African Journal of Advanced Pure and Applied Sciences (AJAPAS)
Online ISSN: 2957-644X
Volume 5, Issue 1, 2026
Page No: 281-291
Website: https://aaasjournals.com/index.php/ajapas/index

ISI 2025: 1.126 **SJIFactor 2024: 6.752** **معامل التأثير العربي: 1.62**

Evaluating the Performance of Time Series Models in Forecasting the Unemployment Rate in Libya

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تقييم أداء نماذج السلاسل الزمنية في التنبؤ بمعدل البطالة في ليبيا

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Received: November 22, 2025	Accepted: February 02, 2026	Published: February 14, 2026

Abstract
This paper aims to analyze and predict the unemployment rate in Libya using a set of time series models. The analysis depended on annual unemployment rate data for the period 1991-2024 and applied trend regression models, exponential smoothing models, a random walk model, and ARIMA models. Series stationarity test were carried out using the ADF test, and the results showed that the series becomes stationary after the 2nd difference. The estimated models were evaluated using residuals diagnostic tests, including Ljung-Box test, Jarque-Bera test, and ARCH-LM test. The results showed that trend regression models failed to meet the residual independence condition while the other models meet all the required conditions. Forecasting performance was evaluated using RMSE, MAE, and MAPE measures. The results showed that Brown exponential smoothing model out-of-sample predictive accuracy was superior. Accordingly, the chosen model was used to predict unemployment rate over the next five years, with confidence intervals at 95% level. The study's results confirm the efficiency of exponential smoothing models, especially the Brown model, in forecasting regular-trend time series in practical applications.

Keywords: Time Series, ARIMA Model, Random Walk, Regression Models, Exponential Smoothing Models, Unemployment Rate, Libya.

الملخص
تهدف هذه الورقة إلى تحليل معدل البطالة في ليبيا والتنبؤ به باستخدام مجموعة من نماذج السلاسل الزمنية. اعتمد التحليل على بيانات سنوية لمعدل البطالة للفترة 1991-2024، وطبقت نماذج الانحدار للاتجاه، ونماذج التمهيد الأسي، ونموذج المشي العشوائي ونماذج ARIMA. أجري اختبار الاستقرارية للسلسلة باستخدام اختبار ADF، وأظهرت النتائج أن السلسلة تصبح مستقرة بعد أخذ الفرق الثاني. قيمت النماذج المقدرة باستخدام اختبارات تشخيص البواقي، بما في ذلك اختبار Ljung-Box، واختبار Jarque-Bera، واختبار ARCH-LM. أظهرت النتائج أن نماذج الانحدار لم تستوف شرط استقلال البواقي، بينما استوفت النماذج الأخرى جميع الشروط المطلوبة. تم تقييم أداء التنبؤ باستخدام المقاييس جذر متوسط مربع الخطأ (RMSE) ومتوسط الخطأ المطلق (MAE) ومتوسط النسبة المئوية للخطأ المطلق (MAPE). بينت النتائج تفوق نموذج براون للتمهيد الأسي في دقة التنبؤ خارج العينة. وبناء على ذلك، استخدم هذا النموذج المختار للتنبؤ بمعدل البطالة خلال السنوات الخمس القادمة، بفترات ثقة عند مستوى 95%. تؤكد نتائج الدراسة كفاءة نماذج التمهيد الأسي، وخاصة نموذج براون في التنبؤ بالسلاسل الزمنية ذات الاتجاه المنتظم في التطبيقات العملية.

الكلمات المفتاحية: السلاسل الزمنية، نماذج ARIMA، المشي العشوائي، نماذج الانحدار، نماذج التمهيد الأسي، معدل البطالة، ليبيا.

Introduction

Time series models are fundamental statistical instruments for analyzing, modeling, and forecasting time-sequential data across numerous applied sciences, including economics, social sciences, engineering, and others. These models allow for the estimation of structural relationships between variable values over time and for the decomposition of time series into trend, seasonal and cyclical components, separating them from random noise [1,2]. Consequently, they provide a basis for developing accurate and reliable predictive models for practical applications. Among the most important models are Autoregressive Integrated Moving Averages (ARIMA) models, which have proven effective in many economics and social applications; and exponential smoothing models which are widely used to analyze trends and seasonality [3]. However, these models become increasing significant when dealing with labor market indicators such as the unemployment rate, which is a dynamic time-dependent variable reflecting changes of economic and social in society. Empirical studies have shown the superiority of exponential smoothing models, such as Brown and Holt models, in short, uniform trending series [4-6]. Moreover, ARIMA models can provide reliable forecasts of the unemployment rate in many countries, in contexts characterized by long-term trends [7-9], whilst, seasonal models such as SARIMA highlighted the importance of addressing seasonal structure when it is present in the data [10]. The ARFIMA models demonstrated the superiority of long-memory models over traditional models in forecasting the unemployment rate [11,12].

