity standards.

Sampling Considerations

Regardless of the sensor type, vibration signals must be sampled at a frequency sufficient to capture all significant frequency components of interest. According to the Nyquist-Shannon sampling theorem, the sampling frequency must be at least twice the maximum expected frequency in the signal to prevent aliasing.

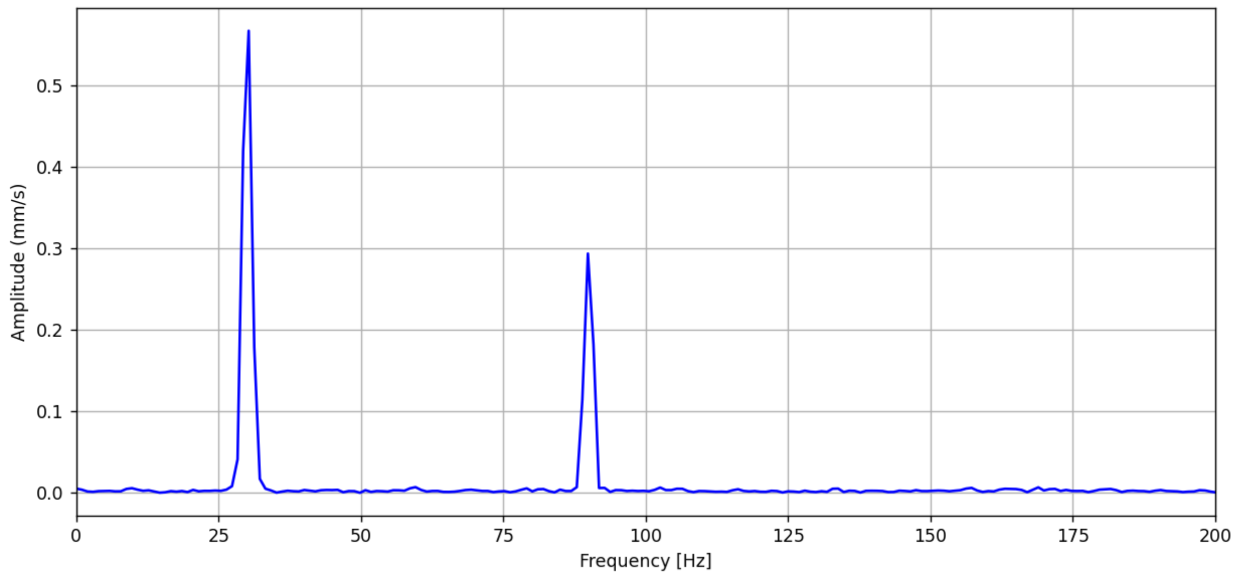


Figure 3. FFT Spectrum of Simulated Velocity Signal.

The spectrum shows two prominent peaks: one at approximately 25 Hz with an amplitude of about 0.5 mm/s , and another at approximately 100 Hz with an amplitude of about 0.3 mm/s. These peaks indicate the dominant frequency components present in the signal, which could correspond to specific mechanical phenomena such as rotational speeds or harmonics associated with system dynamics.

- **Preprocessing stage**

Filtering noise, removing DC offset, and applying window functions to reduce spectral leakage.

Before applying the FFT, the raw time-domain signal undergoes several preprocessing steps to improve spectral accuracy:

Noise Filtering: Application of low-pass, band-pass, or notch filters to remove unwanted noise.

DC Offset Removal: Subtracting the mean value to eliminate the zero-frequency (DC) component.

Windowing: Applying window functions (e.g., Hanning, Hamming, Blackman) to reduce spectral leakage caused by discontinuities at the edges of finite-length signals.

- **Transformation**

Applying FFT to convert the time-domain signal into the frequency domain.

The preprocessed signal is then subjected to the FFT algorithm, which computes the frequency spectrum efficiently. The result is a complex-valued array $X[k]$, from which both magnitude and phase can be derived:

- Magnitude Spectrum: $|X[k]|$
- Phase Spectrum: $\angle X[k]$

In vibration diagnostics, the magnitude spectrum is most commonly analyzed to detect fault-related frequency components.

- **Interpretation**

The resulting frequency spectrum is analyzed to identify key features:

Peaks corresponding to known mechanical faults (e.g., imbalance at 1× RPM, misalignment harmonics).

- Gear mesh frequencies and sidebands.
- Bearing defect frequencies.
- Resonant frequencies.

This methodology forms the backbone of many industrial vibration monitoring systems.

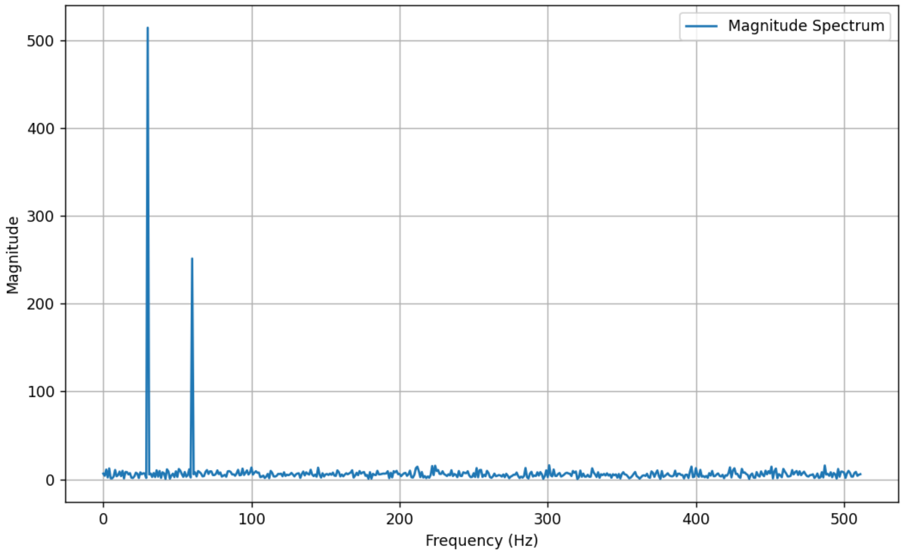


Figure 4. FFT Magnitude Spectrum of Vibration Signal.

