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Solving Linear Ordinary Differential Equations with Constant Coefficients via Inverse Laplace Transform and Residue Theorem

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حل المعادلات التفاضلية العادية الخطية ذات المعاملات الثابتة باستخدام تحويل لابلاس العكسي ونظرية الرواسب

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Abstract:
This research paper discusses the advanced mathematical framework of the Residue Theorem within complex analysis, combined with the Laplace Transform, as a precise analytical methodology for solving linear ordinary differential equations with constant coefficients. The research aims to address the challenges associated with finding the inverse Laplace transform for complex systems with high-order or multivalued poles in the complex plane. The methodology relies on transferring the differential equation from the time domain t to the complex frequency domain s to convert it into a purely algebraic equation, and then utilizing the Bromwich Integral and the Residue Theorem to accurately compute the inverse Laplace transform and achieve closed-form solutions. The study demonstrates that integrating the Residue Theorem with the Laplace Transform not only simplifies mathematical solutions but also provides a crucial tool for understanding the stability and dynamic behavior of engineering and physical systems under various initial conditions.

Keywords: Linear Ordinary Differential Equation, Inverse Laplace Transform, Residue Theorem, Constant Coefficients, Bromwich Integral.

الملخص
تناقش هذه الورقة البحثية الإطار الرياضي المتقدم لـ "مبرهنة الرواسب" في مجال التحليل المركب، مقترناً بـ "تحويل لابلاس"، وذلك باعتبارهما منهجية تحليلية دقيقة لحل المعادلات التفاضلية العادية الخطية ذات المعاملات الثابتة. ويهدف البحث إلى معالجة التحديات المرتبطة بإيجاد تحويل لا بلاس العكسي للأنظمة المعقدة التي تحتوي على أقطاب ذات رتبة عالية أو أقطاب متعددة القيم في المستوى المركب. تعتمد المنهجية على نقل المعادلة التفاضلية من نطاق الزمن(t) إلى نطاق التردد المركب (s) لتحويلها إلى معادلة جبرية بحته، ومن ثم استخدام تكامل "بروميتش" ونظرية الرواسب لحساب تحويل لابلاس العكسي بدقة والتوصل إلى حلول بصيغة مغلقة. تُظهر الدراسة أن دمج "مبرهنة الرواسب" مع "تحويل لابلاس" لا يقتصر على تبسيط الحلول الرياضية فحسب، بل يوفر أيضاً أداةً جوهريةً لفهم الاستقرار والسلوك الديناميكي للأنظمة الهندسية والفيزيائية في ظل ظروفٍ أوليةٍ متنوعة.

الكلمات المفتاحية: المعادلات التفاضلية الخطية العادية، تحويل لابلاس العكسي، نظرية الرواسب، المعاملات الثابتة، تكامل بروميتش.

Introduction

Linear differential equations with constant coefficients serve as the fundamental backbone for modeling a wide range of physical, engineering, and economic phenomena, such as mechanical vibration systems, transient electrical circuits, power transmission networks, and automatic control systems-frameworks rigorously established from a theoretical standpoint by Deutsch (1974) [1]. Despite the efficiency of traditional methods in solving these equations (such as the characteristic roots method or the method of undetermined coefficients, as discussed by Edwards et al, 2017) [2], the complexity of external excitations or non-zero initial conditions necessitates broader and more effective mathematical tools.

The Laplace transform is a crucial transformative tool, as it simplifies complex differential and integral operations in the time domain in to easily manageable algebraic equations in the complex variable domain. However, the primary obstacle frequently encountered by researchers is the inversion phase-returning from the Laplace domain to the original time domain via the inverse Laplace transform. This challenge is particularly prominent when the algebraic solution yields fractional functions whose denominators contain complex roots or high-order poles that are difficult to resolve using traditional algebraic methods, such as partial fractions. To address these complex computational obstacles in the inversion process, advanced techniques relying on residue calculation have been increasingly adopted to provide a direct and efficient framework for obtaining exact solutions (Dobrushkin, 2019 [3]; Sheng et al., 2011) [4]. Recent studies, such as those by Saltas et al. (2023) [5] and Kerr et al. (2023) [6], further highlight the necessity of combining the Laplace transform with residue theory to successfully extract analytical solutions for complex differential equations characterized by high-order poles or delay-induced poles.