Despite the applications of these models in previous studies, the Libyan case remains limited in terms of applications of time series models for forecasting the unemployment rate (UR) data. Therefore, this paper aims to apply and compare the forecasting performance of several time series models, including trend regression models, exponential smoothing models, random walk, and ARIMA models in predicting the UR in Libya. Models performance is evaluated using standard forecasting accuracy measures such as RMSE, MAE, and MPAE. Thus, the study addresses the current research gap in the applied statistical modeling of time series related to Libyan socio-economic indicators.

Literature Review

The applied literature in analyzing and forecasting the UR data has relied on a range of time series models, depending on the series features, objectives and area of study. Exponential smoothing models have been applied in many recent studies for short-term forecasting. Syafwan et al., [6] applied the double exponential smoothing model to predict the UR in North Sumatra from 2000 to 2019. The accuracy of the prediction was assessed using MAD, MSE, and MAPE, and then this model was used for forecasting the UR in 2020.

The Autoregressive Integrated Moving Averages (ARIMA) models have been used to represent the UR. Kurita, [11] studied the modeling and forecasting the monthly UR in Japan using AR and ARFIMA models over the period from January 1995 to August 2008. The results of comparing the forecasting accuracy based on RMSE and MAPE showed that the ARFIMA model performed better than the AR models, especially in short-term forecasting. Adenomon, [7] used ARIMA models to model the annual UR in Nigeria from 1972 to 2014. The results concluded that the ARIMA(2,1,2) model is best suited to representing and modeling the dynamic behavior of the series. Moreover, the selected model was used to obtain future forecasts of UR for the period from 2015 to 2018. Dritsakis and Klazoglou, [10] applied ARIMA model for forecasting monthly UR data in USA from January 1955 to July 2017. The outcomes indicated that the chosen SARIMA(1,1,2)(1,1,1)$_{12}$-GARCH(1,1) model exhibited good statistical fit and acceptable shot-term predictive power. Didiharyono and Syukri, [8] used the ARIMA model to forecast the open UR in South Sulawesi over the period 1986-2018. The results showed that the ARIMA(1,2,1) model was the best, exhibiting the smallest mean squares value compared to other models. The chosen model provided relatively accurate future predictions for the next 12 periods. Ismail et al., [12] used the ARIMA and ARFIMA models for modeling UR in Malaysia from January 2010 to July 2021. These models were evaluated using MAPE, MAE, and RMSE, and the ARFIMA model was superior compared to ARIMA model, indicating the importance of considering long-term memory in modeling UR data. Yan, [9] applied the ARIMA model for forecasting the UR in the UK and the Eurozone from 2014 to 2024. The ARIMA model was used because it is well-suited to non-stationary series, a characteristic of the data in both time series. The results demonstrated that while the ARIMA model effectively predicted the o so inward part of the business cycle, it failed to do so for short-term forecasting after 2023 due to external shocks in the UR data in UK.

In the context of comparing various time series models, Davidescu et al., [4] conducted a comparative study of several univariate time series models including ARIMA and exponential smoothing models, for Romanian monthly UR data from 2000 to 2020. Based on the evaluation of the in-sample forecasts, the error measures RMSE, MAE, and MAPE showed that the multiplicative Holt-Winters model was superior to the other models. Lip et al., [5] compared the double exponential smoothing model, the Holt model, and the ARIMA model for modeling the UR in Malaysia from January 2012 to December 2018. The best performing model was determined using MSE, RMSE, MAPE, and GRMSE, with the results showing that the ARIMA(2,1,3) model was the best fit, as it recorded the lowest value for all error measures.

Previous studies have shown that the ARIMA models are the most common tool for modeling and forecasting the UR series, however, some recent studies have highlighted the superiority of exponential smoothing models, particularly, the Brown and Holt models, in series with regular trend. Therefore, the current study adopted a comparative approach combining trend models, exponential smoothing models, random walk model and ARIMA, focusing on evaluating their predictive performance both in-sample and out-of-sample to ensure the selection of the most suitable model for Libyan UR data.

Methodology

Description of Data

The data used in this paper is an annual time series of the UR in Libya covering the period 1991-2024, obtained from the World Bank Database [13]. The data expressed as the percentage of unemployment individuals in the total labor force, and were split into training set (1991-2018) for model estimation and an out-of-sample test set (2019-2024) for forecasting evaluation.

Time Series Models

This study focused on comparing the performance of several statistical models for forecasting the annual UR in Libya, the time series models include the following

Linear Trend Model

The linear trend model is a statistical model that assumes a variable changes over time in a uniform linear direction, with a random component representing irregular changes. This model is expressed as follows:

$$y_t = \beta_0 + \beta_1 t + \epsilon_t \, , \quad \epsilon_t \sim N(0, \sigma^2), \qquad (1)$$

where y_t is the variable value at time t , β_0 is a constant, β_1 is the slope, and ϵ_t is the white noise error term.

Quadratic Trend Model

The quadratic trend model assumes that a variable changes over time along a quadratic curve, allowing for increasing or decreasing of changes over time. Moreover, this model originated from an expansion of the least squares method, for estimating nonlinear relationship, and it is expressed as follows:

$$y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \epsilon_t \, , \qquad \epsilon_t \sim WN(0, \sigma^2) \qquad (2)$$

where y_t is the variable value at time t , β_0 is a constant, β_1 is the linear term parameter, β_2 is the quadratic term parameter ,and ϵ_t is the white noise error term.