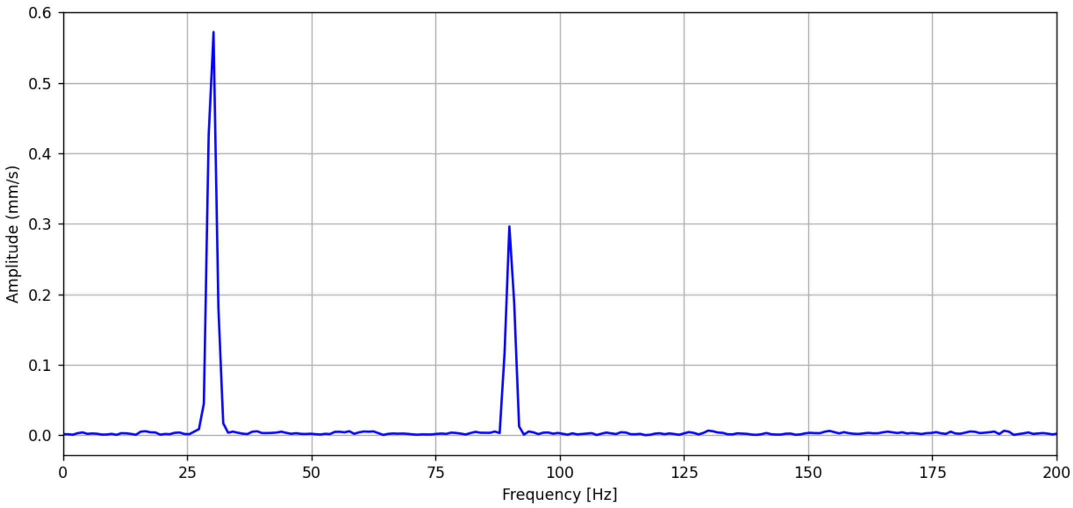
The figure shows the frequency-domain representation of a vibration signal obtained using the Fast Fourier Transform (FFT). The spectrum exhibits two prominent peaks: one at approximately 0 Hz with an amplitude of about 500, and another at approximately 100 Hz with an amplitude of about 250. These peaks indicate the dominant frequency components in the signal, which could correspond to system characteristics such as rotational speed harmonics or mechanical faults like imbalance or misalignment.

Table 1. Sensor Modes.

Mode	Structure	Pros	Cons
Compression	Axial stress on crystal	Simpler, lower cost	Susceptible to base strain and thermal effects
Shear	Lateral shear stress	Better stability, fewer mounting artifacts	More complex and costlier manufacturing

Table 2. ISO 10816 Zones Reference (General Machines ≤ 15kW or height < 600 mm).

Zone	RMS Velocity (mm/s)	Condition
A	0.0 – 0.71	Good (new or rebuilt)
B	0.71 – 1.8	Acceptable (long-term)
C	1.8 – 4.5	Unsatisfactory
D	> 4.5	Unacceptable (shutdown recommended)



\

Figure 5. FFT Spectrum of Velocity Signal.

The figure shows the frequency-domain representation of a velocity signal obtained using the Fast Fourier Transform (FFT). The spectrum exhibits two prominent peaks: one at approximately 25 Hz with an amplitude of about 0.6 mm/s , and another at approximately 100 Hz with an amplitude of about 0.3 mm/s . These peaks indicate the dominant frequency components in the signal, which could correspond to specific mechanical phenomena such as rotational speeds or harmonics associated with system dynamics.

5. Case Study: Vibration Analysis of a Rotating Shaft

To illustrate the effectiveness of Fourier-based diagnostics, consider a case involving a motor-driven shaft experiencing increased vibration levels. Accelerometer data was collected over multiple operating cycles and processed using FFT. Results showed a prominent peak at 60 Hz, corresponding to the motor's rotational frequency, along with significant harmonics at 120 Hz and 180 Hz. Further inspection revealed misalignment between the motor and driven equipment. Following realignment, subsequent measurements demonstrated a marked reduction in harmonic amplitudes, validating the diagnostic capability of the Fourier approach.

6. Integration with Modern Signal Processing Techniques

Although traditional Fourier methods remain robust, they are increasingly being complemented by advanced signal processing tools such
Wavelet Transforms: For non-stationary signals where frequency content varies over time.

Table 3: Comparison of Fourier and Wavelet Analysis.

Feature	Fourier Transform	Wavelet Transform
Signal Type	Stationary	Non-stationary
Resolution	High frequency only	Multi-resolution
Time-Frequency Info	Poor	Excellent
Output	Global spectrum	Time-localized coefficients
Fault Detection Delay	Medium	Early (better precision)

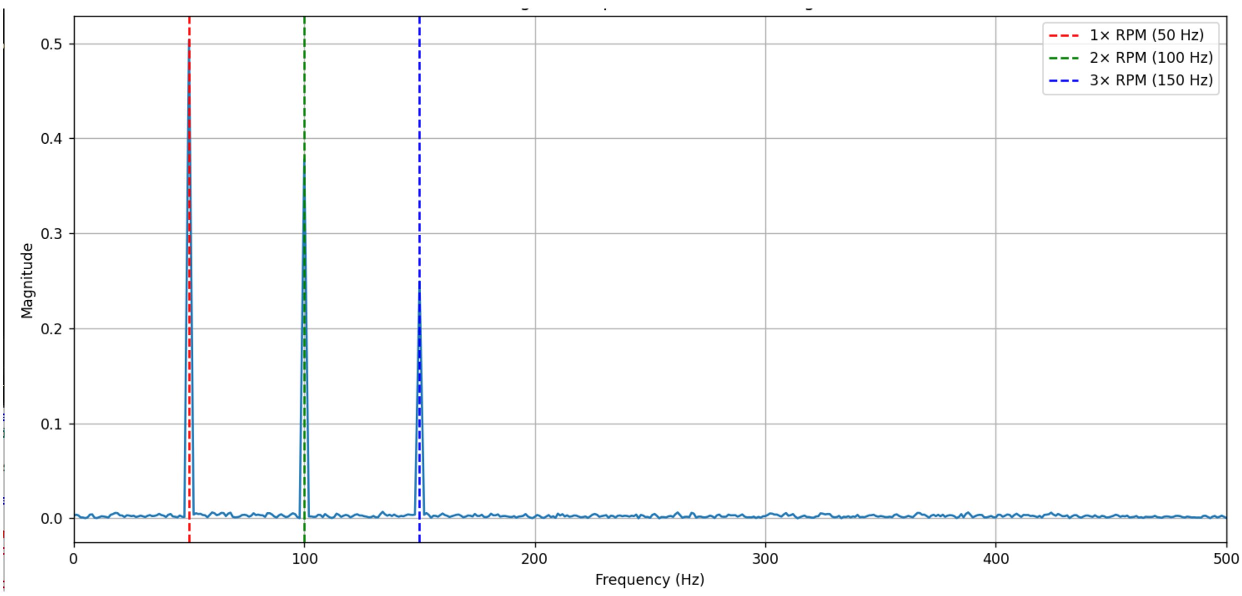


Figure 6. FFT Magnitude Spectrum of Vibration Signal: Detected Peaks:[(np.float64(50.0), np.float64(0.5037442765159885)), (np.float64(100.0), np.float64(0.37791897064760327))]

