

Design Provisions and Structural Behaviour of Reinforced Concrete Deep Beams

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إشتراطات التصميم والسلوك الإنشائي للكمرات الخرسانية المسلحة العميقة

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Abstract:

Reinforced concrete (RC) remains a fundamental material in structural engineering due to its strength, durability, and versatility in diverse applications. Design provisions for RC elements have evolved significantly, particularly in enhancing the reliability of static load-bearing performance. This paper investigates the application of the Strut-and-Tie Model (STM) as outlined in major design codes from Canada, the United States, and Europe for reinforced concrete deep beams. The study analyzes over 300 experimental test results to assess the accuracy of these codes in predicting ultimate strength and failure modes. The results indicate that while the Canadian code generally provides more conservative strength estimates, none of the codes reliably predict the actual failure modes or their locations. In addition, the study examines the influence of parameters such as shear span-to-depth ratio, web reinforcement, and concrete compressive strength on the predicted capacities using STM-based approaches.

Keywords: Deep beam; shear span; FRP reinforcement; strut-and-tie model, Abaqus software, Finite element model.

الملخص

لا تزال الخرسانة المسلحة مادة أساسية في الهندسة الإنشائية نظراً لقوتها ومتانتها وتعدد استخداماتها في تطبيقات متنوعة. وقد شهدت أنظمة تصميم عناصر الخرسانة المسلحة تطوراً ملحوظاً، لا سيما في تعزيز موثوقية أدائها في تحمل الأحمال الساكنة. تبحث هذه الورقة تطبيق نموذج الدعامات والروابط (STM) كما هو موضح في أكواد التصميم الرئيسية في كندا والولايات المتحدة وأوروبا للجسور العميقة الخرسانية المسلحة. تحلل الدراسة أكثر من 300 نتيجة اختبار تجريبي لتقييم دقة هذه الأكواد في التنبؤ بالقوة القصوى وأنماط الانهيار. تشير النتائج إلى أنه في حين أن الكود الكندي يوفر عمومًا تقديرات أكثر تحفظاً للقوة، إلا أن أيًا من الأكواد لا يتنبأ بدقة بأنماط الانهيار الفعلية أو مواقعها. بالإضافة إلى ذلك، تدرس الدراسة تأثير معايير مثل نسبة طول القص إلى العمق، وتسليح الجدار، ومقاومة الخرسانة للضغط على القدرات المتوقعة باستخدام نهج تصميم يعتمد على نموذج الدعامات والروابط.

الكلمات المفتاحية: الكمرة العميقة؛ مدى القص؛ تقوية بألياف البوليمر المقوى؛ نموذج الدعامات والوصلات، برنامج أباقوس، نموذج العناصر المحدودة.

Introduction

The structural behavior of deep beams is influenced by factors such as the span-to-depth ratio, type of loading, reinforcement ratios (both vertical and horizontal), concrete strength, and cross-sectional shape. Due to the nonlinear strain distribution across the depth of the cross section, simple beam theory cannot reliably estimate the capacity of deep beams. To address this, the Strut-and-Tie Model (STM) has been developed and integrated into various modern design codes and standards for more effective deep beam design. The STM, supported by numerous experimental studies, provides a rational and widely accepted design approach. Michael et al. [1] noted that the STM was first introduced in the Canadian Code [2] in the 1980s, followed by its inclusion in the Eurocode [3] for deep beam design. The STM was later incorporated into the American Concrete Institute Code ACI 318 [4] towards the end of the 20th century. However, the implementation of STM varies across these codes. This research aims to compare the provisions of these codes in applying the STM for deep beam design and to assess their accuracy in estimating deep beam capacity based on experimental findings.

This paper examines and compares the design provisions for concrete deep beams outlined in three major design codes for reinforced concrete, all of which utilize the Strut-and-Tie Model (STM). The study evaluates the effectiveness of STM provisions in predicting the strength and failure modes of concrete deep beams, drawing on data from numerous experimental studies. Key factors influencing the capacities predicted by the STM, such as shear span-to-depth ratio, web reinforcement, and concrete compressive strength, were also analyzed. The results suggest that modifying the efficiency factor of bottle-shaped struts as a function of the shear span-to-depth ratio offers a conservative and practical design approach for reinforced concrete deep beams. These findings are anticipated to be valuable for both design professionals and code developers.