Stemming from this synthesis, the profound mathematical significance of residue theory in complex analysis comes to the fore, where recent theoretical developments (such as the contributions of Federico, 2020) [7] have demonstrated that the general form of the residue theorem can be derived through direct integration of the Taylor series without requiring the assumption of bounding circles or the reduction of radii to zero, there by lending greater mathematical rigor to the theorem. By applying the Bromwich integral—the contour integral in the complex plane along a closed path enclosing all poles—the residue theorem enables the systematic calculation of the inverse Laplace transform by computing the "residues" of the function at its singular poles. Accordingly, this paper aims to formulate and elucidate this cognitive integration between residue theory and the Laplace transform to provide accurate and direct solutions for linear differential equations with constant coefficients.

Mathematical Background and Definitions

1. The Laplace Transform

The unilateral Laplace transform maps a time –domain function $f(t), t \geq 0$ in to a complex frequency-domain function $F(s), (s = \alpha + i\beta)$ via the improper integral:

$$F(s) = \int_0^{\infty} f(t)\, e^{-st} dt = \ell\{f(t)\} \qquad \dots\dots\dots \qquad (1)$$

Provided the integral converges for $\mathrm{Re}(s)$ greater than the abscissa of convergence .

Some Important Properties of Laplace Transform:

■ **Linearity**: For any function $f(t)$ and $g(t)$ whose Laplace transforms exist, and any constants c_1 and c_2 :

$$\ell\{c_1 f(t) + c_2 g(t)\} = c_1 \ell\{f(t)\} + c_2 \ell\{g(t)\} \qquad \dots\dots\dots \qquad (2)$$

■ **Transforms of Derivatives**: Let $f, \acute{f}, \dots, f^{(n-1)}$ be continuous for all $t \geq 0$ and let $f^{(n)}$ be piecewise continuous on every finite interval on the semi –axis $t \geq 0$. Then the transform of $f^{(n)}$ is given by:

$$\ell\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} \acute{f}(0) - \cdots - f^{(n-1)}(0) \dots\dots (3)$$

■ **Laplace Transformation of Some Elementary Function**:

1) $\ell\{1\} = \dfrac{1}{S}$, $s > 0$

2) $\ell\{t^n\} = \dfrac{n!}{S^{n+1}}$ were $n = 0,1,2,3, \dots$

3) $\ell\{e^{at}\} = \dfrac{1}{s-a}$, $s > a$

4) $\ell\{\sin at\} = \frac{a}{s^2+a^2}$, $s > 0$

5) $\ell\{\cos at\} = \frac{s}{s^2+a^2}$, $s > 0$

■ **Existence Theorem** : According to this theorem $\int_0^{\infty} e^{-st} f(t)\,dt$ exists if $\int_0^{\lambda} e^{-st} f(t)\,dt$ Can actually be evaluated and its limit as $\lambda \to \infty$ exists. Otherwise we may use the following theorem :

If $f(t)$ is continuous and $\lim_{t\to\infty}[e^{-at} f(t)]$ is finite ,then Laplace transform of $f(t)$ i.e. $\int_0^{\infty} e^{-st} f(t)\,dt$ exists for $s > a$.

It should however ,be kept in that the above foresaid conditions are sufficient but not necessary.

■ **Inverse Laplace Transformation** : The inverse Laplace transform of $F(t)$ is $f(t)$ Which is defined by

$$\ell^{-1}\{F(s)\} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{st}\,F(s)ds = f(t)\,,\quad t > 0 \;\ldots\ldots \qquad (4)$$

2-Fundamentals of Complex Analysis

■ **Open Sets:** Let $D \subset C$ be open (every point in D has a small disc around it which still is in D). [8]

■**Differentiability** :The differentiable functions $D \to C$ is Denoted by C'(D).This mean that for $f(z) = f(x+iy) = u(x+iy) + iv(x+iy)$, the partial derivatives $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$. Are continuous, real-valued functions on D.

