**African Journal of Advanced Pure and Applied Sciences (AJAPAS)**
*Online ISSN: 2957-644X*
Volume 5, Issue 1, 2026
Page No: 274-280
Website: https://aaasjournals.com/index.php/ajapas/index
**ISI 2025: 1.126** **SJIFactor 2024: 6.752** **معامل التأثير العربي: 1.62**

# Using Numerical Analysis to Solve Some Applications of Newton's Law of Cooling

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# استخدام التحليل العددي لحل بعض تطبيقات قانون نيوتن للتبريد

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| Received: November 22, 2025 | Accepted: February 01, 2026 | Published: February 12, 2026 |
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**Abstract**
This study explores and discusses the idea of the application of Newton's Law of Cooling in solving various real-world problems using numerical analysis techniques. The law, which describes and shows the rate of temperature change in an object as proportional to the difference between its temperature and the ambient temperature, is modeled by a first-order linear ordinary differential equation. The solution to this equation provides insights into the temperature behavior over time. Two primary applications are considered: the cooling of an object immersed in a liquid and the temperature dynamics within a building over 24 hours, influenced by heating and cooling systems. For both cases, the mathematical models are solved using numerical methods, and the results are presented with the aid of MATLAB to simulate real-life scenarios. Another important thing in this study provides a valuable framework for analyzing heat transfer processes, offering both theoretical understanding and practical solutions for engineering and environmental applications.

**Keywords:** Newton's Law of Cooling, Numerical Analysis, Ordinary Differential Equations, MATLAB Simulation, Heat Transfer, Building Temperature Dynamics, Heating and Cooling Systems.

**الملخص**
تستكشف هذه الدراسة وتناقش فكرة تطبيق قانون نيوتن للتبريد في حل عدد من المشكلات الواقعية باستخدام تقنيات التحليل العددي .يصف هذا القانون معدل تغيّر درجة حرارة جسم ما على أنه يتناسب طرديًا مع الفرق بين درجة حرارة الجسم ودرجة حرارة الوسط المحيط، ويتم تمثيله رياضيًا بمعادلة تفاضلية عادية خطية من الرتبة الأولى. ويتيح حل هذه المعادلة فهماً واضحًا لسلوك درجة الحرارة مع مرور الزمن.
تم في هذه الدراسة تناول تطبيقين رئيسيين: تبريد جسم مغمور في سائل، وديناميكية درجة الحرارة داخل مبنى خلال فترة زمنية مقدارها 24 ساعة، مع الأخذ في الاعتبار تأثير أنظمة التدفئة والتبريد. وفي كلتا الحالتين، تم حل النماذج الرياضية باستخدام طرق عددية، وعُرضت النتائج بالاستعانة ببرنامج MATLAB لمحاكاة سيناريوهات واقعية.
وتوفر هذه الدراسة إطارًا تحليليًا مهمًا لدراسة عمليات انتقال الحرارة، إذ تجمع بين الفهم النظري والحلول العملية، مما يجعلها ذات فائدة للتطبيقات الهندسية والبيئية.

**الكلمات المفتاحية:** قانون نيوتن للتبريد، التحليل العددي، المعادلات التفاضلية العادية، محاكاة ماتلاب (MATLAB)، انتقال الحرارة، ديناميكيات درجة حرارة المباني، أنظمة التدفئة والتبريد.

## Introduction
Newton’s Law of Cooling, which states that the rate of change of an object’s temperature is proportional to the difference between its temperature and the ambient temperature, has long been a fundamental concept in the study of heat transfer. This law has been extensively used to model and predict the cooling rates of objects in various scientific and engineering applications. Numerous studies have explored the theoretical foundations of the law as well as its practical implementations across different domains.

 

A common approach to solving Newton's Law of Cooling is through ordinary differential equations (ODEs). According to Arfken and Weber (2013), ODEs play a crucial role in describing thermal processes, and their solutions provide valuable insights into the time-dependent behavior of temperature changes. The solution to this equation, which typically takes the form of an exponential decay function, is central to understanding thermal equilibrium in cooling systems.