Reinforced concrete (RC) is among the most widely used construction materials, valued for its versatility, durability, and structural efficiency. To ensure safety, functionality, and cost-effectiveness, it is essential to understand its design principles and behavior under different loading conditions. This review explores the development of RC design codes, the performance of RC under various stress conditions, and recent advancements in RC technology. Over the last century, RC design codes have undergone substantial evolution. Contemporary standards, including the American Concrete Institute (ACI 318), Eurocode 2, and the Canadian Standards Association (CSA A23.3), offer detailed guidelines for designing RC components across a range of loading conditions. These codes are founded on a combination of theoretical models, empirical observations, and experimental findings to ensure the structural safety and reliability of RC designs.

Most modern reinforced concrete (RC) design codes utilize the limit state design (LSD) methodology, which ensures structures meet performance standards under both ultimate and serviceability conditions. The ultimate limit state (ULS) aims to prevent structural failure, while the serviceability limit state (SLS) addresses factors like deflection, cracking, and vibration control. For instance, ACI 318 uses a load factor approach to account for uncertainties in loads and material properties, providing adequate safety margins. Conversely, Eurocode 2 employs partial safety factors for materials and load combinations, reflecting regional variations in design philosophy. Contemporary design standards also prioritize durability to enhance the service life of RC structures. Specifications for minimum reinforcement cover, crack width limits, and concrete quality address challenges such as corrosion, freeze-thaw damage, and chemical degradation. For example, CSA A23.3 includes durability classifications tailored to Canada's varied environmental conditions.

The behavior of reinforced concrete (RC) under different loading conditions—such as tension, compression, shear, and bending—is shaped by its composite nature. The interaction between concrete and steel reinforcement allows the material to combine the strengths of both components, resulting in a durable and ductile structural system. The flexural performance of RC beams depends on the proportions of concrete and reinforcement. Under bending, steel resists tensile forces while concrete withstands compressive forces. Failure modes can vary from tension-controlled (ductile) to compression-controlled (brittle), depending on the reinforcement ratio. Experimental research, including studies by Park and Paulay (1975), has shown that well-designed RC beams exhibit substantial ductility, enabling stress redistribution before failure. This ductile behavior is crucial in the design of continuous beams and frames, where moment redistribution plays a significant role.

Shear failure in reinforced concrete (RC) elements tends to be more brittle and less predictable compared to flexural failure. The inclusion of shear reinforcement, such as stirrups, enhances shear strength and helps prevent sudden, catastrophic failures. Research by Kani (1964) and others has emphasized the complex mechanisms involved in resisting shear forces, including diagonal tension, aggregate interlock, and dowel action. RC structures are also affected by long-term phenomena such as creep and shrinkage, which can result in increased deflections and cracking over time, compromising both serviceability and durability. To address these issues, design codes incorporate provisions that account for the time-dependent deformation behavior of concrete. Recent advancements in reinforced concrete (RC) technology aim to improve performance, sustainability, and ease of construction. These innovations include:

- High-Performance Concrete (HPC): Known for its exceptional strength, durability, and workability, HPC is well-suited for challenging applications like high-rise buildings and bridges.

- Fiber-Reinforced Concrete (FRC): The addition of fibers enhances crack resistance, toughness, and impact resistance, improving overall durability.
- Self-Consolidating Concrete (SCC): SCC flows freely under its own weight, eliminating the need for vibration during placement, thereby increasing efficiency and ensuring a higher-quality finish.
- Reinforcement Innovations: The use of materials like fiber-reinforced polymers (FRPs) offer advantages such as corrosion resistance and lighter weight compared to traditional steel reinforcement.
- Sustainability Measures: Using recycled materials, such as fly ash and slag, in concrete mixtures helps minimize the environmental impact of RC structures. The design principles and behavior of reinforced concrete play a vital role in its extensive use in modern construction. Ongoing advancements in design methodologies and material technologies are continuously enhancing the performance and sustainability of RC structures. Future research should prioritize improving analytical models, advancing material properties, and developing more efficient construction techniques to meet the increasing demands of the construction industry.