Cubic Trend Model

The cubic trend model is an extension of the quadratic trend model allowing for the capture of complex patterns of change over time using a cubic curve. This model is expressed as follows:

$$y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + \epsilon_t \, , \quad \epsilon_t \sim WN(0, \sigma^2) \qquad (3)$$

where y_t is the variable value at time t , β_0 is a constant, β_1 is the linear term parameter, β_2 is the quadratic term parameter , β_3 is the cubic term parameter, and ϵ_t is is the white noise error term.

Exponential Trend Model

The exponential trend model assumes that the variable changes exponentially over time, meaning the change is relative rather than constant. This allows the model to capture rapid growth or gradual decline in data. This model is expressed as follows:

$$y_t = \beta_0 e^{\beta_1 t} + +\epsilon_t, \qquad \epsilon_t \sim WN(0, \sigma^2) \qquad (4)$$

or in logarithmic form $\ln(y_t) = \ln(\beta_{0)} + \beta_1 t + \epsilon_t,$

where y_t is the variable value at time t , β_0 is a constant, β_1 is the growth rate, and ϵ_t is the white noise error term [14] [15].

Simple Exponential Smoothing Model

Simple Exponential Smoothing model uses the exponential weighting method to predict future values, giving more recent observations greater weight than older ones. It is typically used to predict data that do not show a clear trend or seasonality. This model is expressed as

Smoothing Equation $S_t = \alpha \, y_t + (1 - \alpha) \, S_{t-1}$

Forecasting Equation $\hat{y}_t = y_{t-1}$ (5)

where α is the smoothing parameter $(0 < \alpha < 1)$, S_t is the smoothed value at time t, and S_{t-1} is the smoothed value at time $t-1$, $\hat{y}_t$ is the forecasted value at time t, and y_{t-1} is the variable value at time $t-1$.

Brown's Linear Exponential Smoothing Model

Brown's Linear Exponential Smoothing model uses to forecast date exhibiting a linear trend. This model gives greater weight to recent observations while adding a trend component and it is represented as follows:

$$\text{Single smoothing} \quad \grave{S}_t = \alpha\, y_t + (1-\alpha)\, \grave{S}_{t-1}$$
$$\text{Double smoothing} \quad \grave{\grave{S}}_t = \alpha\, \grave{S}_t + (1-\alpha)\, \grave{\grave{S}}_{t-1}$$
$$\text{Forecasting Equation} \quad \hat{y}_{t+h} = h\, T_t + L_t \qquad (6)$$
$$T_t = \left(\frac{\propto}{1-\propto}\right)(\grave{S}_t - \grave{\grave{S}}_t), \qquad L_t = 2\,\grave{S}_t - \grave{\grave{S}}_t,$$

where $\grave{S}_t$ is the single smoothed value at time t, $\grave{\grave{S}}_t$ is the double smoothed value at time t, y_t is the actual observed value, α is the smoothing parameter $(0 < \alpha < 1)$, L_t is the estimated level, T_t is the estimated trend, h is the forecast horizon, and $\hat{y}_{t+h}$ is the forecast value for period $t+h$.

Holt's Linear Exponential Smoothing Model

Holt's Linear Exponential Smoothing model uses to model trend-driven time series without a seasonal pattern. This model is unique in its ability to estimate both series level and trend independently, making it suitable for time series data exhibiting consistent growth or decline over time. This model is expressed as follows:

$$\text{Level Equation} \quad S_t = \alpha\, y_t + (1-\alpha)(S_{t-1} + b_{t-1})$$
$$\text{Trend Equation} \quad b_t = \beta(S_t - S_{t-1}) + (1-\beta) b_{t-1}$$
$$\text{Forecast Equation} \quad \hat{y}_{t+h} = h\, b_t + S_t \qquad (7)$$

where y_t is the variable value at time t, S_t is the estimated level at time t, b_t is the estimated trend at time t, α and β are smoothing parameters, $\hat{y}_{t+h}$ is the forecast value for period $t+h$, and h is the forecast horizon [16].

Random Walk Model

The stochastic random walk model is a probability model that assumes successive values of a variable change randomly and independently and it is expressed as follows:

$$y_t = y_{t-1} + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma^2) \qquad (8)$$

where $y_t$is the variable value at time t, y_{t-1} is the variable value at time $t-1$, and ϵ_t is the white noise errot term.

Autoregressive Integrated Moving Average ARIMA(*p*,*d*,*q*) Model

The autoregressive integrated moving average model introduced by Box and Jenkins [17], is a widely used in applications. This model combines three components: an autoregressive (AR) term, where the current value depends on its previous values; a differencing (I-integrated) term to remove non-stationarity, and a moving average (MA) term based on previous residuals. The ARIMA models are denoted as ARIMA(*p*,*d*,*q*) and written in the following form:

$$\varphi_p(B)\,(1-B)^d\, y_t = \varphi_0 + \theta_q(B)\epsilon_t\,, \qquad (9)$$

where p is the order of autoregressive (AR) term, d is the order of differencing, q is the order of moving average (MA) term, B is the backshift operator, φ_0 is a constant, $\varphi_p(B) = \left(1 - \varphi_1 B - \varphi_2 B^2 - \cdots - \varphi_p B^p\right)$ and $\theta_q(B) = \left(1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q\right)$ represent the AR(p) and MA(q) models polynomials, and ϵ_t denotes a with noise error term.