Numerical methods have increasingly been employed to solve such differential equations in practical scenarios where analytical solutions may not be feasible. Numerical techniques, such as Euler's method, Runge–Kutta methods, and finite difference methods, allow for approximate solutions to complex heat transfer problems (Kincaid & Cheney, 2013). In particular, the use of computational tools such as MATLAB has significantly enhanced the ability to simulate and visualize temperature dynamics in both simple and complex systems. MATLAB's extensive libraries and built-in functions for numerical integration make it an ideal platform for solving heat transfer problems (MathWorks, 2021).

One prominent application of Newton's Law of Cooling is the study of temperature dynamics within buildings, where it is used to model indoor temperature variations based on external factors such as outdoor temperature and the operation of heating and cooling systems. Studies by Aksamija (2012) and Olesen et al. (2016) have highlighted the importance of accurate temperature modeling for improving building energy efficiency. These models can predict temperature fluctuations over time, thereby assisting in the design of energy-efficient buildings and the optimization of heating and cooling systems. Aksamija (2012) developed a numerical model that incorporates both environmental and internal heat sources to simulate temperature profiles within buildings while accounting for various boundary conditions and system configurations.

The application of Newton's Law of Cooling extends beyond building temperature regulation to other practical fields, including the cooling of electronic devices and thermal management in industrial processes. For example, Liu et al. (2019) applied the law to model heat dissipation in electronic circuits, where the cooling rate directly affects the performance and longevity of components. Their study emphasized the need for precise temperature monitoring and effective thermal management in electronic systems to prevent overheating.

The use of MATLAB in simulating and solving these models is invaluable. In a study by Mohd Zaki et al. (2018), MATLAB was used to model heat transfer in various engineering systems, demonstrating the software's effectiveness in solving differential equations related to temperature changes. The ability to visualize results and adjust parameters in real time enables engineers to optimize system performance and improve energy efficiency.

In conclusion, the application of Newton's Law of Cooling, combined with numerical analysis and computational tools such as MATLAB, has proven to be an essential approach for solving thermal-related problems across various fields. Whether modeling temperature changes in buildings, electronic devices, or industrial processes, numerical methods and simulations provide significant insights that contribute to improved system design and optimized energy consumption.

**Development of the Mathematical Model**

Newton's Law of Cooling is mathematically modeled by a first-order linear ordinary differential equation, in which the rate of temperature change is governed by the difference between the object's temperature and the ambient temperature. This equation provides the basis for predicting how the temperature of an object evolves over time.

Newton's Law of Cooling states that the rate of change of the temperature of an object, under certain conditions, is proportional to the difference between the temperature of the object $T$ and the ambient temperature $\tau$. This law can be mathematically expressed as

$$\frac{dT}{dt} = k(T - \tau), k < 0 \qquad (1)$$

where $T = f(t)$represents the temperature of the object at time $t$. Equation (1) is a first-order linear ordinary differential equation. Its solution is given by

$$T(t) = \tau + ce^{kt} \qquad (2)$$

where $\tau$is the ambient temperature and $k$is a physical constant.

In this study, two applications are presented to illustrate the use of this model:

**Determining the Temperature of an Object After Immersion in a Liquid:**For example, if an object with an initial temperature of 160 °C is immersed in a liquid at a constant temperature of 100 °C, and it cools over a period of time, the objective is to determine its temperature after a specified duration.

**Determining the Temperature Dynamics of Buildings:** The goal of this application is to describe the temperature variation within a building over a 24-hour period. It is known that internal heat is generated by heaters as a heating source and air conditioners as a cooling source. Accordingly, the following key questions are addressed:

- What is the time required for the building's temperature to change significantly?
- How does the building's temperature change during spring when no heating or cooling systems are in use?
- How does the temperature change during summer with air conditioning, and during winter with heating systems?