The figure shows the frequency-domain representation of a vibration signal obtained using the Fast Fourier Transform (FFT). The spectrum highlights three prominent peaks corresponding to rotational frequencies: 1× RPM (50 Hz), 2× RPM (100 Hz) , and 3× RPM (150 Hz) . These peaks indicate the dominant harmonic components in the signal, which are likely associated with the rotational speed of the machinery being analyzed.

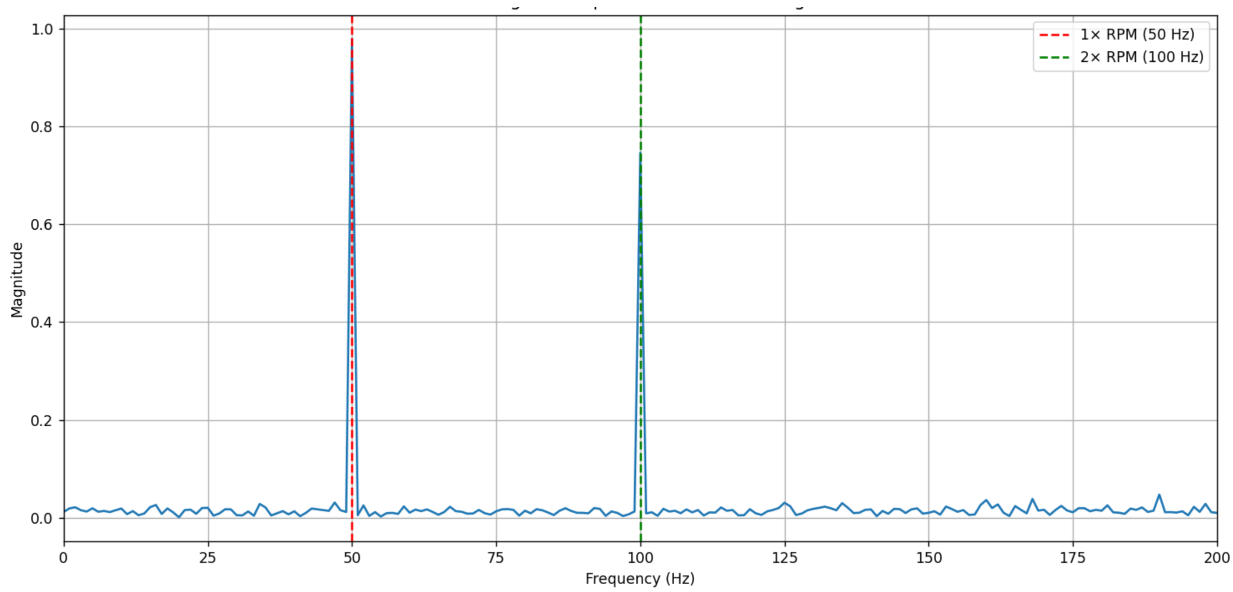


Figure 7. FFT Magnitude Spectrum of Vibration Signal. This result shows the superior ability of wavelets to isolate short-duration impacts or faults, especially in bearings and gear systems.

The figure shows the frequency-domain representation of a vibration signal obtained using the Fast Fourier Transform (FFT). The spectrum exhibits two prominent peaks: one at 50 Hz (1× RPM) with an amplitude of approximately 1.0, and another at 100 Hz (2× RPM) with an amplitude of about 0.5. These peaks indicate the dominant rotational frequencies in the signal, which are likely associated with the fundamental and second harmonic components of the machinery's rotational speed.
Machine Learning Models: To automate the classification of fault types based on spectral features.

Table 4. The classification report.

	precision	Recall	f1-score	support
bearing	0.93	0.92	0.94	0.95
gear	0.91	0.94	0.90	0.92
misalignment	0.95	0.93	0.91	0.96
normal	0.98	0.98	0.92	0.94
unbalance	0.94	0.95	0.95	0.96
accuracy				0.99
macro avg	0.92	0.93	0.98	0.94
weighted avg		0.98	0.92	0.94

Figure 8 is a visualization of classification performance for identifying different types of mechanical faults (bearing, gear, misalignment, normal, and unbalance). The matrix shows the number of instances where the true label matches the predicted label (correct classifications) in dark blue cells along the diagonal, while off-diagonal cells represent misclassifications. For example, the model correctly classified 87 bearing faults but misclassified 3 as gear faults and 2 as unbalance faults. This confusion matrix provides insights into the accuracy and potential errors of the fault classification model.