Preliminary Statistical Tests

Preliminary statistical tests aim to investigate the basic features of the UR time series data before applying forecasting models. However, these tests include visual inspection, stationarity tests, and autocorrelation analysis.

Visual Inspection of Time Series

Visual inspection is performed by plotting the UR data to identify the overall behavior of the series and detect any temporal trends or seasonal or cyclical fluctuations, thus providing an initial understanding of the nature of the series.

Stationarity Tests

Given the importance of the statonarity condition in ARIMA models, the Augmented Dickey-Fuller (ADF) test [18] is used to verify the stationarity of the series. This test aims to test the null hypothesis that a unit root exists in the series. If the null hypothesis is not rejected, the series is considered non-stationary, and first or second differencing must be performed to confirm stationarity before estimating ARIMA models.

Autocorrelation Analysis

Autocorrelation functions (ACF) and partial autocorrelation functions (PACF) are important tools used to study the time-dependent structure of time series and to help determine the initial orders of AR and MA components (p and q).

Model Estimation

The models used in this study were estimated using appropriate statistical methods for each model. Regression models were estimated using the Ordinary Least Squares (OLS) method, while exponential smoothing models were estimated by selecting optimal smoothing parameters that minimize the sum of squared forecast errors using nonlinear least squares. For ARIMA models, parameters were estimated using the Maximum Likelihood Method (MLM). Moreover, the suitability of each time series model in-sample was assessed using several statistical criteria, including the following:

- Parameter significance tests to ensure that all main parameters of the model are statistically significant.
- Model fit criteria such as the Akaike Information Criterion (AIC) [19], and the Schwarz Information Criterion (SIC) [20], especially for ARIMA models, are used to select the most efficient model that gives the lowest values of these criteria. These criteria are defined as:

$$AIC = -2\,ln(L) + 2k\,, \qquad (10)$$

$$SIC = 2\,ln(L) + k\,ln(T) \qquad (11)$$

where L denotes the likelihood function of the model, k is the estimated parameters number, and T is the sample size.

Residual Examination

Examining residuals is a crucial step in evaluating time series models. Its primary aim is to verify assumptions about the error term, as follows:

Independence: This is a key assumption in time series analysis; the residuals should be time-independent. To confirm this, the visual inspection of ACF is used, along with Ljung-Box test [21].

Homogeneity of variance: The residuals should exhibit homogeneity of variance over time. This can be verified using the ARCH-LM test [22].

Normal distribution: The convergence of the normal distribution of the residuals can be confirmed using the histogram and the Jarque-Bera test [23].

Forecasting Evaluation

The forecasting accuracy of the models used is evaluated using the following measures: the root mean square error (RMSE), the mean absolute error (MAE), and the mean absolute percentage error (MAPE). The model that shows the lowest value of these measures is considered the most accurate and reliable to predict the UR data in the future.

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{y}_t - y_t)^2}, \qquad (12)$$

$$MAE = \frac{1}{T}\sum_{t=1}^{T}|\hat{y}_t - y_t|, \qquad (13)$$

$$MAPE = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{\hat{y}_t - y_t}{\hat{y}_t}\right| \times 100 \qquad (14)$$

where $\hat{y}_t$ is the predictive value, y_t is the actual value, and T is the size of sample.

Results and Discussion

Figure 1 shows the historical time series behavior of the UR in Libya during the period 1991-2024. The series exhibits limited fluctuation a relative stable general level without a clear upward or downward trend in the long term. Furthermore, the annual changes lack regular cyclical patterns, indicating the non-seasonal nature of the time series data.

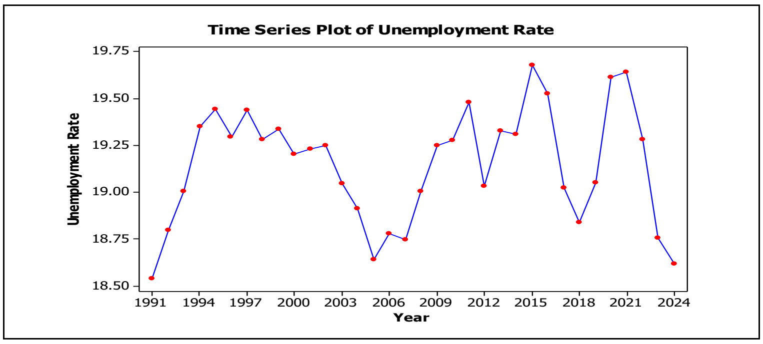

Figure 1. Time Series Plot of Unemployment Rate from 1991 to 2024**.**

Figure 2 illustrate the histogram and descriptive statistics for the UR data. The summary statistics indicate that the UR ranges between a low of 18.53% and a high of 19.67%, with a mean of 19.15%. The low value of standard deviation (0.31) indicates limited dispersion around the mean. On the other hand, the negative skewness coefficient (-0.23) indicates a slight bias towards the lower values, while the kurtosis value (2.15) reflects a distribution closer to normal distribution. The result of the Jarque-Bera test also supports the acceptance of the normality hypothesis (*p*-value > 0.05).

