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Solving EM Fields Equations using Finite Difference Method for a 5G Microstrip Patch Antenna Operating at 28 GHz

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حل معادلات المجالات الكهرومغناطيسية باستخدام طريقة الفروق المنتهية لهوائي ميكروشريطي لتطبيقات الجيل الخامس يشتغل عند 28 جيجا هرتز

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Abstract:
This study aims to analyze and solve the electromagnetic field equations for a rectangular microstrip patch antenna operating at 28 GHz, designed for 5G applications, using both analytical and numerical approached. The study begins with precise mathematical modeling based on Maxwell's equations to derive a representative equation, applying appropriate boundary conditions for perfect conductors. The analytical solution is obtained using separation of variables, providing a detailed description of the electric and magnetic field distributions within the cavity. On the numerical side, the finite difference method (FDM) is employed to solve the mathematical model of the rectangular microstrip patch antenna, with the Gauss-Seidel method applied for iterative solution. The electric field component E_z is computed using a second-order central difference approximation, but the magnetic field component H_x is calculated using a first-order central difference approximation. The numerical results show good agreement with the analytical solution, confirming the accuracy of the implemented algorithm. Furthermore, the study of the effect of segment numbers on accuracy reveals that increasing the number of segments significantly reduces the error. Additionally, the numerical properties are analyzed, including truncation error, accuracy, consistency, convergence, and stability. The analyses indicate that the numerical solution achieves second-order accuracy $O(\Delta h^2)$, is fully consistent, stable, and converges to the analytical solution as the segment size decreases.

Keywords: Maxwell's equations, finite difference method, microstrip antenna, consistency, convergence.

الملخص
تهدف هذه الدراسة إلى تحليل وحل معادلات المجالات الكهرومغناطيسية لهوائي ميكروشريطي مستطيل يشتغل عند تردد 28GHz مخصص لتقنيات الجيل الخامس (5G)، باستخدام كل من الحلول التحليلية والعددية. تبدأ الدراسة بنمذجة رياضية دقيقة باستخدام معادلات ماكسويل للوصول إلى معادلة نموذجية مع تطبيق شروط الحدود المناسبة للموصلات المثالية. تم الحصول على الحل التحليلي باستخدام طريقة فصل المتغيرات مما أتاح وصفًا دقيقًا لتوزيع المجالات الكهربائية والمغناطيسية داخل التجويف. على الجانب العددي، تم استخدام طريقة الفروق المحدودة لحل النموذج الرياضي للهوائي الميكروشريطي المتسطيل، مع تطبيق طريقة جاوس-سايدل للحل التكراري، حيث تم حساب مركبة المجال الكهربائي E_z باستخدام تقريب فروق مركزية من الرتبة الثانية، في حين تم حساب مركبة المجال المغناطيسي H_x باستخدام تقريب فروق مركزية من الرتبة الأولى. أظهرت النتائج العددية توافقًا جيدًا مع الحل التحليلي، مما يؤكد دقة الخوارزمية المنفذة. كما أظهرت دراسة

تأثير عدد المقاطع على الدقة أن زيادة عدد المقاطع يقلل الخطأ بشكل ملحوظ، بينما يزيد عدد التكرارات المطلوبة للوصول إلى التقارب. أيضًا تم تحليل الخصائص العددية، بما في ذلك خطأ الاقتطاع والدقة والاتساق والتقارب والثبات. أظهرت التحليلات أن الحل العددي يحقق دقة من الدرجة الثانية $O(\Delta h^2)$، ومتسق تمامًا، ومستقر، ويتقارب نحو الحل التحليلي عند تصغير حجم المقاطع.

الكلمات المفتاحية: معادلات ماكسويل، طريقة الفروق المحدودة، هوائي ميكروشريطي، الاتساق، التقارب.

Introduction

Microstrip patch antennas are considered one of the most prominent technical solutions used in modern communication network, especially in 5G networks, due to their small size, simplicity, and efficient performance at high frequencies [1], as well as their suitability as compact and integrable elements [2]. One the other side, an accurate understanding of the electric and magnetic fields of the antenna and the surrounding medium is required in order to evaluate and analyze the performance of these antennas. This understanding necessitates advanced mathematical tools that include analytical and numerical methods [3]. However, the choice of mathematical tools and methods for problem analysis depends on the state of the problem itself and suitability of the method to it [4]. In this paper, the electromagnetic field equations are solved within a defined medium representing a 5G rectangular microstrip patch antenna operating in the TM_{10} mode and at a frequency of 28 GHz. The mathematical model is highlighted through the solution of Maxwell's equations. For the analytical method, the separation of variables technique is used under fixed boundary conditions. Then, the finite difference method is applied as a general numerical solution, which can be used in more complex scenarios where it is difficult to obtain a closed – form solution. This paper aims to link applied mathematical concepts with practical modern communication systems.