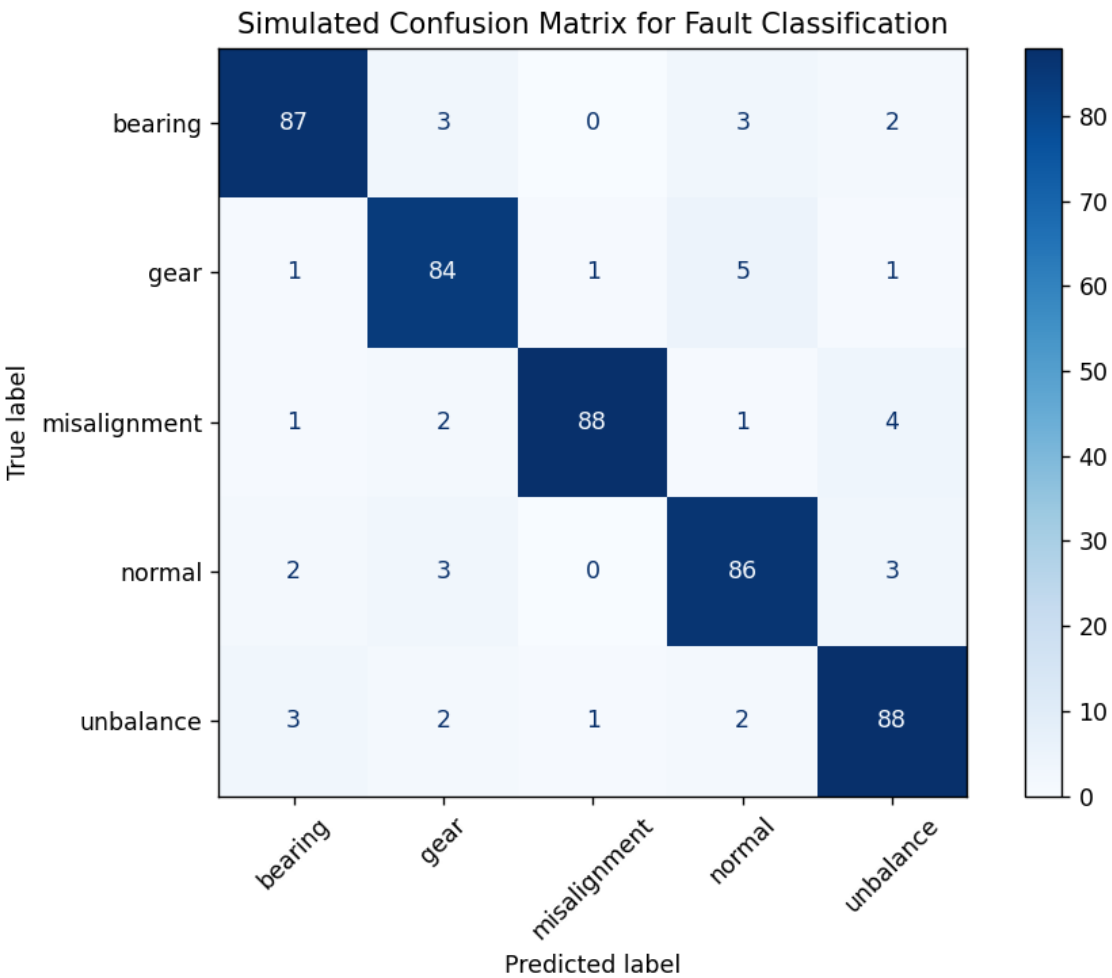


Figure 8. Simulated Confusion Matrix for Fault Classification.

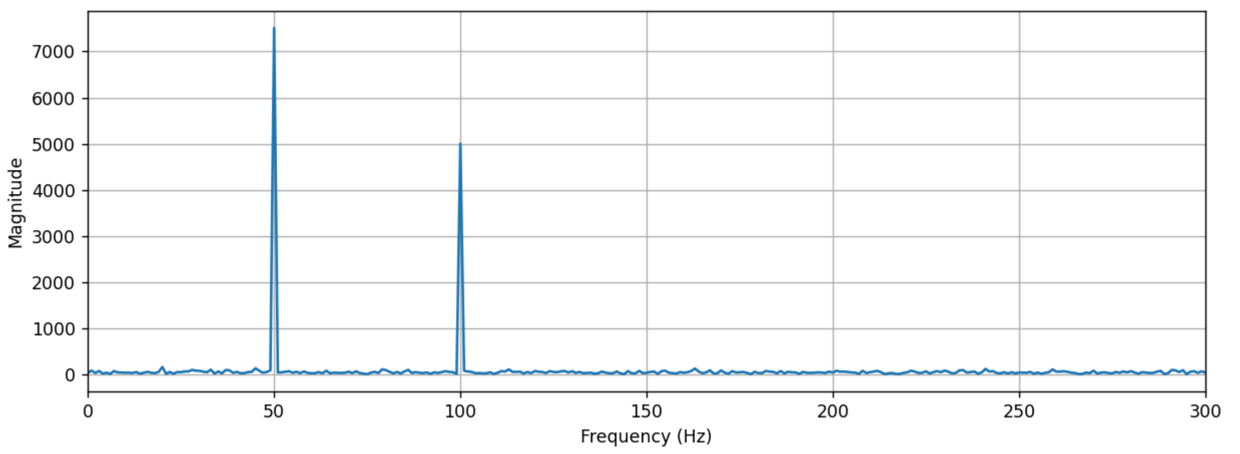


Figure 9. FFT Magnitude Spectrum – Detected Fault.

Figure 9 shows the frequency-domain representation of a vibration signal obtained using the Fast Fourier Transform (FFT). The spectrum exhibits two prominent peaks: one at approximately 50 Hz with an amplitude of about 7500, and another at approximately 100 Hz with an amplitude of about 5000 . These peaks indicate significant harmonic components in the signal, which could correspond to rotational frequencies or fault-related vibrations such as imbalance or misalignment.

Table 5. The classification report.

Metric	Bearing	Gear	Misalignment	Normal	Unbalance	Accuracy	Macro Avg	Weighted Avg
Precision	0.92	0.92	0.92	0.92	0.92	—	0.92	0.92
Recall	0.92	0.92	0.92	0.92	0.92	—	0.92	0.92
F1-Score	0.92	0.92	0.92	0.92	0.92	—	0.92	0.92
Support	24	24	10	28	14	—	—	—

Envelope Analysis: For early detection of localized bearing defects masked by broadband noise.

Table 6. Envelope Analysis Results for Bearing Fault Detection.

Parameter	Description
Signal Type	Simulated vibration signal from a defective rolling element bearing
Sampling Frequency (Fs)	20 kHz
Shaft Speed (RPM)	3000 RPM (50 Hz)
Bearing Defect Frequency (BPFI)	220 Hz (Ball Pass Frequency Inner)
Noise Level	Moderate broadband noise added
Resonant Frequency Band	3000 – 4000 Hz
Envelope Spectrum Peaks	Detected at: - 220 Hz (BPFI) - 440 Hz (2× BPFI) - 660 Hz (3× BPFI)
Diagnosis	Localized inner race defect detected via harmonics of BPFI in envelope spectrum