■**Complex Line Integral**: Let $\gamma:(a,b) \to D$ be a differentiable curve in D. Define the complex line integral:

$$\oint_{\gamma} f(z)dz = \int_a^b f(\gamma(t), \acute{\gamma}(t))dt \ldots\ldots \qquad (5)$$

The integral for piecewise differentiable curves γ is obtained by adding the integrals of the piece. We always assume from now on that all curve γ are piecewise differentiable.

Example: let $D = \{|z| < r\}$ with $r > 1$, $\gamma:[0,2\pi k] \to D$, $\gamma(t) = (\cos(t), \sin(t))$, $f(z) = z^n$ with $k \in N$, $n \in z$.

$$\int_{\gamma} z^n dz = \int_0^{2\pi k} e^{int}\,.\,e^{it} i dt$$

If $n \neq -1$, we get $\int_{\gamma} z^n dz = 0$. If $n = -1$, we have

$$\int_{\gamma} z^n dz = \int_0^{2\pi k} i dt = 2\pi k i$$

■**Taylor Series and Laurent Series:** The Taylor series is the function

that represents the sum of infinite terms expressed by a function's derivatives at a single point. The Laurent series, which includes negative power words, has some similarities to it.[9]

The general equation of the Taylor Series:

$$f(z) = \sum_{n=0}^{\infty} \frac{f^n(x_0)}{n!}(x - x_0)^n \ldots\ldots \qquad (6)$$

The general equation of the Laurent series:

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n + \sum_{n=0}^{\infty} \frac{b_n}{(z - z_0)^n} \ldots \ldots \ldots \qquad (7)$$

■ **Analytic Functions**: The class of analytic functions is formed by the complex functions variable which possess a derivative wherever the function is defined. The term holomorphic function is used with identical meaning.[10]

■ **Singularities**: The singularities simply mean the points where $f(x)$ is not analytic in the complex plane.

■ **Residue**: The residue of $f(z)$ at an isolated singularity a is unique complex number R which makes $f(z) - \frac{R}{z-a}$, the derivative of a single-valued analytic function in an annulus $0 < |z - a| < \delta$.

3.Classification of Singularities and Methods for Calculating Residues in Complex Analysis:

■**First: Removable Singularities:** A removable singularity is a point z_0 where the function $f(z_0)$ appears to be undefined but with the knowledge that $\lim_{z \to z_0} f(z) = w_0$ and the author assigns $f(z_0)$ the value w_0.Due to the regular behavior of the function after removing the singularity and the absence of negative powers in its corresponding Laurent series, the residue at a removable singularity is always zero $(Res\,(f, z_0)) = 0$.

For example: the point $z = 0$ would be a singularity of the function $f(z) = \frac{\sin(z)}{z}$.

■**Second: Isolated Singularities and Poles:** An isolated singularity is a point $z_0$where the function is differentiable on the simple closed contour $0 < |z - z_0| < r$. However, this function is undefined at the point $z = z_0$, always say isolated singularities poles. In other words, $f(z)$ has a singularity at z_0 that is not a limit point of other singularities of $f(z)$. Particularly , the residue of poles can be expressed in two different ways.

If z_0 is a first-order pole, the residue could be expressed as:

$$\mathrm{Res}\,[f(z), z_0] = \lim_{z \to z_0} (z - z_0) f(z) \ldots \ldots \ldots \qquad (8)$$

If z_0 is a m-th order pole, the residue could be expressed as :

$$\mathrm{Res}\,[f(z), z_0] = \frac{1}{(n-1)!} \lim_{z \to z_0} \frac{d^{n-1}}{dz^{n-1}} [(z - z_0) f(z)] \ldots \ldots \ldots \qquad (9)$$

■ **For example**: the point $z = i$ is the isolated singularity of the function $f(z) = \frac{z}{z-i}$.