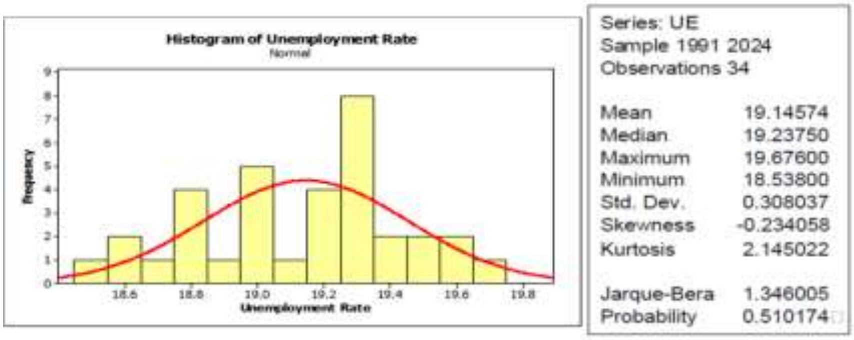

Figure 2. Histogram and Descriptive Statistics of Unemployment Rate.

Table 1 displays the results of the ADF test for the UR time series. The results indicate that the series at the level is non-stationary in most cases, with (*p*-value > 0.05) when both an intercept and trend are included and in the case of none. This implies the presence of a unit root and the series non-stationary over the long-term. However, for the first difference series, the results show partial stability of the series when the intercept and trend are excluded, while stationarity is not achieved when either the trend or the intercept is included (*p*-value > 0.5). This suggests that the first difference does not guarantee the series' complete stationary. Looking at the second difference, all cases show clear stationaruty, with the null hypothesis being rejected at a significance level 1% (*p*-values < 0.01). This confirms that the series becomes completely stationary after the second difference. Based on these results, it is clear that the original series data is non-stationary, and ARIMA models should be estimated after the second difference (*d*=2).

Table 1. Results of Unit Root Test.

Augmented Dickey-Fuller (ADF) Test		**Unemployment Rate Series**		
		At Level	**First Difference**	**Second Difference**
None	Test statistic	-0.4620	-2.0887	-9.4876
	p-value	0.5059	0.0373	0.0000
Intercept	Test statistic	-3.3489	-1.9957	-9.3768
	p-value	0.0220	0.2869	0.0000
Intercept and linear trend	Test statistic	-3.4218	-1.9132	-9.1956
	p-value	0.0687	0.6210	0.0000
The null hypothesis: data has a unit root.				

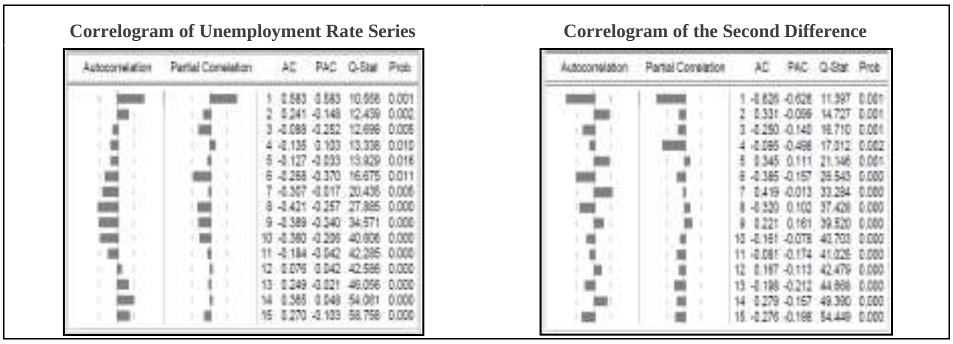

Figure 3. The ACF and PACF of Unemployment Rate and the Second Difference Series.

Figure 3 shows the ACF and PACF plots for the original series and the second difference series of the UR. For the original series, strong autocorrelation was observed at lag 1 (AC=0.583, p-value < 0.01), with weak to moderate correlations persisting in subsequent periods. The PACF showed that the dependence decreased rapidly after lag 1. This pattern reflects instability in the original series and consistent with the ADF test results, which indicated the presence of a unit root. In contrast, the ACF and PACF of the differenced series showed limited short-term dependence with a rapid decline in most periods. Moreover, values at lags 1 and 2 decreased significantly, indicating that the second difference successfully stabilized the series.

Model Estimation and Residual Diagnostics

This section presents the estimation results of the time series models used for modeling the UR in Libya, in order to compare their forecasting performance. The modeling and estimation process included regression models, exponential smoothing models, random walk model, and ARIMA models. Foe each model, the estimated coefficients are presented, followed by a diagnostic tests for residuals.

Table 2. Estimation Results and Diagnostic Tests for Regression Models.