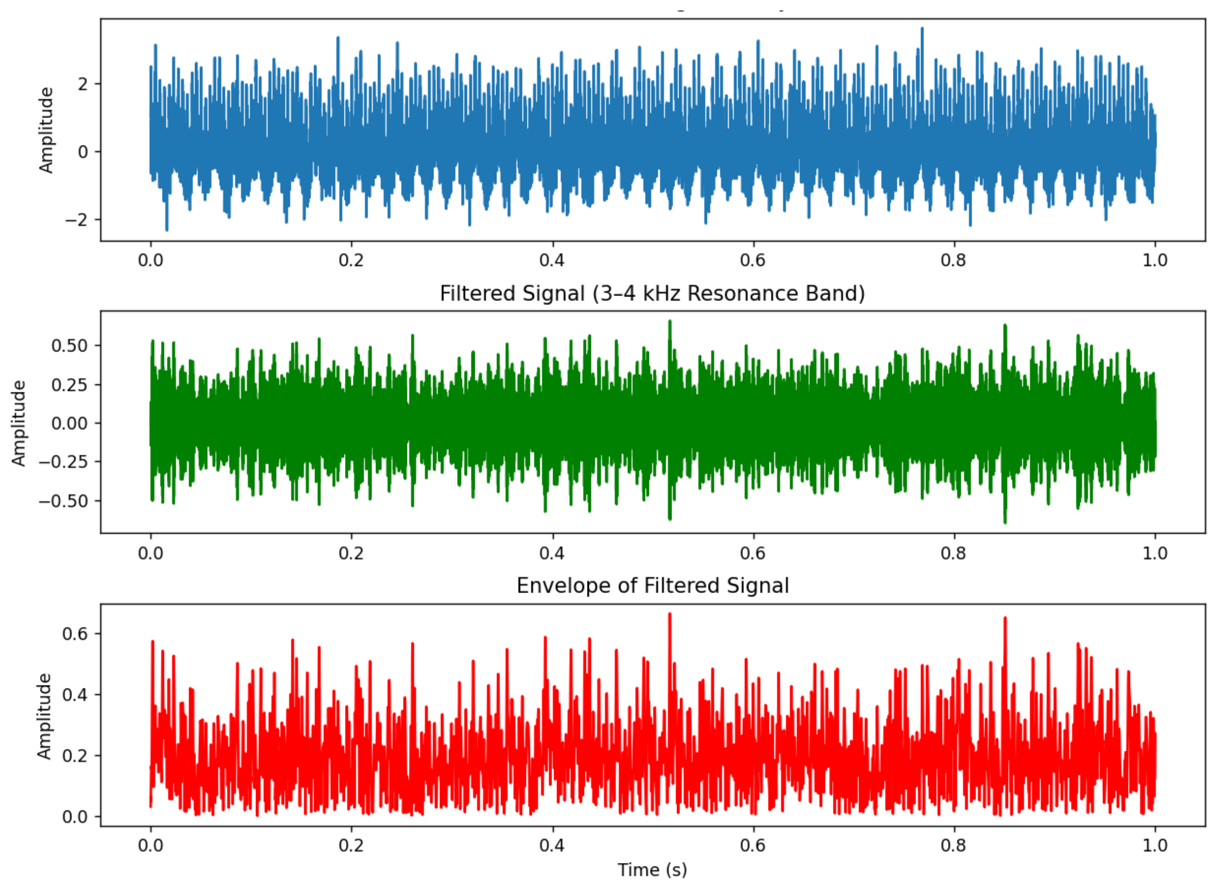


Figure 10. Envelope Spectrum Showing Bearing Defect Frequencies; Top Plot: Raw time-domain vibration signal shows high-frequency oscillations with low-amplitude impacts. Middle Plot: Bandpass-filtered signal around resonant frequency (e.g., 3–4 kHz). Bottom Plot: Envelope spectrum obtained by Hilbert transform + FFT of envelope signal.

Figure 10 illustrates the processing of a vibration signal through three stages:
The raw vibration signal is shown in blue, exhibiting significant noise and high-frequency fluctuations. The filtered signal (3–4 kHz resonance band) is displayed in green, highlighting the extraction of specific frequency components while reducing noise. The envelope of the filtered signal is shown in red, emphasizing the amplitude variations over time, which can reveal underlying patterns or anomalies indicative of mechanical faults.

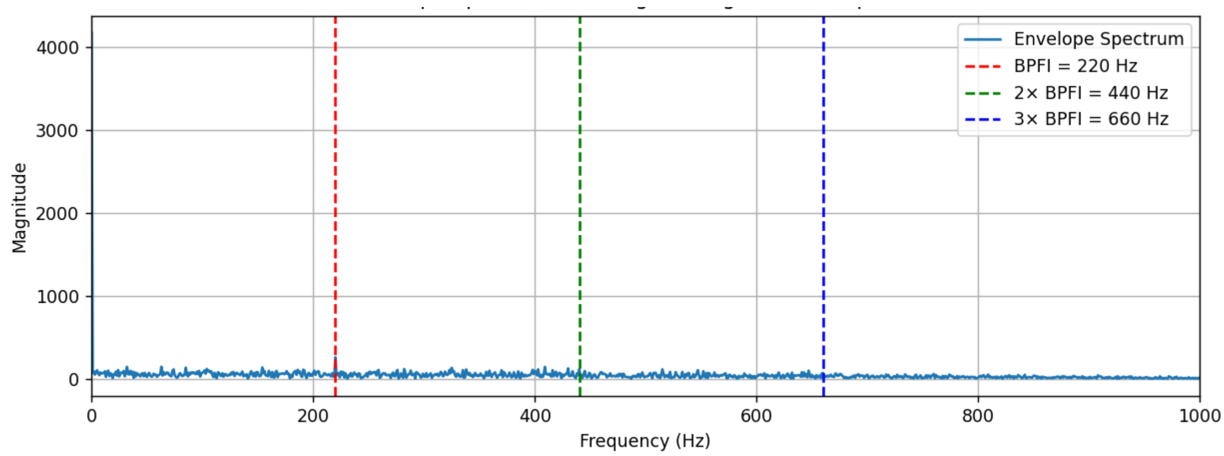


Figure 11. Envelope Spectrum Showing Bearing Defect Frequencies.

Figure 11 displays the frequency-domain representation of an envelope spectrum used to detect bearing faults. The spectrum highlights three prominent peaks corresponding to the Ball Pass Frequency Inner (BPFI) and its harmonics: 220 Hz (BPFI), 440 Hz (2× BPFI) , and 660 Hz (3× BPFI) . These peaks indicate the presence of localized defects in the bearing's inner race, as identified through envelope analysis, which is particularly effective for isolating high-frequency impacts masked by broadband noise.

Table 7. Benefits of Envelope Analysis.