This paper provides a comprehensive review of the design provisions governing RC structures, examining their evolution, key principles, and effectiveness in ensuring structural integrity. It also explores the behavior of RC elements under various conditions, analyzing factors such as material properties, load resistance, failure mechanisms, and durability considerations. Furthermore, this study highlights gaps in current design approaches and areas requiring further research to enhance the safety, efficiency, and sustainability of RC structures. By synthesizing findings from existing literature and design standards, this review aims to provide valuable insights for researchers, engineers, and policymakers, contributing to the ongoing development of reinforced concrete design and construction practices.

Methodology

2.1. Overview of the Design Provisions for Concrete Deep Beams

The definition of reinforced concrete deep beams varies across design codes. In ACI 318-08 [5], a deep beam is described as a beam loaded on one face and supported on the opposite face, with a clear span (l_n) that is less than or equal to four times the overall depth. Additionally, regions with concentrated loads must fall within a height of ($a \leq 2h$), where a and h represent the shear span and beam depth, respectively. In contrast, the Eurocode (EN 1992-1-1-2004-E) [6] defines a deep beam as having a clear span (l_n) less than or equal to three times the beam depth. Meanwhile, the Canadian code (CSA A23.3-04) classifies a deep flexural member as a beam with a clear span-to-depth ratio of less than 2.

These differences in definitions primarily stem from variations in how the codes address the nonlinear strain distribution across the depth of the beam's cross section. Key recommendations for deep beams provided in these codes are summarized below:

- In the design of deep beams, the nonlinear distribution of strain needs to be considered. Strut-and-Tie Models may be used. The ACI 318-08 [5], Eurocode [6] and CSA A23-3-04 [7] provide slightly different versions of the STM.
- Lateral buckling shall be considered when a beam is very thin; such that the h/b ratio is large (b is the width of the beam). This phenomenon has been investigated by many researchers [8, 9, 10] to determine the size effect on the failure of shear strength.
- The definition of the nominal shear strength, V_n , for a deep beam varies from one code to another.
 - ACI 318-08 defines V_n as $0.83\sqrt{f'_c}bw_d$.
 - In the Euro code6, V_n is determined by the reaction V_{Ed} , which is equal to $0.5bw_dv_{fcd}$. This value may be multiplied by $\beta = a/2d$ if the shear-span is between $0.5d$ and $2d$.
 - CSA7 does not specify any limitation on the ultimate shear force, which is calculated from the STM.
- The minimum horizontal and vertical reinforcements in the side faces of a deep flexural member should satisfy the code requirements as discussed below.
- The provisions of the ACI 318-08 [5] specify that the minimum web reinforcement should satisfy the equation (A-4) of the code Eq (1) or the area of the vertical (AV) and horizontal reinforcements (AH) should not exceed $0.0025bwS_1$ and $0.0015bwS_2$ respectively. As shown in Figure 1, S_1 and S_2 are spacings of the bars in the respective directions. The bar spacing S_1 and S_2 should be less than $d/5$ and 12 in or 305 mm.
- The Eurocode6 provides that the area of skin reinforcements in the form of the orthogonal mesh should be 0.1% of the beam cross-sectional area, but not less than $0.071 \text{ in}^2/\text{feet}$ ($150 \text{ mm}^2/\text{m}$) in each face and direction; and the bar spacing, S , should be less than $2b$ and 300 mm.
- CSA A23-3-04 [7] specifies that web reinforcements are required if the height of a deep beam exceeds 750 mm and shall be uniformly distributed along the exposed side faces for a distance of $[0.5h-(2h-d)]$. In such a case, the area of reinforcement should not exceed $0.002A_c$ in each direction, and the bar spacing, S , should not exceed 300 mm.