■ **Third: Branch Singularities:** A branch singularity is a point z_0 through which all possible branch cuts of a multi-valued function can be drawn to produce a single-valued function. **An example** of such a point would be the point $z = 0$ for $f(z) = \log(z)$.[11]

■ **Fourth: Essential Singularities:** There also a special kind of isolated singularity, called an essential singularity. The canonical **example** of an essential singularity is $z = 0$ for the function $f(z) = e^{\frac{1}{z}}$.

The easiest way to define an essential singularity of a function involve Laurent Series.[12]

4. The Residue Theorem and Inverse Laplace Transform

■ **The Residue Theorem**: Cauchy's residue theorem is developed by Cauchy's integral formula

$$f(z_0) = \frac{1}{2\pi i} \oint_c \frac{f(z)}{(z - z_0)} dz \ldots \ldots \ldots \qquad (10)$$

Then the Cauchy residue theorem claims that the integral of the function f around C is given by the sum of the residues of f at the singular points inside C times $2\pi i$. Namely ,

$$\int_c f(z)\,dz = 2\pi i \sum \operatorname{Res}[f(z), z = z_k] \ldots\ldots\ldots \qquad (11)$$

■ **Inverse Laplace Transform via Residue Theorem:** [13]

If complex frequency domain function $F(s)$ has isolated singularities (poles) at points s_k $(\text{for } k = 1,2,3 \cdots, n)$ then the inverse Laplace transform $f(t)$ can be evaluated as the sum of $F(s)$ at all its poles :

$$f(t) = \sum_{k=1}^{n} \operatorname{Res}(e^{st}F(s), s_k), \qquad \text{for } t > 0 \qquad \ldots\ldots.. \qquad (12)$$

5.Solving the linear Differential Equations

■ **Linear Differential Equation:** An nth order linear Differential Equation is an Equation of the form:

$$a_0(t)\frac{d^n y}{dt^n} + a_1(t)\frac{d^{n-1}y}{dt^{n-1}} + \cdots + a_{n-1}(t)\frac{dy}{dt} + a_n(t)y \;= f(t) \ldots\ldots\ldots \;(13)$$

We assume that the function $a_0, \ldots, a_n$ and are continuous real-Valued functions on some interval I , and that a_0 is nowhere zero in this interval.

■ **The Initial Condition:** To determine a unique solution, it is necessary to specify n initial conditions,

$$y(t_0) = y_0 , \qquad y'(t_0) = y'_0 , \qquad \ldots, y^{(n-1)}(t_0) = y_0^{(n-1)} \ldots\ldots\ldots \quad (14)$$

Where t_0 may be any point in the interval I and y_0 , y'_0 , $\ldots, y_0^{(n-1)}$ is any set of prescribed real constants.

The three does exist a solution of the initial value problem(13), (14) and that it is unique are guaranteed by the following existence and uniqueness theorem.

■ **Existence and Uniqueness Theorem**:

If the functions $a_0, \ldots, a_n$, and G are continuous on the open interval I , then there exists exactly one solution $y = f(t)$ of the differential (13) that also satisfies the initial conditions (14). This solution exists throughout the interval .

■ **Existence and Uniqueness Theorem Solution of Linear Differential Equations Using Laplace Transform and Residue Theorem:**

Applying the Laplace transform to both sides of the equation (13) and using the linearity property (2), we obtain

$$a_0(t)\ell\left\{\frac{d^n y}{dt^n}\right\} + a_1(t)\ell\left\{\frac{d^{n-1}y}{dt^{n-1}}\right\} + \cdots + a_{n-1}(t)\ell\left\{\frac{dy}{dt}\right\} + a_n(t)\ell\{y\} = \ell\{f(t)\}.$$

Substituting the transforms of derivatives(3) and initial conditions (14), we find:

$$K(s)\,Y(s) - Q(s) \;= F(s)$$

where $k(s) = a_n s^n + \cdots + a_1 s + a_0$

is the characteristic polynomial obtained from the derivatives,

and $Q(s)$ is a polynomial determined by the initial conditions.