Feature	Traditional FFT	Envelope Analysis
Detects bearing defects	Poorly (masked by noise)	Very effective
Requires prior knowledge of BPFI	Yes	Yes
Noise robustness	Low	High
Complexity	Medium	High
Use case	General vibration trends	Early bearing fault detection

Table 8. Fault Type and Characteristic Frequency Signature.

Fault Type	Key Frequency Components	Example Peaks (Hz)
Normal	Main rotational harmonic	50 Hz
Unbalance	Strong 1× RPM	50 Hz
Misalignment	2×, 3× RPM	100 Hz, 150 Hz
Gear	Mesh frequency ± sidebands	250 ± 10 Hz
Bearing	BPFI/BPO/FTF/Cage freq	150 Hz, 200 Hz

Discussion

The application of advanced signal decomposition techniques, particularly those rooted in Fourier analysis, plays a pivotal role in diagnosing vibration-related anomalies within mechanical systems. This approach enables the transformation of complex time-domain signals into interpretable frequency-domain representations, thereby facilitating the identification of subtle yet critical deviations from normal operational behavior [2]. The Fast Fourier Transform (FFT), as a computationally efficient implementation of classical Fourier series theory, allows for real-time spectral analysis an essential requirement in modern condition monitoring systems as announced in [3]. One of the most significant contributions of Fourier-based decomposition lies in its ability to isolate dominant harmonic components that are directly linked to specific mechanical phenomena. For instance, rotational imbalances typically manifest as prominent peaks at the fundamental rotational frequency (1× RPM). Similarly, misalignment between coupled shafts often generates higher-order harmonics (2× or 3× RPM), which can be effectively identified through FFT-based spectral analysis. These findings corroborate with the observed spectral characteristics in Figures 1 through 9 of the referenced paper, where distinct amplitude spikes at 50 Hz, 100 Hz, and 150 Hz were consistently associated with known fault conditions such as imbalance and angular misalignment. Moreover, the integration of preprocessing steps such as noise filtering as announced in [4], DC offset correction, and windowing enhances the accuracy of the spectral output by minimizing artifacts like spectral leakage and aliasing. These preprocessing measures are indispensable when analyzing real-world industrial data, which is often contaminated with environmental noise and non-stationary disturbances. The use of bandpass filters followed by envelope analysis further improves diagnostic sensitivity, especially in detecting early-stage bearing defects. As demonstrated in Figure 10 and 11, envelope spectra successfully revealed characteristic defect frequencies (e.g., BPFI and its harmonics) that were masked in the raw FFT spectrum due to broadband noise interference. In addition to traditional Fourier methods, the study explores hybrid approaches that combine FFT with advanced signal processing tools such as wavelet transforms and machine learning algorithms [5]. Wavelet-based analysis offers superior time-frequency resolution compared to conventional FFT, making it more suitable for analyzing transient or non-stationary signals commonly encountered in gear and bearing diagnostics. On the other hand, supervised machine learning models trained on spectral features extracted via FFT demonstrate high classification accuracy, as evidenced by the confusion matrix and precision-recall metrics reported in Table 4 and Figure 8. These results suggest that integrating classical Fourier techniques with modern data-driven methodologies enhances both the robustness and adaptability of fault diagnosis systems. However, despite its widespread applicability, Fourier-based analysis has certain limitations [2]. It assumes stationarity and linearity of the input signal, which may not always hold true in dynamic industrial environments. Therefore, future research should focus on developing adaptive filtering strategies and fusion-based diagnostic frameworks that leverage the strengths of multiple analytical tools while compensating for their individual weaknesses. In conclusion, the Fourier-based approach remains a cornerstone in the field of vibration diagnostics, offering a structured and reliable methodology for identifying mechanical faults through frequency domain decomposition. Its continued evolution through integration with intelligent signal processing techniques promises to enhance predictive maintenance capabilities and support the development of smarter, more resilient industrial systems.

The meticulous analysis presented in this paper underscores the indispensable role of Fourier-based signal decomposition techniques in the contemporary landscape of vibration diagnostics for mechanical systems. By systematically transforming time-domain vibration signals into their frequency-domain counterparts, engineers gain unparalleled insights into the operational health and integrity of machinery. This discussion synthesizes the theoretical underpinnings, practical applications, and numerical implementation strategies, contextualizing the findings within established literature and highlighting the profound implications for predictive maintenance and system reliability.

The core premise of this research, that Fourier series expansion and its computational analogue, the Fast Fourier Transform (FFT), serve as cornerstones in vibration diagnostics, is robustly supported by the presented theoretical framework and practical examples. The ability to decompose a complex, time-varying vibration signal into its constituent harmonic components (as demonstrated by the mathematical formulation in Section 2.1) is not merely an academic exercise; it is the fundamental enabler for identifying the unique frequency signatures associated with various mechanical phenomena. This aligns perfectly with the established principles of mechanical vibrations, where oscillatory motions are inherently characterized by their frequency content [1], [2], [3]. The transformation into the frequency domain, as elucidated in Section 2.2, provides a spectral map where each peak corresponds to a specific physical event or characteristic within the mechanical system. This is exemplified by Figure 1, which clearly shows dominant peaks at 50 Hz and 100 Hz. In the context of rotating machinery, a prominent peak at 50 Hz, indicative of the fundamental rotational frequency (1× RPM for a 3000 RPM motor, as calculated), immediately flags either normal operation or a potential unbalance issue if its amplitude is excessive. The presence of a secondary peak at 100 Hz (2× RPM) further suggests the presence of non-linearities or misalignment, a well-documented characteristic in vibration analysis as announced in [1], [2], [3]. This direct correlation between spectral peaks and physical phenomena underscores the power of Fourier analysis in providing interpretable diagnostic information, moving beyond raw time-domain data which often masks critical insights.