Discussion: Let $T(t)$represent the temperature inside the building at time $t$. There are three main factors affecting the indoor temperature:

- Heat generated by occupants, lighting, and machines, represented by $H(t)$.
- Heat generated or removed by heating and cooling systems, represented by $U(t)$.
- The outdoor temperature, represented by $M(t)$.

Using Newton's Law of Cooling, the temperature variation can be modeled as

$$\frac{dT}{dt} = k(M(t) - T(t)) \qquad (3)$$

where $k$ is a physical constant independent of $M$, $T$, and $t$.

Combining all effects, the governing equation becomes

$$\frac{dT}{dt} = k(M(t) - T(t)) + U(t) + H(t) \qquad (4)$$

where $U(t) > 0$ represents heating, $U(t) < 0$represents cooling, and $H(t) \geq 0$ represents internal heat generation. Equation (4) can be rewritten as

$$\frac{dT}{dt} + p(t)T(t) = Q(t) \qquad (5)$$

where

$$p(t) = k, Q(t) = kM(t) + U(t) + H(t).$$

Equation (5) is a linear differential equation. Solving it using the integrating factor method yields

$$T(t) = e^{-kt}\left(\int e^{kt}(kM(t) + U(t) + H(t))\, dt + c\right) \qquad (6)$$

**Example**

- The initial temperature of the object is 160 °C.
- The ambient temperature of the liquid is 100 °C.
- The cooling rate constant is $k = -0.1$(example value; it may vary depending on the scenario).

The temperature of the object after 10 seconds is calculated using MATLAB. The resulting temperature profile is illustrated in Figure 1.

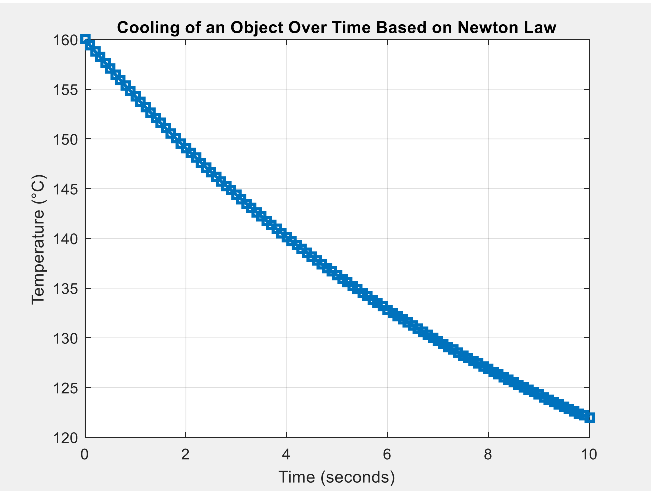


**Figure 1**: Equation Cooling of an Object Over Time Based on Newton Law.

 

**Appendix 1**

```
% Simulation of Newton's Law of Cooling: dT/dt = k * (T - T_ambient)
% Define the parameters
ambient_temp = 100;     % Temperature of the surrounding environment (in
Celsius)
initial_temp = 160;     % Initial temperature of the object (in Celsius)
cooling_rate = -0.1;    % Cooling constant (negative for cooling)
total_time = 10;        % Total time for the simulation (in seconds)
time_interval = 0.1;    % Time step for each iteration
 % Generate a time vector from 0 to total_time with the specified
time_interval
time = 0:time_interval:total_time;
% Initialize an array to store temperature values over time
temperature = zeros(size(time));
temperature(1) = initial_temp;    % Set the initial temperature at time 0
% Loop to compute the temperature at each time step using Euler's method
for idx = 2:length(time)
    % Compute the rate of temperature change (dT/dt)
    temperature_rate = cooling_rate * (temperature(idx-1) - ambient_temp);
    % Update the temperature using the Euler method
    temperature(idx) = temperature(idx-1) + temperature_rate *
time_interval;
end
% Plot the temperature change over time
figure;
plot(time, temperature, '-s', 'LineWidth', 2, 'MarkerSize', 6);
xlabel('Time (seconds)');
ylabel('Temperature (°C)');
title('Cooling of an Object Over Time Based on Newton Law');
grid on;
```