Mathematical Modelling of EM Fields

In this section, we derive the general differential equation of the electromagnetic fields inside a rectangular Microstrip patch cavity. Appropriate assumptions about the medium must be made in order to derive a wave equation specific to the TM_{10} mode. The derivation begins by applying Maxwell's equations to the particular case and its specific conditions.

It is easier to work with the phasor form (Frequency domain form) of Maxwell's equations, and for isotropic, linear, and source – free dielectric medium, these equations take the following form [5]:

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \tag{1}$$

$$\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} \tag{2}$$

Where **E** and **H** are the electric and magnetic field vectors, respectively. ε and μ are the permittivity and permeability of the medium, respectively. ω is the angular frequency [5].

We consider a rectangular patch antenna of dimensions $L \times W$, mounted over a dielectric substrate with relative permittivity ε_r and non-magnetic materials ($\mu_r = 1$). The cavity is bounded by perfect electric conductors (PECs), which lead us to the following boundary conditions [6]:

$$\mathbf{E}_{||} = 0 \quad \text{on all walls} \tag{3}$$

For transverse magnetic (TM) modes, including fundamental mode (TM_{10} mode), the field components satisfy the following conditions:

$$E_x = 0 \ , \ E_y = 0 \ , \ E_z \neq 0 \ , \ H_x \neq 0 \ , \ H_y \neq 0 \ , \ H_z = 0 \tag{4}$$

This indicates that the electric field has a single component in the z-direction, while the magnetic field has two components in the x- and y- directions.

From Maxwell's curl equations, we eliminate **H** to obtain a second-order equation for **E**. Taking the curl of the first equation:

$$\nabla \times (\nabla \times \mathbf{E}) = -j\omega\mu\nabla \times \mathbf{H} = -j\omega\mu(j\omega\varepsilon\mathbf{E}) = \omega^2\mu\varepsilon\mathbf{E} \tag{5}$$

Using the vector identity [5]:

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla.\mathbf{E}) - \nabla^2\mathbf{E} \tag{6}$$

In a source-free, dielectric and homogeneous medium, we have $\nabla.\mathbf{E} = 0$ [5], so

$$\nabla^2\mathbf{E} + \omega^2\mu\varepsilon\mathbf{E} = 0 \tag{7}$$

Equation (7) is known as the *vector Helmholtz equation* [5]. The equation can be reduced by considering only the single non-zero component E_z:

$$\nabla^2 E_z + \omega^2\mu\varepsilon E_z = 0 \tag{8}$$

The operator ∇^2 is the *Laplacian operator*, which represents a second – order differential operator acting on the scalar wave equation [6]. Then, the scalar wave equation becomes:

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + k^2 E_z = 0 \qquad ; \qquad k = \frac{\omega}{v_p} \tag{9}$$

Where: $v_p = 1/\sqrt{\mu\varepsilon}$ is the phase velocity in the medium [5]. Equation (9) represented Helmholtz equation for the longitudinal electric field component inside the rectangular cavity.

Finally, a *Dirichlet boundary value problem* will be assumed, which considers the conductors to be perfect electric conductors (PEC) [6], meaning that the tangential electric field is equal to zero.

$$E_z = 0 \quad at \quad x = 0\,, x = L\,; y = 0\,, y = W \tag{10}$$

In the following two sections, the solution to the obtained mathematical model will be determined through an analytical approach using the method of separation of variables, and a numerical approach using the finite difference method.

Analytical Solution using Separation of Variables

To solve the scalar Helmholtz equation in Eq. (9) subject to the boundary conditions of Eq. (10), the *separation of variables* method can be applied as analytical solution. This method is suitable for problems with rectangular or orthogonal boundaries [6], where the field component can be expressed as the product of functions of independent coordinates:

$$E_z = E_0 \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi y}{W}\right) \tag{11}$$

Where $m, n = 0, 1, 2, \ldots$ represent the mode indices.