Linear

$$y_t = 19.0636 + 0.0059\,t + \epsilon_t$$

[0.0000] [0.3899]

Ljung-Box $Q(20) = 60.587$ ($p = 0.000,\ p < 0.05$)	Ljung-Box $Q^2(20) = 14.897$ ($p = 0.782,\ p > 0.05$)	Jarque-Bera test = 1.6803 ($p = 0.4316,\ p > 0.05$)	ARCH test= 8.5516 ($p = 0.5751,\ p > 0.05$)

Quadratic

$$y_t = 19.0388 + 0.0116\,t - 0.0002\,t^2 + \epsilon_t$$

[0.0000] [0.6651] [0.8244]

Ljung-Box $Q(20) = 62.244$ ($p = 0.000,\ p < 0.05$)	Ljung-Box $Q^2(20) = 16.831$ ($p = 0.664,\ p > 0.05$)	Jarque-Bera test = 1.5472 ($p = 0.4613,\ p > 0.05$)	ARCH test = 8.7528 ($p = 0.5757,\ p > 0.05$)

Cubic

$$y_t = 18.8831 + 0.0878\,t - 0.0074\,t^2 + 0.0002\,t^3 + \epsilon_t$$

[0.0000] [0.1730] [0.1864] [0.1925]

Ljung-Box $Q(20) = 54.234$ ($p = 0.000,\ p < 0.05$)	Ljung-Box $Q^2(20) = 11.526$ ($p = 0.931,\ p > 0.05$)	Jarque-Bera test = 1.1634 ($p = 0.5589,\ p > 0.05$)	ARCH test = 5.8773 ($p = 0.8255,\ p > 0.05$)

Exponential Trend

$$y_t = e^{2.9477} e^{0.0003\,t} + \epsilon_t, \quad \text{or } \ln(y_t) = 2.947673 + 0.000307t + \epsilon_t$$

[0.0000] [0.3900]

Ljung-Box $Q(20) = 60.918$ ($p = 0.000,\ p < 0.05$)	Ljung-Box $Q^2(20) = 14.928$ ($p = 0.780,\ p > 0.05$)	Jarque-Bera test = 1.7273 ($p = 0.4216,\ p > 0.05$)	ARCH test = 5.8773 ($p = 0.5885,\ p > 0.05$)

Table 2 shows the results of estimating different trend regression models for the UR data, along with diagnostic tests of the residuals of these models. The results of Ljung-Box at lag 20 indicate a significant autocorrelation in the residuals of all estimated models ($p < 0.05$), demonstrating that these models fail to adequately represent the time dependence in the data. In contrast, the results of Ljung-Box at lag 20 for squared residuals and the ARCH tests at lag 10 show no correlation or significant heterogeneity in the variance of the residuals ($p > 0.05$). The results of the Jarque-Bera tests indicate that the normality hypothesis is accepted ($p > 0.05$) in all estimated models. Therefore, although the assumption of normality and homogeneity of variance are satisfied, the trend models remain limited in terms of describing the dynamic structure of time series data, which necessitates dependence on the other models.

Table 3. Estimation Results and Diagnostic Tests for Exponential Smoothing Models.

Simple Exponential Smoothing

$\alpha = 1,\ p = 0.000,\ p < 0.05$

Ljung-Box $Q(20) = 23.175$ ($p = 0.280,\ p > 0.05$)	Ljung-Box $Q^2(20) = 11.792$ ($p = 0.923,\ p > 0.05$)	Jarque-Bera test = 0.9229 ($p = 0.6303,\ p > 0.05$)	ARCH test= 12.425 ($p = 0.260,\ p > 0.05$)

Brown's Linear Exponential Smoothing

$\alpha = 0.648,\ p = 0.000,\ p < 0.05$

Ljung-Box $Q(20) = 62.244$ ($p = 29492,\ p > 0.05$)	Ljung-Box $Q^2(20) = 12.912$ ($p = 0.881,\ p > 0.05$)	Jarque-Bera test = 1.3287 ($p = 0.5146,\ p > 0.05$)	ARCH test = 11.019 ($p = 0.035,\ p > 0.05$)

Holt's Linear Exponential Smoothing

$\alpha = 1,\ p = 0.000,\ p < 0.05\ \ ,\beta = 1.688\text{E} - 6,\ p = 1.000,\ p > 0.05$

Ljung-Box $Q(20) = 23.538$ ($p = 0.263,\ p > 0.05$)	Ljung-Box $Q^2(20) = 11.327$ ($p = 0.937,\ p > 0.05$)	Jarque-Bera test = 0.9389 ($p = 0.6253,\ p > 0.05$)	ARCH test = 12.579 ($p = 0.250,\ p > 0.05$)

The estimation results in Table 3 show that the smoothing parameters in all models were statistically significant at the 1% level. Moreover, the Ljung-Box tests indicate that the models do not exhibit significant serial correlations, either in the residuals or their squares (Ljung-Box tests, $p > 0.05$), suggesting the independenc of residuals and the absence of nonlinear dependenncy. The Jarque-Bera accepted the null hypothesis that the residuals are normally distributed, while the ARCH test provided evidence of no conditional variance (ARCH tests, $p > 0.05$).