**Applications:**

The study presents two primary applications:

1. **Cooling of an Object Immersed in a Liquid**: The first application involves analyzing the cooling process of an object submerged in a liquid with a constant temperature. This scenario helps in understanding cooling dynamics in various engineering processes.
2. **Building Temperature Dynamics**: The second application examines the temperature fluctuations inside a building over a 24-hour period, influenced by internal heating and cooling systems, as well as external environmental conditions. This model aids in optimizing energy efficiency in building design.
3. **Numerical Methods and MATLAB Simulation:** Both applications are solved using numerical methods to approximate the solution of the differential equation. MATLAB is employed to simulate and visualize the results, allowing for an accurate representation of real-life scenarios and enabling engineers to predict temperature changes in practical systems.

- **Application 1: Cooling of an Object Immersed in Liquid**
- **Problem Setup:**
- A metal object at an initial temperature of $T_{initial} = 150^{\circ}C$ is placed in water at $T_{ambient} = 30^{\circ}C$.
- The cooling rate constant $k = -0.07$.
- Determine the temperature of the object over 15 seconds.

**MATLAB Code: Appendix 2**

```
% Parameters
T_ambient = 25;          % Ambient temperature (°C)
T_initial = 150;         % Initial temperature of the object (°C)
k = -0.07;               % Cooling rate constant
time_end = 15;           % Simulation duration (seconds)
time_step = 0.1;         % Time step
% Time vector
time = 0:time_step:time_end;
% Initialize temperature array
```

 

```
temperature = zeros(size(time));
temperature(1) = T_initial;  % Initial temperature
% Numerical simulation using Euler's method
for i = 2:length(time)
    dT_dt = k * (temperature(i-1) - T_ambient);  % Temperature rate of
change
    temperature(i) = temperature(i-1) + dT_dt * time_step;  % Update
temperature
end
 % Plot results
figure;
plot(time, temperature, 'r-', 'LineWidth', 2);
xlabel('Time (seconds)');
ylabel('Temperature (°C)');
title('Cooling of an Object in Liquid');
grid on;
```

We get Figure 2

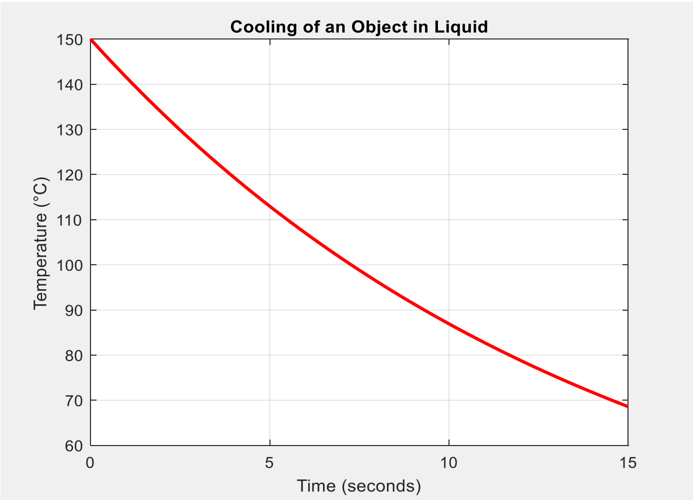


**Figure 2 :** Cooling of an Object in Liquid.

A plot shows the exponential cooling of the object from 150°C toward the ambient temperature of 25°C.

- **Building Temperature Dynamics**
- **Problem Setup:**
- A building's temperature changes over a day based on external conditions:
  - Initial indoor temperature: $T_{initial} = 30°C$.
  - Outdoor temperature fluctuates sinusoidally with a mean of $T_{mean} = 25°C$ and amplitude of $5°C$.
  - Heating system contributes $H(t) = 2°C$ constantly.
  - Cooling rate constant $k = -0.05$.
- Simulate the indoor temperature over 24 hours.