For the fundamental TM_{10} mode, where $m = 1, n = 0$, so this leading us to the analytical solution for this mode:

$$E_z = E_0 \sin\left(\frac{\pi x}{L}\right) \tag{12}$$

The analytical solution shown in Eq. (12) indicates that the electric filed for TM_{10} is constant along the y-axis, while it takes a sinusoidal form along the x-axis.

By Obtaining the electric field component, Eq. (1) can be applied to derive the magnetic field components, which yields a single non-zero component:

$$H_x = \frac{-E_0}{j\eta} \cos\left(\frac{\pi x}{L}\right) \tag{13}$$

Where $\eta = \sqrt{\mu/\varepsilon}$ is the *intrinsic impedance* of the substrate material [7]. The corresponding resonant frequency is given by [1]:

$$f_{10} = \frac{c}{2L\sqrt{\varepsilon_r}} \quad (14)$$

Numerical Solution using Finite Difference Method

To obtain a numerical solution for the scalar Helmholtz equation in Eq. (9), the *finite difference method* (FDM) is employed. This method has been widely used for solving complex electromagnetic problems due to its flexibility in modeling intricate structures. With the significant advancements in computational power, its applications have expanded to an even broader range of fields [3], demonstrating consistency and high effectiveness in modeling three-dimensional problems [8].

In this method, the object under investigation is directly discretized within its surrounding environment, a region referred to as the *computational volume.* This volume is divided into a large number of small elements known as *cells*. Within each cell, the six electromagnetic field components $(E_x, E_y, E_z, H_x, H_y, H_z)$ are positioned at specific spatial points [3].

Discretizing the Helmholtz equation in form at Eq. (9) using central difference approximations along the three spatial axes yields:

$$\frac{E_z(i+1,j,k) - 2E_z(i,j,k) + E_z(i-1,j,k)}{\Delta x^2} + \frac{E_z(i,j+1,k) - 2E_z(i,j,k) + E_z(i,j-1,k)}{\Delta y^2} + k^2 E_z(i,j,k) = 0 \quad (15)$$

Where: $E_z(i,j,k)$ denoted the electric field at the grid point (i,j,k). Δx and Δy represent the spatial discretization steps along the x- and y- directions, respectively. The number of discretization segments along the length is denoted by N_{segment}, such that $\Delta x = L/N_{\text{segment}}$. For the common practical simplification, $\Delta x = \Delta y = \Delta h$.

The discretized Helmholtz equation is solved using *Gauss-Seidel iterative method* due to its simplicity, low computational cost, and fast convergence [9]. It is particularly effective for large systems that typically arise from the finite difference discretization of partial differential equations [10].

Then applied Eq. (1) and using first central-difference approximation at (i,j,k) given:

$$H_x(i,j,k) = \frac{1}{-j\omega\mu} \frac{E_z(i,j+1,k) - E_z(i,j-1,k)}{2\Delta y} \quad (16)$$

The material parameters (μ, ε) are assumed to be constant in the computational domain.

Antenna Specifications

In this study, the considered antenna is a rectangular Microstrip patch antenna. The design parameters used in this study are values for practical antennas that were specifically designed to meet the requirements of 5G applications [1]. The key parameters of the antenna are summarized in Table 1.

Table 1 Design parameters of the 5G rectangular Microstrip patch antenna.

Parameter	Symbol	Value	Unit
Patch Length	L	3.44	Mm
Patch Width	W	4.40	Mm
Substrate Height	h	0.20	Mm
Permittivity	ε_r	4.4	--
Ground Length	L_g	4.65	Mm
Ground Width	W_g	5.60	Mm
Operating Frequency	f	28	GHz

This configuration enables a resonant behavior at 28 GHz, which lies within the mmWave band allocated for 5G systems.

Results

A) Analytical Solution

The analytical solution for the TM_{10} mode of the 5G rectangular microstrip patch antenna is illustrated in Figure 1, representing the exact field distribution. The E_z component exhibits a sinusoidal variation along the x-axis while remaining uniform along the y-axis. Similarly, the H_x component demonstrates an absolute cosine variation along the x-axis. It should be noted that the absolute value was employed in the representation of the H_x component due to its complex-values nature.

B) Numerical Results

Table 2 presents samples of the numerical results using FDT for EM fields, for the TM_{10} mode of the 5G rectangular microstrip patch antenna. The computations were implemented in MATLAB. The results demonstrate close agreement between the exact analytical solution and the finite-difference numerical solution, validating the accuracy of the implemented algorithm. The numerical errors, reported in Table 2 as relative errors, reflect local pointwise discrepancies between the FDT samples and the exact analytical solution. These errors tend to be larger near the center of the field distribution and decrease toward the edges. Such discrepancies are primarily influenced by local discretization effects and numerical dispersion arising from the finite grid spacing. The overall accuracy can be further improved by increasing N_{segment}, as will be explained in the following discussion.