For applying ARIMA(p,2,q) models, several ARIMA models with $p \leq 1$ and $q \leq 2$ were estimated. The initial values of p and q were guided by the patterns observed in the ACF and PACF presented in Figure 3. The competing ARIMA specifications were subsequently evaluated using AIC and BIC to identify the most appropriate model. The results of this comparison are reported in Table 4.

Table 4. Comparison of ARIMA Models.

Criteria	ARIMA (1,2,0)	ARIMA (0,2,1)	ARIMA (1,2,1)	ARIMA (0,2,2)	ARIMA (1,2,2)
AIC	0.1882	0.1751	0.2506	0.2513	0.3259
BIC	0.2849	0.2719	0.3957	0.3964	0.5195

The results in Table 4 indicate that the ARIMA(0,2,1) model achieves the lowest values for both the AIC and the BIC compared to all other models. Accordingly, the ARIMA(0,2,1) model was chosen as the preferred ARIMA model. Then, the estimation of parameters and diagnostic tests for both random walk model and ARIMA(0,2,1) model are reported in Table 5.

Table 5. Estimation Results and Diagnostic Tests for Random Walk and ARIMA Models.

Random Walk

$$y_t = \underset{[0.0004]}{y_{t-1}} + \epsilon_t \ ,$$

Ljung-Box $Q(20) = 22.956$ ($p = 0.291,\ p > 0.05$)	Ljung-Box $Q^2(20) = 13.301$ ($p = 0.864,\ p > 0.05$)	Jarque-Bera test = 0.9752 ($p = 0.6141,\ p > 0.05$)	ARCH test = 9.7096 ($p = 0.4663,\ p > 0.05$)

ARIMA(0,2,1)

$$(1 - B)^2\, y_t = (1 - \underset{[0.0003]}{0.8815}\, B)\epsilon_t \qquad \hat{\sigma}^2 = \underset{[0.0064]}{0.0565}$$

Ljung-Box $Q(20) = 20.828$ ($p = 0.346,\ p > 0.05$)	Ljung-Box $Q^2(20) = 14.552$ ($p = 0.801,\ p > 0.05$)	Jarque-Bera test = 0.3159 ($p = 0.8539,\ p > 0.05$)	ARCH test = 9.7157 ($p = 0.4658,\ p > 0.05$)

The results shown in Table 5 indicate that the coefficients of both random walk and ARIMA(0,2,1) models are significant at the 1% level ($p < 0.01$). Moreover, the residuals in both models behave as white noise (Ljung-Box tests, $p > 0.05$), and the Ljung-Box at lag 20 demonstrating the absence of serial correlations and nonlinear depending of the squared residuals. The Jarque-Bera tests indicate that the residuals are nearly normally distributed, while the ARCH tests provided no evidence of conditional heteroscedasticity.

Forecasting Performance Evaluation

To compare the performance of time series models in forecasting the UR, both in-sample and out-of-sample accuracy measures including RMSE, MAE, and MAPE were used, and the results of the comparison are summarize in Table 6.

Table 6. Forecasting Accuracy Measures of Employed Models.

Model	In-Sample			Out-of-Sample		
	RMSE	MAE	MAPE	RMSE	MAE	MAPE
Linear Trend	0.2765	0.2412	1.2632	0.4080	0.3559	1.8680
Quadratic Trend	0.2762	0.2396	1.2547	0.3927	0.3519	1.8419
Cubic Trend	0.2664	0.2236	1.1703	0.8163	0.6059	3.2196
Exponential Trend	0.2765	0.2415	1.2647	0.4078	0.3559	1.8679
Simple Exponential Smoothing	0.2260	0.1809	0.9446	0.3860	0.2700	1.4060
Brown's Exponential Smoothing	0.2460	0.1900	0.9910	0.3520	0. 2470	1.2900
Holt's Exponential Smoothing	0.2300	0.1800	0.9420	0.3740	0.2800	1.4550
Random Walk	0.2258	0.1866	0.9749	0.3643	0.3052	1.5942
ARIMA (0,2,1)	0.2404	0.1919	1.0016	0.3779	0.3140	1.6378

The results of the in-sample and out-of-sample predictive performance measures in Table 6 indicate noticeable variation in predictive performance across the considered models. However, in-sample performance, the random walk and the simple exponential smoothing achieved the lowest values for RMSE, MAE, and MAPE, reflecting a high ability to fit historical data. The Holt model and ARIMA model also performed very closely, while the linear, quadratic and exponential trend models relatively weaker. Moreover the cubic model achieved the lowest sample error among the trend regression models, this suggests a potential for overfitting. In contrast, exponential smoothing models demonstrate superior out-of-sample predictive accuracy. Specifically, the Brown's exponential smoothing achieves the lowest RMSE, MAE, and MAPE values among all competing models, highlighting its effectiveness in capturing the underlying trend dynamics of the data. Therefore, based on the criteria for out-of-sample forecasting accuracy, Brown's linear exponential smoothing model was chosen as the most suitable prediction model for the dynamics of the UR in Libya, outperforming trend regression models, random walk model, and ARIMA model. Figure 4 shows the residuals plots of Brown's linear exponential smoothing model.