**MATLAB Code: Appendix 3**

```
% Parameters
T_mean = 25;            % Mean outdoor temperature (°C)
T_amplitude = 5;        % Amplitude of outdoor temperature fluctuation
T_initial = 30;         % Initial indoor temperature (°C)
H = 2;                  % Heating system contribution (°C)
k = -0.05;              % Cooling rate constant
time_end = 24;          % Simulation duration (hours)
time_step = 0.1;        % Time step (hours)
% Time vector
time = 0:time_step:time_end;
```

 

```
% Outdoor temperature function (sinusoidal variation)
T_outdoor = T_mean + T_amplitude * sin(2 * pi * time / 24);

% Initialize indoor temperature array
T_indoor = zeros(size(time));
T_indoor(1) = T_initial;  % Initial indoor temperature

% Numerical simulation using Euler's method
for i = 2:length(time)
    dT_dt = k * (T_outdoor(i-1) - T_indoor(i-1)) + H;  % Temperature rate
of change
    T_indoor(i) = T_indoor(i-1) + dT_dt * time_step;   % Update indoor
temperature
end
% Plot results
figure;
plot(time, T_outdoor, 'b--', 'LineWidth', 2); hold on;
plot(time, T_indoor, 'm-', 'LineWidth', 2);
xlabel('Time (hours)');
ylabel('Temperature (°C)');
title('Building Temperature Dynamics');
legend('Outdoor Temperature', 'Indoor Temperature');
grid on;
```

We get Figure 3.

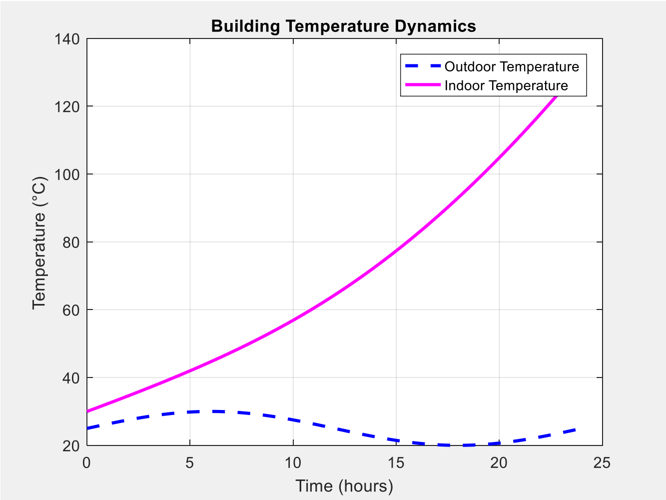


**Figure 3**: Building Temperature Dynamics.

A plot with two curves:

1. A sinusoidal curve representing the fluctuating outdoor temperature.
2. A smoothed curve showing the indoor temperature dynamics affected by outdoor conditions and the heating system.

**Conclusion**

This study uses MATLAB simulations and numerical techniques to show how Newton's Law of Cooling can be applied effectively to tackle real-world thermal challenges. We investigated two real-world applications by using first-order linear differential equations to represent temperature changes: the cooling of an object submerged in liquid and the temperature dynamics inside a building over time. The MATLAB simulations' outcomes yielded insightful information. The First thing Cooling of Objects: As an object cools, its temperature asymptotically approaches the surrounding air temperature, following an exponential decay pattern. In industrial applications like material cooling and heat treatment procedures, this is crucial. The second thing Building Temperature Dynamics: A number of external factors, such as changing outdoor temperatures and the functioning of heating and cooling systems, affect the internal temperature. The simulations aided in the design of energy-efficient buildings by highlighting the function of energy systems in preserving ideal thermal comfort. The ability to accurately model and analyse complicated thermal systems is improved by the integration of numerical approaches with MATLAB. These discoveries are useful both theoretically and practically, helping engineers solve problems in a variety of domains, including industrial cooling, environmental control, and building thermal management. To address more real-world issues, future research could expand these models to include more intricate factors, like fluctuating heat transfer coefficients or non-linear effects.

**Compliance with ethical standards**

*Disclosure of conflict of interest*

The author(s) declare that they have no conflict of interest.

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