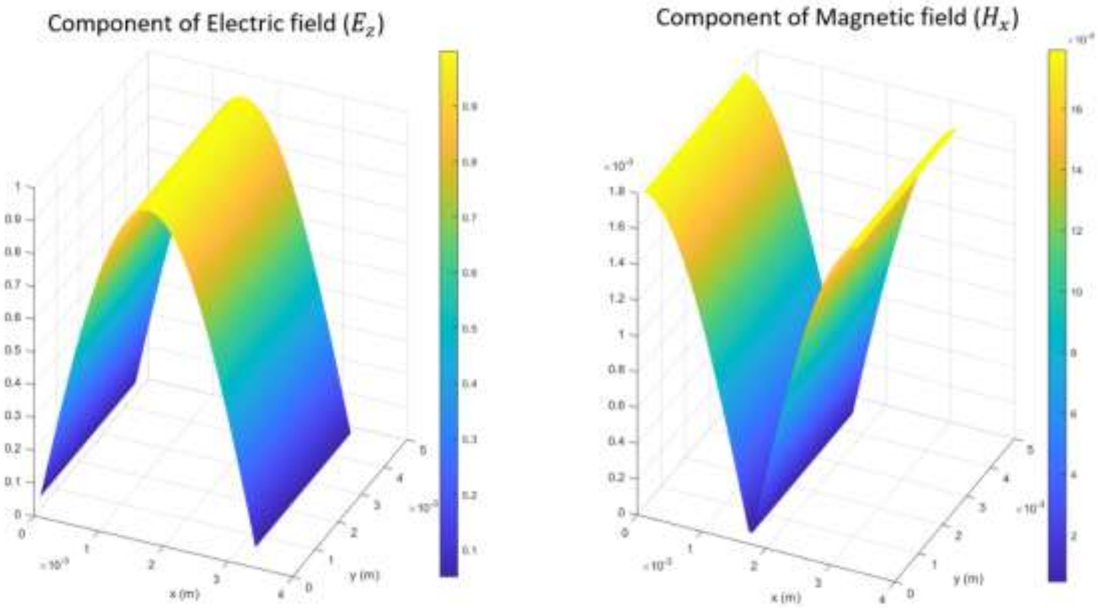


Figure 1: Exact electromagnetic field distribution for a 5G rectangular Microstrip patch antenna at TM_{10} mode.

Table 2 Sample of the results and errors at (N_{segment} =80)

x	y	Electric Field (E_z)			Magnetic Field (H_x)		
		Exact	FDM	Error	Exact	FDM	Error
0.0022509	0.0021405	0.88476	0.59725	0.32496	-0.000837j	-0.0012991j	0.55204
0.0022933	0.0021405	0.86603	0.58460	0.32496	-0.000898j	-0.0013937j	0.55199
0.0023358	0.0021405	0.84599	0.57108	0.32496	-0.000957j	-0.0014862j	0.55193
0.0023783	0.0021405	0.82468	0.55670	0.32495	-0.001015j	-0.0015764j	0.55185
0.0024207	0.0021405	0.80212	0.54148	0.32495	-0.001072j	-0.0016643j	0.55175

x	y	Electric Field (E_z)			Magnetic Field (H_x)		
		Exact	FDM	Error	Exact	FDM	Error
0.0024632	0.0021405	0.77836	0.52544	0.32494	-0.001127j	-0.0017484j	0.55161
0.0025057	0.0021405	0.72737	0.50862	0.32493	-0.001181j	-0.0018321j	0.55144
0.0025481	0.0021405	0.72737	0.49104	0.32491	-0.001233j	-0.0019119j	0.55122
0.0025906	0.0021405	0.70022	0.47272	0.32489	-0.001282j	-0.0019886j	0.55093
0.0026331	0.0021405	0.67201	0.45370	0.32486	-0.001330j	-0.0020623j	0.55055
0.0026756	0.0021405	0.64279	0.43400	0.32482	-0.001376j	-0.0021326j	0.55006
0.0027180	0.0021405	0.61260	0.41365	0.32476	-0.001419j	-0.0021995j	0.54942
0.0027605	0.0021405	0.58149	0.39269	0.32468	-0.001461j	-0.0022627j	0.54859
0.0028030	0.0021405	0.54951	0.37116	0.32456	-0.001501j	-0.0023221j	0.54749
0.0028454	0.0021405	0.51670	0.34909	0.32438	-0.001537j	-0.0023773j	0.54605
0.0028879	0.0021405	0.48311	0.32652	0.32413	-0.001573j	-0.0024282j	0.54416
0.0029304	0.0021405	0.44880	0.30349	0.32377	-0.001605j	-0.0024743j	0.54167
0.0029729	0.0021405	0.41381	0.28006	0.32323	-0.001635j	-0.0025153j	0.53838
0.0030154	0.0021405	0.37820	0.25626	0.32242	-0.001662j	-0.0025505j	0.53405
0.0030579	0.0021405	0.34202	0.23216	0.32120	-0.001687j	-0.0025794j	0.52833