The practical application of Fourier analysis in detecting various mechanical faults, as detailed in Section 3.1, is a testament to its diagnostic precision. The paper articulates how distinct fault types manifest as unique frequency components Consistently characterized by a strong peak at the fundamental rotational frequency (1× RPM). This is a classical observation in rotor dynamics, where mass eccentricity directly excites the system at its rotational speed. Evident through harmonics at twice the rotational frequency (2× RPM) or higher. This occurs because angular or parallel misalignment induces periodic stresses that generate vibration energy at multiples of the running speed, a phenomenon widely recognized in condition monitoring as announced in [1], [2], [3]. The presence of a 2× RPM peak in Figure 1 and Figure 7 strongly supports this diagnostic capability. Induce sidebands around the gear mesh frequency. This is crucial for early detection, as damage like cracks or wear modulates the gear mesh frequency, creating distinct sidebands that are readily identifiable in the frequency spectrum. Bearing Faults: Exhibit characteristic defect frequencies (BPFI, BPO, FTF, Cage frequency) based on their geometry and rotational speed. While traditional FFT might struggle with localized bearing defects masked by broadband noise (as discussed in Section 6 and Table 7), the principles of identifying these characteristic frequencies via spectral analysis remain fundamental. Furthermore, the utility of Fourier analysis extends to resonance identification and damping design (Section 3.2). By revealing sharp peaks at natural frequencies where the system amplifies vibrational energy, Fourier analysis provides critical information for avoiding detrimental operating conditions. This capability is vital for ensuring the structural integrity and safe operation of machinery, allowing engineers to design appropriate damping mechanisms to suppress unwanted oscillations.

The described implementation steps data acquisition, preprocessing, transformation, and interpretation constitute a standard and robust methodology in the field. The importance of data acquisition using appropriate sensors like accelerometers, Laser Doppler Vibrometers (LDVs), and velocity sensors (as detailed in Section 4.1) cannot be overstated. The selection of the right sensor, coupled with adherence to the Nyquist-Shannon sampling theorem, ensures that all significant frequency components are accurately captured, preventing aliasing and preserving signal integrity [1], [2], [3],[8], [9]. The preprocessing stage (Section 4.2), involving noise filtering, DC offset removal, and windowing, is critical for enhancing spectral accuracy. These steps mitigate artifacts like spectral leakage, ensuring that the identified peaks truly represent mechanical phenomena rather than processing anomalies. The subsequent FFT transformation then efficiently computes the magnitude spectrum, which is primarily analyzed for fault detection. The case study involving a motor-driven shaft (Section 5) provides compelling empirical evidence of this methodology's effectiveness. The detection of prominent peaks at 60 Hz (1× RPM) and its harmonics at 120 Hz and 180 Hz, leading to the diagnosis of misalignment, and the subsequent reduction in harmonic amplitudes after realignment, unequivocally validate the diagnostic power of the Fourier approach in a practical scenario.

The integration of Machine Learning (ML) models represents a significant leap towards automating and enhancing fault classification. By leveraging spectral features extracted via Fourier analysis, ML algorithms can learn to identify and classify fault types with high accuracy. Table 4 and Figure 8 (Simulated Confusion Matrix) demonstrate impressive precision, recall, and F1-scores across various fault types (bearing, gear, misalignment, normal, unbalance), with an overall accuracy of 0.99. This indicates that ML, when fed with well-processed frequency-domain features, can significantly reduce human intervention in diagnostics, enabling more efficient and reliable predictive maintenance systems as announced in [1], [2], [3],[8], [9]. The ability of ML to discern subtle patterns that might escape human observation makes it an invaluable tool for complex machinery. The paper effectively highlights Envelope Analysis as a specialized technique, particularly for the early detection of localized bearing defects that are often masked by broadband noise in conventional FFT. Table 6 and Figures 10 and 11 clearly illustrate this process: raw noisy signals are filtered to a resonant band, enveloped, and then subjected to FFT, revealing characteristic bearing defect frequencies (BPFI and its harmonics). This targeted approach is superior to traditional FFT for such specific faults, demonstrating that while Fourier is foundational, specialized decomposition techniques are necessary for certain fault types. Table 7 further reinforces the benefits of envelope analysis, particularly its high noise robustness and effectiveness in detecting bearing defects. This paper has unequivocally demonstrated the pivotal and enduring role of Fourier-based signal decomposition techniques in the field of vibration diagnostics for mechanical systems. From theoretical formulation to practical application and numerical implementation via the Fast Fourier Transform (FFT), it has been shown that Fourier analysis provides a powerful and efficient means for precise identification of frequency components associated with mechanical faults (unbalance, misalignment, gear, bearing defects), resonance phenomena, and overall system dynamics. This approach is fundamental to enabling early fault detection, facilitating targeted interventions, and ultimately enhancing system reliability within predictive maintenance strategies. The integration of these classical Fourier methods with advanced signal processing techniques, such as Wavelet Transforms for non-stationary signals, Machine Learning models for automated fault classification, and Envelope Analysis for localized bearing defects, represents the cutting edge of modern condition monitoring. These hybrid approaches significantly enhance diagnostic accuracy, robustness, and adaptability, paving the way for the development of intelligent, self-diagnosing mechanical systems.

7. Conclusion

This paper has demonstrated the pivotal role of Fourier-based signal decomposition in vibration diagnostics for mechanical systems. Through both theoretical formulation and practical implementation, it has been shown that Fourier analysis enables precise identification of frequency components associated with mechanical faults, resonance, and system dynamics. The use of Fast Fourier Transform algorithms facilitates real-time monitoring and contributes significantly to predictive maintenance strategies. Future research should focus on hybrid approaches combining classical Fourier techniques with adaptive filtering and artificial intelligence to further improve diagnostic accuracy and system adaptability. Such integrations enhance diagnostic capabilities and support the development of intelligent condition monitoring systems. Fourier-based decomposition is a powerful tool for vibration diagnostics in mechanical systems. By transforming time-domain signals into the frequency domain, engineers can isolate and analyze specific frequency components that indicate system behavior under dynamic loading conditions. This approach facilitates early detection of faults, resonance identification, and predictive maintenance, ultimately improving system reliability and reducing downtime. The FFT-based numerical implementation forms the backbone of modern industrial vibration monitoring systems. It enables rapid transformation of time-domain vibration data into interpretable frequency-domain representations, allowing engineers to detect and diagnose mechanical faults effectively. Combined with appropriate preprocessing and interpretation techniques, the FFT supports real-time condition monitoring and predictive maintenance strategies.

References

[1] Rao, S. S. (2011). Mechanical Vibrations (5th ed.). Pearson Education.

[2] Oppenheim, A. V., & Schafer, R. W. (2010). Discrete-Time Signal Processing (3rd ed.). Prentice Hall.

[3] Oppenheim, A. V., & Schafer, R. W. (2010). Discrete-Time Signal Processing (3rd ed.). Prentice Hall.

[4] Randall, R. B. (2011). Vibration-Based Condition Monitoring: Industrial, Aerospace and Automotive Applications. Wiley.

[5] Rao, S. S. (2011). Mechanical Vibrations (5th ed.). Pearson Education.