- The equations provided in the codes for calculating and analyzing beam capacity suggest that web reinforcements do not influence the strength of the struts. An exception is found in ACI 318-08 [5], where, specifically for bottle-shaped struts, the reduction factor β_s is adjusted to 0.75 when $f'_c \leq 6000$ psi or 40 MPa, provided the web reinforcements meet the requirements of equation (A-4) of the ACI 318-08 code. However, experimental research, such as that conducted by Zhang and Tan [10] and Michael and Oguzhan [11], indicates that web reinforcements can play a significant role in improving the capacity of concrete deep beams.

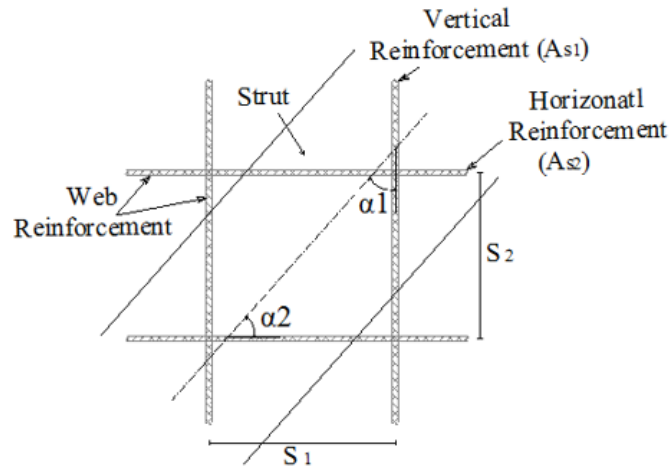


Figure 1: Reinforcement crossing a strut ACI 318-08 [5]

2.2. Review of the Strut-and-tie models in Various codes

ACI 318-08[5] provisions:

Appendix A of ACI 318-08 provides a methodology for determining the nominal capacities of the Strut-and-Tie Model (STM) elements, which include the strut (compression member), the nodal zone, and the tie (tension member). The design of these components is governed by the equation $\phi F_n \geq F_u$, where F_u represents the maximum force in the element under all loading scenarios, F_n is the nominal strength, and ϕ is a strength reduction factor specified in Section 9.3.2.6 of the code. The procedure defines two types of struts. The first type has a uniform cross-sectional area along its length, from the applied load to the support plate. The nominal capacity of this strut is calculated using the formula $f_{ce} = 0.85 \beta_s f'_c$, where β_s is the efficiency factor. This factor accounts for the reduction in strength due to the concrete's ability to resist loads after cracking occurs or to transfer compression across cracks in a tension zone. The value of β_s varies from 0.4 to 1, depending on the type of strut. The second type is a bottle-shaped strut, as illustrated in Figure 2. The nominal capacity of this type is calculated in the same way as the straight strut, but with a different value for β_s . For this type of strut, β_s is taken as 0.75 if the concrete compressive strength f'_c does not exceed 6000 psi (40 MPa) and if the web reinforcement meets the requirements of Equation (A-4) in ACI 318-08.

$$\sum A_{s_i} / (b_s S_{i_i}) \sin \alpha_i \geq 0.003 \quad (1)$$

Michael and Oguzhan [11] have argued that using this type of strut is not recommended when web reinforcement is insufficient, as it cannot effectively prevent the diagonal tension crack from propagating. In situations where f'_c exceeds 6000 psi (40 MPa), and the code offers no specific guidance, Carlos et al. [12] proposed a shallower slope of 6:1 for the distribution of compressive forces in the strut, to reduce the need for excessive web reinforcement.

For struts without web reinforcement or cases that do not meet the aforementioned conditions, the efficiency factor is taken as 0.6λ , where λ refers to modification factor that reflects the reduced mechanical properties of lightweight concrete. Park and Aboutaha [13] compared efficiency factors across various models and found that the values derived from ACI 318-08 [5] tend to be unconservative compared to experimental results in many instances.