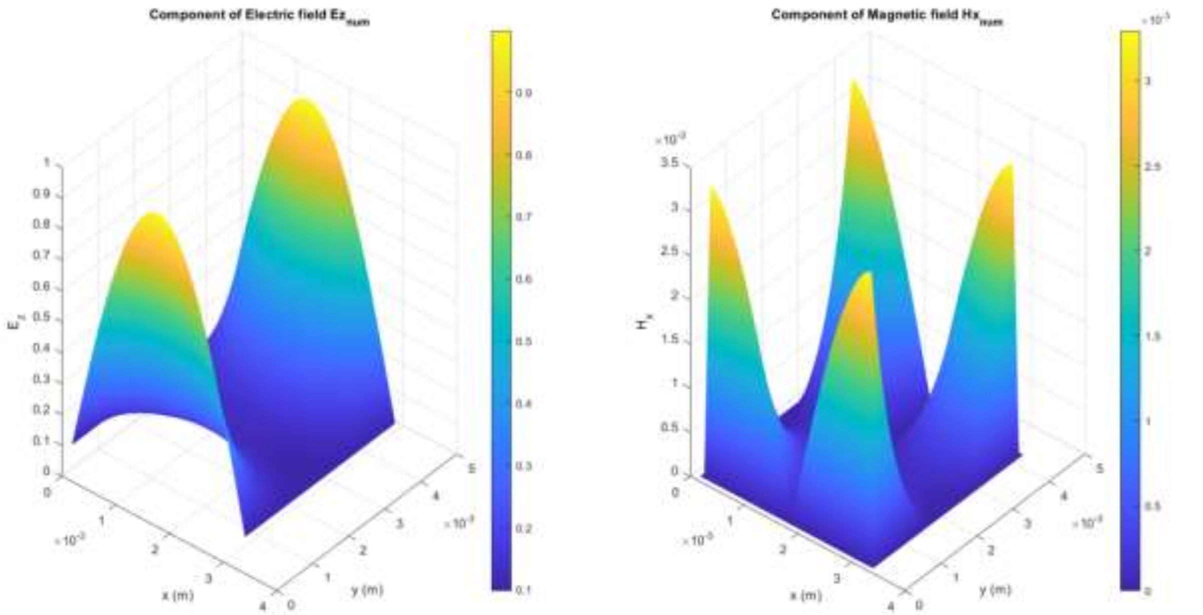
C) Effect of Segment Number on Accuracy and Iterations

The results summarized in Table 3 demonstrate the effect of increasing the number of segments N_{segment} on the accuracy of the numerical solution and the required number of iterations to converged. As the number of segments increases (or Δh decreases), the maximum error decreases significantly, indicating improved alignment of the numerical solution with the analytical solution. For instance, when using only 30 segments, the maximum error reached 0.8736, while with 200 segments, the error dropped to just 0.0648. This behavior highlights the enhanced precision obtained with finer discretization. However, this improvement in accuracy is accompanied by an increase in the number of iterations required to achieve convergence, peaking at 833 iterations for 80 and slightly decreasing with 120 segments.