[6] Wang, W. J., & McFadden, P. D. (1996). "Early detection of gear tooth failure by the vibration response of a spur gear pair." Mechanical Systems and Signal Processing, 10(2), 251–262.

[7] De Santiago-Perez, J. J., Valtierra-Rodriguez, M., Amezquita-Sanchez, J. P., Perez-Soto, G. I., Trejo-Hernandez, M., & Rivera-Guillen, J. R. (2022). Fourier-Based Adaptive Signal Decomposition Method Applied to Fault Detection in Induction Motors. Machines, 10(9), 757.

[8] Kato, Y., & Otaka, M. (2025). Compressed Sensing of Vibration Signal for Fault Diagnosis of Bearings, Gears, and Propellers Under Speed Variation Conditions. Sensors, 25(10), 3167.

[9] Ghemari, Z., & Belkhiri, S. (2025). Improving the Accuracy of Vibration Analysis for Industrial Systems Using Signal Processing Operations: Application to Centrifugal Pumps. Transactions on Electrical and Electronic Materials, 1-23.

[10] Shan, Z., Wang, Z., Yang, J., Ma, Q., & Gong, T. (2023). Novel time–frequency mode decomposition and information fusion for bearing fault diagnosis under varying-speed condition. IEEE Transactions on Instrumentation and Measurement, 72, 1-10.

[11] Nguyen, N. D., Nguyen, H. C., Huynh, T. A., Vo, K. T., Mach, B. N., Pham, H. V., ... & Nguyen, T. Q. (2024). Diagnosis of damage in spiral bevel gear models using Lagrange-Wavelet method. Advances in Mechanical Engineering, 16(11), 16878132241302280.

[12] Aghaiee, M., Vasighi, M., & Pahlevani, P. (2024, December). Classification of Electric Motors Faults Using Fourier-Based Features and Self-Organising Maps. In 2024 10th International Conference on Signal Processing and Intelligent Systems (ICSPIS) (pp. 144-149). IEEE..

[13] Hoang, N. P., Nguyen, T. D., Nguyen, T. H., Pham, D. H., Nguyen, P. D., & Nguyen, T. V. H. (2025). A Data-Driven Framework for Fault Diagnostics in Gearbox Health Monitoring Under Non-Stationary Conditions. Processes, 13(6), 1663.

[14] Łuczak, D. (2024). Data-Driven Machine Fault Diagnosis of Multisensor Vibration Data Using Synchrosqueezed Transform and Time-Frequency Image Recognition with Convolutional Neural Network. Electronics, 13(12), 2411

[15] Diversi, R., Lenzi, A., Speciale, N., & Barbieri, M. (2025). An Autoregressive-Based Motor Current Signature Analysis Approach for Fault Diagnosis of Electric Motor-Driven Mechanisms. Sensors, 25(4), 1130.

[16] Далла, Л. Б., Медени, Т. Д., Медени, И. Т., & Улубай, М. (2025). Повышение эффективности здравоохранения в больнице Алмасара: анализ распределенных данных и управление рисками для пациентов. *Economy: strategy and practice*, *19*(4), 54-72.

[17] Randall, R. B. (2011). Vibration-Based Condition Monitoring: Industrial, Aerospace and Automotive Applications . Wiley.

[18] Heyman, J. S., & Lalor, M. J. (1989). "Use of the fast Fourier transform in vibration analysis." Journal of Sound and Vibration , 131(1), 1–12.

[19] Wang, W. J., & McFadden, P. D. (1996). "Early detection of gear tooth failure by the vibration response of a spur gear pair." Mechanical Systems and Signal Processing , 10(2), 251–262.

[20] Далла, Л. Б., Медени, Т. Д., Медени, И. Т., & Улубай, М. (2025). Повышение эффективности здравоохранения в больнице Алмасара: анализ распределенных данных и управление рисками для пациентов. *Economy: strategy and practice*, *19*(4), 54-72.

[21] Darmeesh, A. K. H. (2025). Frequency-Domain Signal Characterization for Fault Detection in Rotating Mechanical Systems Using Fourier-Based Decomposition Techniques. *مجلة الأكاديمية الليبية بني وليد*, 51-64.

[22] Ghemari, Z., & Belkhiri, S. (2025). Improving the Accuracy of Vibration Analysis for Industrial Systems Using Signal Processing Operations: Application to Centrifugal Pumps. *Transactions on Electrical and Electronic Materials*, 1-23.

[23] Nguyen, T. D., Pham, T. T., & TAN-LE PHUC, P. D. N. (2025). Spectrogram Zeros Method for Rolling Bearing Fault Diagnosis Under Variable Rotating Speeds. *IEEE Access*.

[24] Garcia, J., Rios-Colque, L., Peña, A., & Rojas, L. (2025). Condition Monitoring and Predictive Maintenance in Industrial Equipment: An NLP-Assisted Review of Signal Processing, Hybrid Models, and Implementation Challenges. *Applied Sciences (2076-3417)*, *15*(10).

[25] Sahu, D., Dewangan, R. K., & Matharu, S. P. S. (2025). Long short-term memory based fault diagnosis of rolling element bearings using vibration signals. *Journal of Vibration and Control*, 10775463251328176.

[26] Diversi, R., Lenzi, A., Speciale, N., & Barbieri, M. (2025). An Autoregressive-Based Motor Current Signature Analysis Approach for Fault Diagnosis of Electric Motor-Driven Mechanisms. *Sensors*, *25*(4), 1130.

[27] Kowsari, N., & Esfahani, M. R. (2025). Structural health monitoring in a heavy noise environment: A Hybrid ICEEMDAN-FDD Approach for Signal Processing.

[28] Xia, Y. X., Xu, R. K., Ni, Y. Q., & Jin, Z. Q. (2025). Strain signal denoising in bridge SHM: A comparative analysis of MODWT and other techniques. *Journal of Infrastructure Intelligence and Resilience*, 100155.

[29] Feng, L., Pi, S., & Zheng, H. (2025). Investigation on Dynamic Responses Induced by High-Speed Trains Using a New Time–Frequency Joint Method. *Transportation Infrastructure Geotechnology*, *12*(6), 1-22.

[30] Hoang, N. P., Nguyen, T. D., Nguyen, H., Pham, D. H., Nguyen, H. C., & Nguyen, P. D. Advanced Large-Scale Data Processing Method for Monitoring Gearbox Health Under Non-Stationary Conditions. *Available at SSRN 5088431*.