The nominal compressive strength of a strut is calculated using the formula $F_{ns} = f_{ce} \cdot A_{cs}$, where A_{cs} represents the cross-sectional area of the strut, obtained by multiplying the strut width (w_s) by the beam width, and f_{ce} is the effective compressive strength of the concrete. The value of F_{ns} is determined at both ends of the strut, with the lesser value being used. The strength of the nodal zone is checked at the minimum face area of the node using the equation $F_{nn} = f_{ce} A_{nz}$ where A_{nz} is the nodal zone face area subjected to F_u acts and $f_{ce} = 0.85 \beta_n f'_c$. The

efficiency factor β_n is assigned values of 0.6, 0.8, and 1 for CTT, CCT, and CCC nodes, respectively, where C indicates compression and T indicates tension in the interacting members at the node. In the CTT and CCT cases, the nodal strength is adjusted based on the strut's strength. The strength of ties is determined using the equation $F_{nt} = A_t f_y + A_{tp}(f_{se} + \Delta f_p)$, with the second term applying specifically to pre-stressed members. ACI 318-08 [5] provides recommendations for tie reinforcement, such as ensuring that the axis of the reinforcement aligns with the tie's axis and that ties are anchored using mechanical devices, standard hooks, or straight bars. The code also specifies that the angle θ between the strut and tie axes must not be less than $\arctan \frac{1}{2} = 26.5^\circ$ to minimize cracking and avoid incompatibilities.

Euro code EN 1992-1-1-2004(E) 6 provisions:

The compressive force within a strut is determined by the compressive strength of cylindrical concrete, f_{ck} . According to the Eurocode, only one type of strut is defined with a uniform cross section—similar to the CSA [7] code. However, unlike the CSA code, the Eurocode specifies two different efficiency factors based on the presence of transverse tension within the strut. In the first case, for struts without transverse tension, the efficiency factor is given as $1/\gamma_c$, where γ_c is a partial safety factor for concrete. For transient design situations, γ_c is equal to 1.5, while for accidental design situations, it is equal to 1.2. In the second case, for struts with transverse tension, the efficiency factor is calculated as $\nu = 1 - f_{ck}/250$. Compressive strength significantly impacts the efficiency factor in this category. For example, when $f_{ck}=3625.94$ psi [25 N/mm²], $\nu=0.9$, whereas for $f_{ck}=5801.51$ psi [40 N/mm²], ν decreases to 0.84. The Eurocode also defines three nodal zones—CCC, CCT, and CTT—based on the node region and the direction of anchoring by the tie. The compressive stress in these nodes $\sigma_{Rd,max}$ must not exceed the compressive strength of concrete, which is reduced by factors K_1, K_2 and K_3 for each node type. These factors are specified as $K_1=1, K_2=0.85$ and $K_3=0.75$, respectively.

Canadian code A23-3-04 [7] provisions:

The CSA [7] code specifies that all struts must be considered as having uniform cross sections. The compressive force in a strut is restricted to $\phi_c f_{cu} A_{cs}$, where $f_{cu} = f'_c / (0.8 + 170\epsilon_1) \leq 0.85f'_c$ and $\epsilon_1 = \epsilon_s + (\epsilon_s + 0.002) \cot 2\theta_s$. Here, θ_s represents the smallest angle between the strut and the adjoining tie. In this equation, the nominal capacity of the strut is reduced by a factor of $\phi_c / (0.8 + 170\epsilon_1)$, which is capped at $0.85\phi_c$. The CSA A23-3-04 [7] standard also identifies three types of nodal zones based on the node region and the direction of tie anchorage: CCC, CCT, and CTT. The compressive stress in these nodes is limited to f'_c , reduced by factors of $0.85\phi_c$, $0.75\phi_c$ and $0.65\phi_c$, respectively. Additionally, the tensile force in a tie must not exceed $\phi_s f_y A_{st}$. The Canadian code emphasizes that tie reinforcements should be adequately anchored and that the angle θ between the strut and tie axes should be no less than $29 + 7000\epsilon_x$. Table 1 provides a comparison of the reduction factors for the effective compressive strength of struts and nodal zones as defined in the various codes. These reduction factors are expressed as $\mu = \beta\phi$, where β is the efficiency factor and ϕ is the strength coefficient.