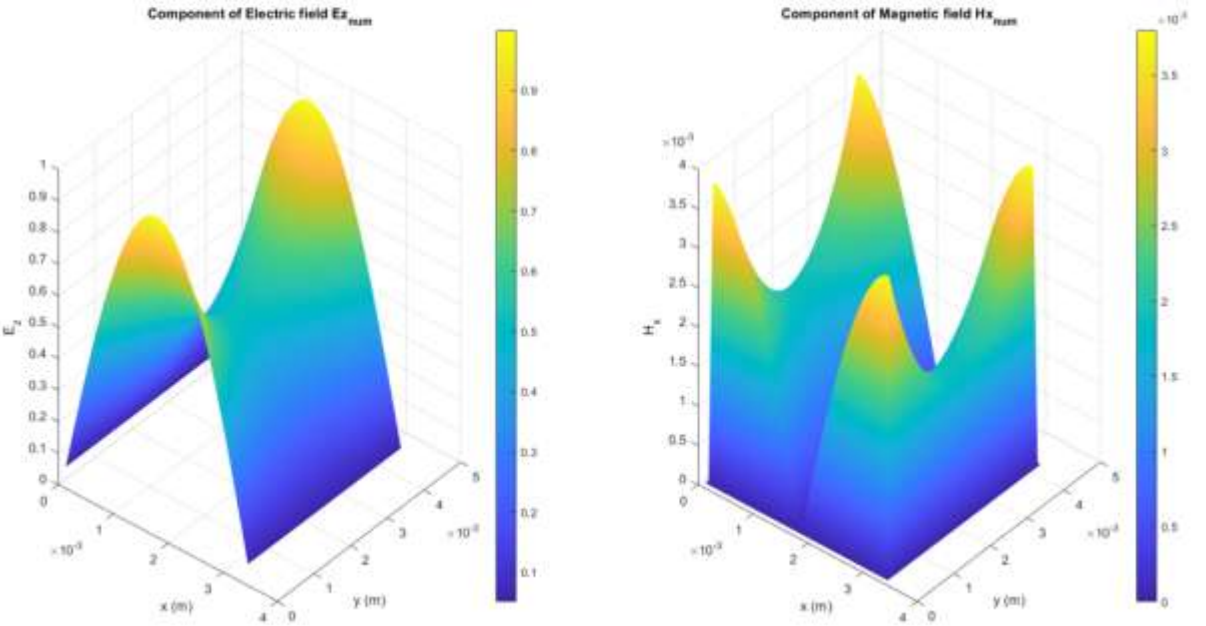
Table 3 Maximum error and convergence iterations for different N_{segment}

N_{segmetn}	Max of error	N. Iteration
30	0.8736	502
60	0.4866	786
80	0.3250	833
120	0.1662	832
200	0.0648	760

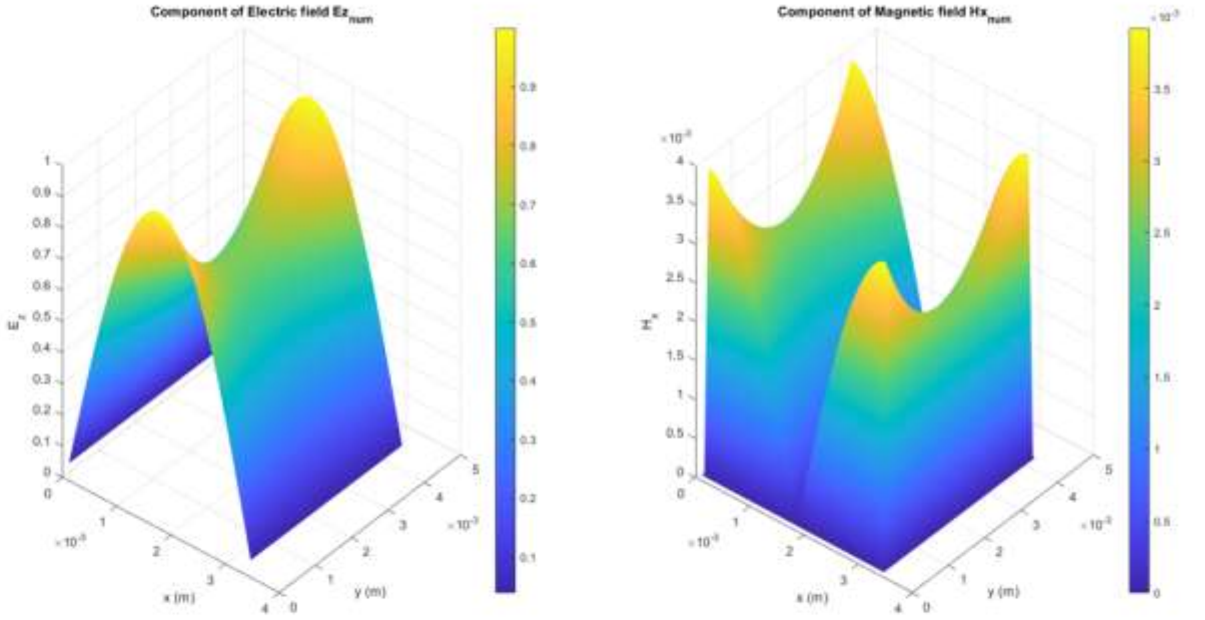
Figure 2 presents the numerical solution for both the electric and magnetic fields at different values of segments. It can be observed that the numerical solutions using FDM approach the form obtained from the exact analytical solution as the number of segments increases (or Δh decreases).