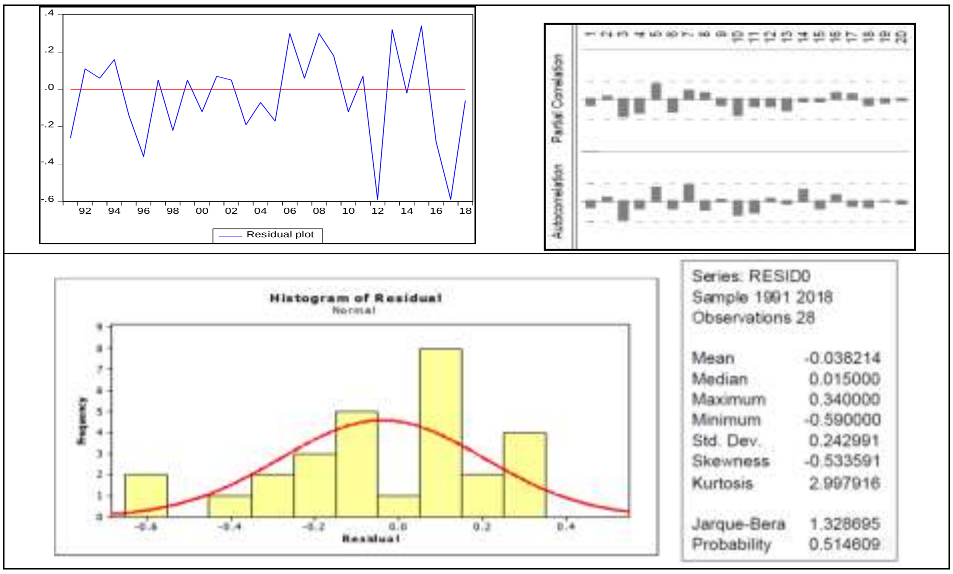

Figure 4. Residual plots for Brown's Linear Exponential Smoothing Model.

Based on the forecasting performance, Brown's exponential smoothing model was employed to produce five-year-ahead forecasts (2025-2029) for the UR series, and the results are reported in Table 7 and Figure 5.

Table 7. Five-Year Ahead Forecasts of the Unemployment Rate using Brown's Model.

Year	Forecasted value	95% Forecast Interval	
		Lower	Upper
2025	18.34	17.74	18.94
2026	18.09	17.05	19.13
2027	17.83	16.28	19.38
2028	17.57	15.44	19.70
2029	17.31	14.55	20.07

The results of point forecasts in Table 7 indicate a moderate downward trend, with the forecasted unemployment rate declining from 18.34% in 2025 to 17.31 in 2029, suggesting a gradual improvement in labor market conditions. Moreover, the 95% confidence interval associated with the forecast widen from (17.74%, 18.94%) in 2025 to (14.55%, 20.07%) in 2029.

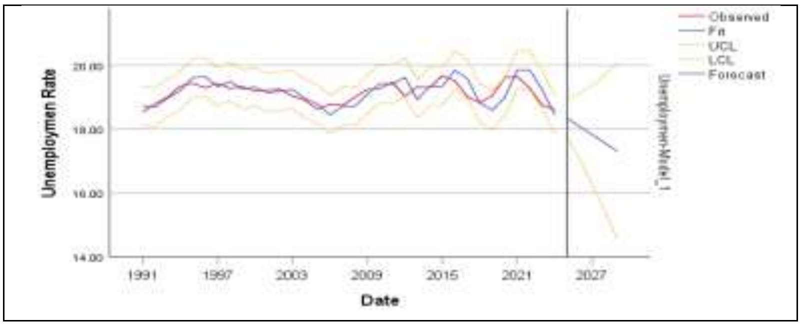

Figure 5. Five-Year Ahead Forecasts of the Unemployment Rate using Brown's Model.

Conclusion

This paper investigated the modeling and forecasting of the unemployment rate in Libya over the period 1991-2024 through a comparative analysis of several time series models. The findings indicate that traditional trend regression models, despite their simplicity, suffer from the lack of residual independence, which limits their suitability foe reliable forecasting. In contrast, exponential smoothing models demonstrated strong statistical adequacy by satisfying all residual diagnostic test and achieving superior forecasting performance, particularly, in the out-of-sample evaluation. The ARIMA (0,2,1) model also provided an acceptable statistical fit, however, its predictive accuracy was lower to that of some exponential smoothing models. Based on an assessment of the accuracy of out-of-sample forecasting, Brown's exponential smoothing model was selected as the optimal model for predicting the unemployment rate. The findings of the future forecasts from 2025 to 2029 with 95% confidence intervals indicate a moderate downward trend in the unemployment rate during this period suggesting a gradual improvement in labor market conditions. The study recommends adopting exponential smoothing models in statistical applications related to relatively short time series with regular trend, and suggests expanding future work to include more complex time series models or higher-frequency data.

Compliance with ethical standards

Disclosure of conflict of interest

The author(s) declare that they have no conflict of interest.

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