Thus, we have

$$Y(s) = \frac{F(s) + Q(s)}{K(s)} \quad \ldots\ldots\ldots \qquad (15)$$

By applying the Residue Theorem, we get

$$Y(t) = \sum_{k=1}^{m} Res(Y(s)\,e^{st}, s_k) \;\dots\dots\dots \qquad (16)$$

where $s_1, s_2, \dots, s_m$ are the poles of the function lying inside the integration contour C.

6.Illustrative Examples

Example 1 : Solve the equation

$\acute{y} + y = cost + sint\ , \qquad y(0) = 0$

Solution:

Taking Laplace Transformation on both sides

$\ell\{\acute{y}\} + \ell\{y\} = \ell\{cost\} + \ell\{sint\}$

$$\left(S\,Y(s) - y(0)\right) + Y(s) = \frac{s}{s^2+1} + \frac{1}{s^2+1}$$

Using initial condition

$$S\,Y(s) + Y(s) = \frac{s}{s^2+1} + \frac{1}{s^2+1}$$

$$Y(s)\,(s+1) = \frac{s+1}{s^2+1}$$

Then

$$Y(s) = \frac{1}{s^2+1}$$

Using the Residue Theorem, we get

$$Y(t) = \sum Res\left(\frac{1}{s^2+1}e^{st}, s_k\right) \dots\dots\dots \qquad (17)$$

There are Two simple poles at $s_1 = i\ ,\ s_2 = -i$

$$Res\left(\frac{1}{s^2+1}e^{st}, i\right) = \lim_{s\to i}(s-i)\frac{1}{(s-i)(s+i)}e^{st} = \frac{e^{it}}{2i}$$

And

$$Res\left(\frac{1}{s^2+1}e^{st}, -i\right) = \lim_{s\to -i}(s+i)\frac{1}{(s-i)(s+i)}e^{st} = \frac{e^{-it}}{-2i}$$

Hence the final solution is given by:

$$Y(t) = \tfrac{1}{2i}\left[e^{it} - e^{-it}\right] = sint$$

Example 2: Solve

$\acute{\acute{y}}(t) + 3\acute{y}(t) + 2y(t) = 0\ \ with\ condition\ , \acute{y}(0) = 0\ , y(0) = 1$

Solution:

Taking Laplace Transformation on both sides

$\ell\{\acute{\acute{y}}(t)\} + 3\ell\{\acute{y}(t)\} + 2\ell\{y(t)\} = \ell\{0\}$

$$s^2y(s) - sy(0) - \acute{y}(0) + 3[sy(s) - y(0)] + 2y(s) = 0$$

Using initial condition

$$s^2y(s) - s + 3sy(s) + 3 + 2y(s) = 0$$

$$y(s) = \frac{s+3}{(s+1)(s+2)}$$

Using the Residue Theorem, we get

$$Y(t) = \sum_{k=1}^{m} Res\left(\frac{s+3}{(s+1)(s+2)}e^{st}, s_k\right) \quad \ldots\ldots\ldots \quad (18)$$

There are two simple poles $s_1 = -1$, $s_2 = -2$

$$Res\left(\frac{s+3}{(s+1)(s+2)}e^{st}, -1\right) = \lim_{s\to -1}(s+1)\frac{s+3}{(s+1)(s+2)}e^{-t} = 2e^{-t}$$

And

$$Res\left(\frac{s+3}{(s+1)(s+2)}e^{st}, -2\right) = \lim_{s\to -2}(s+2)\frac{s+3}{(s+1)(s+2)}e^{-2t} = e^{-2t}$$

Hence the final solution is given by :

$$Y(t) = 2e^{-t} + e^{-2t}$$

Example 3: Find the Solution of the equation

$$\acute{\acute{y}}(t) + 2\acute{y}(t) + y(t) = e^{-t}$$

With the initial conditions $\acute{y}(0) = 1, y(0) = 0$

Solution:

Taking Laplace Transformation on both sides

$$\ell\{\acute{\acute{y}}(t)\} + 2\ell\{\acute{y}(t)\} + \ell\{y(t)\} = \ell\{0\}$$

$$s^2y(s) - sy(0) - \acute{y}(0) + 2[sy(s) - y(0)] + y(s) = \frac{1}{s-1}$$

By applying the initial condition , we get

$$s^2y(s) - 1 + 2sy(s) + y(s) = \frac{1}{s-1}$$

$$y(s) = \frac{s+2}{(s+1)^3}$$

Using the Residue Theorem, we get

$$Y(t) = \sum_{k=1}^{m} Res\left(\frac{s+2}{(s+1)^3}e^{st}, s_k\right) \cdots\cdots\cdots \quad (19)$$

There are poles of order 3 at $s = -1$

$$Res\left(\frac{s+2}{(s+1)^3}e^{-t}, -1\right) = \frac{1}{(3-1)!}\lim_{s\to -1}\frac{d^2}{ds^2}(s+1)^3\frac{s+2}{(s+1)^3}e^{-t} = \frac{1}{2}(t^2+2t)e^{-t}$$

Hence the final solution is given by :

$$Y(t) = \left(\frac{t^2}{2} + t\right) e^{-t}$$

Example 4: Consider the initial value problem

$$\acute{\acute{y}}(t) + 4y(t) = \cos t$$

With the initial conditions $\acute{y}(0) = y(0) = 0$

Solution:

Taking Laplace Transformation on both sides

$$\ell\{\acute{\acute{y}}(t)\} + 4\ell\{y(t)\} = \ell\{\cos t\}$$

$$s^2 y(s) - sy(0) - \acute{y}(0) + 4y(s) = \frac{s}{s^2+1}$$

Using initial condition

$$s^2 y(s) + 4y(s) = \frac{s}{s^2+1}$$

$$y(s) = \frac{s}{(s^2+1)(s^2+4)}$$

$$y(s) = \frac{s}{(s+i)(s-i)(s+2i)(s-2i)}$$

Using the Residue Theorem, we get

$$Y(t) = \sum_{k=1}^{m} Res\left(\frac{s}{(s+i)(s-i)(s+2i)(s-2i)} e^{st}, s_k\right) \cdots\cdots \qquad (20)$$

There are 4 simple poles at $s = \pm i$, $s = \pm 2i$

Calculating the residue at the simple poles $s = \pm i$

$$2Res\left(\frac{s}{(s+i)(s-i)(s+2i)(s-2i)} e^{it}, i\right) = 2\lim_{s \to i}(s-i)\frac{s}{(s+i)(s-i)(s+2i)(s-2i)} e^{it}$$

$$= \frac{1}{3} e^{it}$$

And calculating the residue at the simple poles $s = \pm 2i$

$$2Res\left(\frac{s}{(s+i)(s-i)(s+2i)(s-2i)} e^{2it}, 2i\right) = 2\lim_{s \to 2i}(s-2i)\frac{s}{(s+i)(s-i)(s+2i)(s-2i)} e^{2it} = -\frac{1}{3} e^{2it}$$

Hence the final solution is given by :

$$Y(t) = \frac{1}{3} e^{it} - \frac{1}{3} e^{2it}$$

$$Y(t) = \frac{1}{3}(\cos t - \cos 2t)$$

7.Conclusion:

The integration of the applications of the Residue Theorem with the Laplace Transform represents a qualitative leap in treating linear differential equations with constant coefficients, particularly for complex systems that classical methods fail to solve efficiently. This methodology empowers researchers to handle system poles and irregular functions with utmost flexibility, thereby contributing precise and stable insights into the behavior of dynamic systems across applied engineering and physics.

Compliance with ethical standards

Disclosure of conflict of interest

The authors declare that they have no conflict of interest.

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