Component of Electric field Ez num
Component of Magnetic field Hx num
E z
H x
x (m)
y (m)

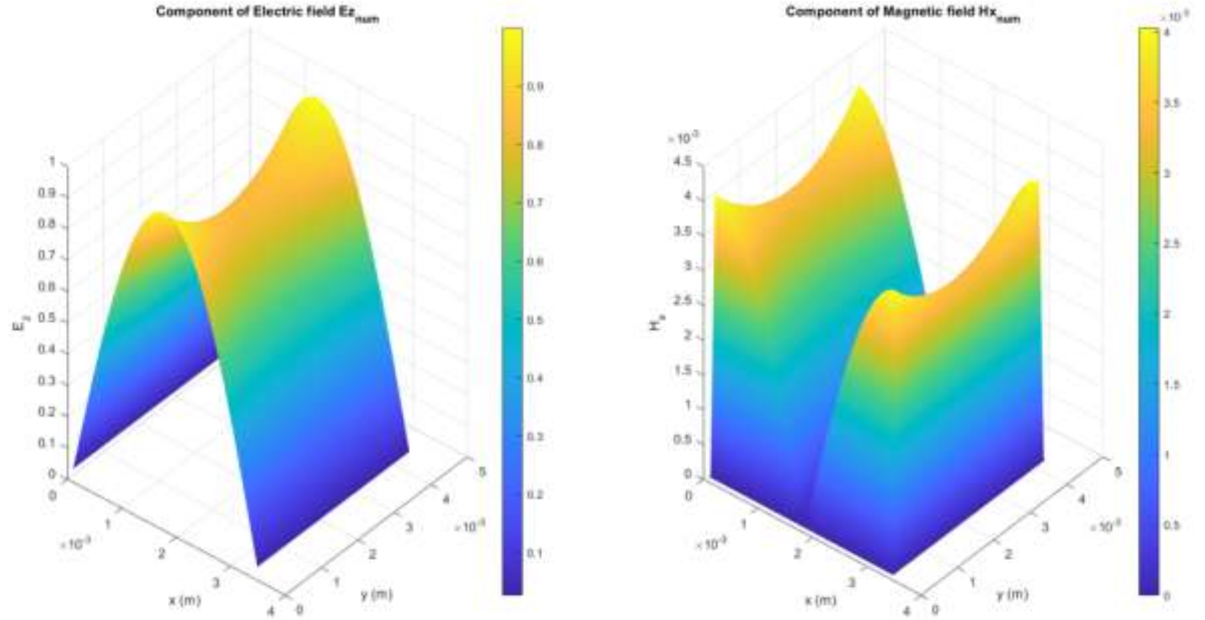
(a)


Component of Electric field Ez num
Component of Magnetic field Hx num
E z
H x
x (m)
y (m)

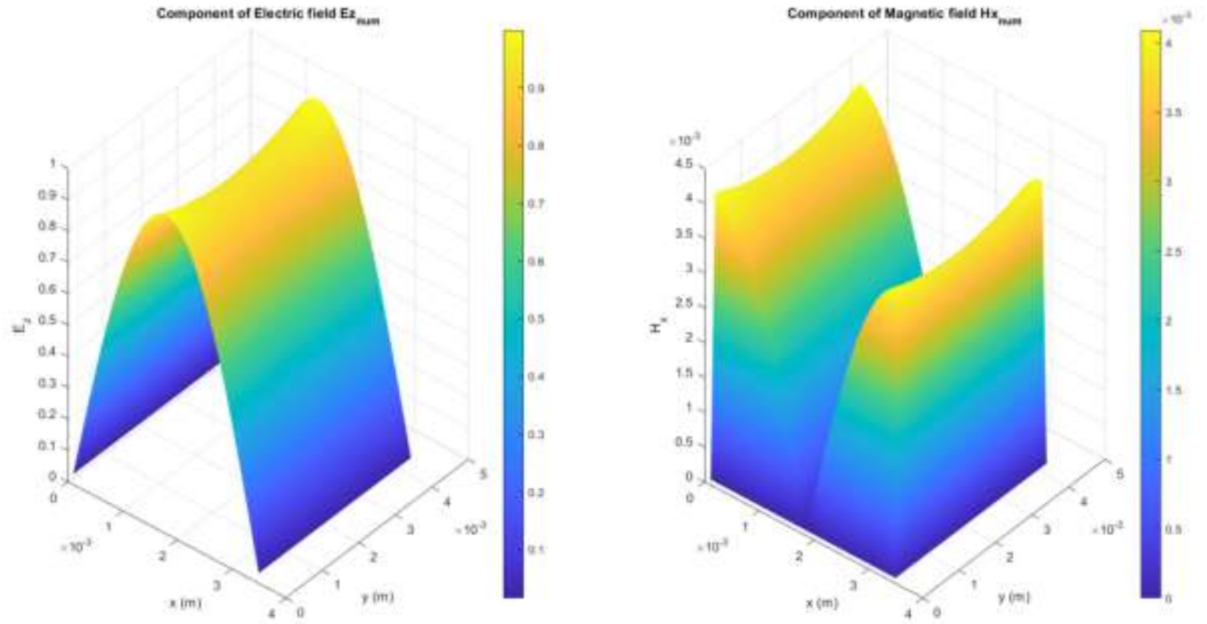
(b)


Component of Electric field Ez_num
x (m)
y (m)
Component of Magnetic field Hx_num
x (m)
y (m)

(c)


Component of Electric field Ez_num
x (m)
y (m)
Component of Magnetic field Hx_num
x (m)
y (m)

(d)



(e)

Figure 2: Numerical electromagnetic field distribution for a 5G rectangular Microstrip patch antenna at TM_{10} mode using finite difference method (FDM) for different segment, (a) 30, (b) 60, (c) 80, (d) 120, (e) 200.

D) Numerical Properties Analysis

To evaluate the numerical performance of the implemented finite-difference scheme, the numerical properties of both the electric field E_z and magnetic field H_x equations were analyzed. These properties include truncation error, accuracy, consistency, convergence, and stability. The truncation error for E_z arises from the central second-approximation of the second derivatives. Each second derivative introduces a leading term of $(\Delta h^2/12)(\partial^4 E_z/\partial x^4)$ (and similarly in y) [11], giving a combined truncation error:

$$T_E \approx \frac{\Delta h^2}{12}\left(\frac{\partial^4 E_z}{\partial x^4} + \frac{\partial^4 E_z}{\partial y^4}\right) \tag{17}$$

For H_x, which is obtained using the central first-difference approximation, so the truncation error:

$$T_E \approx \frac{\Delta h^2}{6}\frac{\partial^3 E_z}{\partial y^3} \tag{18}$$

Both numerical solutions achieve second-order accuracy $O(\Delta h^2)$. The scheme is mathematically consistent with Maxwell's equations [11]. Furthermore, the scheme demonstrates convergence as $\Delta h \to 0$, ensuring that the numerical solution approach the exact analytical solution, and stability is maintained under the chosen discretization. Table 4 summarized these properties.

Table 4 Summary of Numerical properties for E_z and H_x

Property	**Numerical Solution of Electric Field E_z**	**Numerical Solution of Magnetic Field H_x**
Truncation Error	$T_E \approx \frac{\Delta h^2}{12}(\frac{\partial^4 E_z}{\partial x^4} + \frac{\partial^4 E_z}{\partial y^4})$	$T_E \approx \frac{\Delta h^2}{6}\frac{\partial^3 E_z}{\partial y^3}$
Accuracy	$O(\Delta h^2)$	$O(\Delta h^2)$
Consistency	Fully consistent	Fully consistent
Convergence	Convergence as $\Delta h \longrightarrow 0$	If E_z converges at $O(\Delta h^2)$, then H_x computed from central difference also converges.
Stability	Stable	Stable

Conclusion

This study demonstrated the feasibility of employing the finite difference method as a numerical solution for calculating the EM fields of microstrip antenna used in 5G application, combined the Gauss-Seidel iteration. The approach provided accurate numerical results that were consistent with the analytical model. Furthermore, the evaluation of the numerical properties confirmed the method's second-order accuracy, stability, and convergence. These findings highlight the reliability and effectiveness if the proposed approach for modeling high-frequency microstrip antennas in advanced wireless communication systems.

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