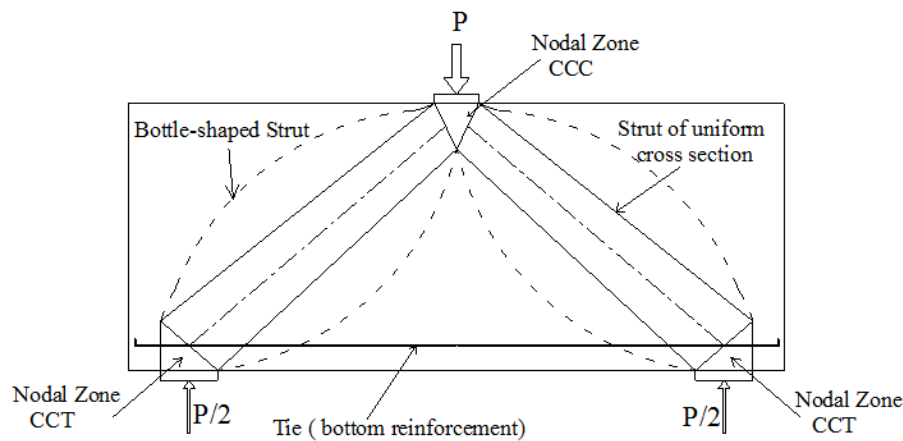


Figure 2: Description of Strut and Tie model

Table 1 The reduction factor of the effective compressive strength for struts and nodes in STM models provided in different codes.

Type		ACI318-08 [5]		Euro code EN 1992-1-1-2004(E) [6]			Canadian code A23-3-04[7]	
		Features	The reduction factor	Features		The reduction factor	Features	The reduction factor
	Uniform cross section	occur in compression zone	0.85	With transv. tension	Transient	0.67	wherever occurs	0.55
		occur in tension zone	0.34					
Strut	bottle-shaped	satisfying equation (A-4)	0.64		Accidental	0.83		
		Not satisfying equation (A-4)	0.51					
	not clarifying	others than above	0.51	without transverse tension for $f_{ck}=145.0377psi$ [40 N/mm ²]		0.84		
	CCC	compression reign	0.85	compression reign for $f_{ck}=145.0377psi$ [40N/mm2]		0.84	CCC	0.55
Node	CCT	compression reign + one tie	0.68	compression reign + one tie for $f_{ck}=145.0377psi$ [40 N/mm ²]		0.714	CCT	0.49
	CTT	compression reign + two tie or more	0.51	compression reign + two tie or more for $f_{ck}=145.0377psi$ [40 N/mm ²]		0.63	CTT	0.43

2.3. Predicting the shear strength of deep beams according to the code's provisions:

A total of 347 experimental test results from existing literature have been compiled and analyzed to evaluate the accuracy and effectiveness of the STM provisions in the three codes under consideration. Table 2 provides a detailed description of all collected specimens. These specimens include deep beams subjected to either one or two concentrated loads. The database covers a wide range of concrete compressive strengths, from 16.5 MPa to 120 MPa, and shear-span depth ratios between 0.27 and 3.5, adhering to the definitions of deep beams provided in various codes. Table 2 also includes the mean, standard deviation, and coefficient of variation for the shear capacity predicted by the provisions of the codes. The calculations of the capacities are based on simple truss models. Figures 3-a and 3-b depict a typical deep beam and its corresponding STM. The geometry of the strut-and-tie model is determined by maximizing the lever arm, which balances the internal compressive force C and tensile force T in the steel. For instance, in the Canadian code, the height of the bottom node is calculated as 1.13 times the height of the top node by dividing the compressive stress of the CCT node by that of the CCC node. Similarly, the height factors are 1.25 and 1.34 for the ACI and Eurocode, respectively. Based on the truss geometry, the dimensions of all struts, nodes, and ties can then be determined.

Using truss geometry and the dimensions of the nodes and members, the angles of the struts can be determined and compared against the allowable minimum strut angle specified by the codes. If the calculated angle exceeds the permitted limit, the truss geometry is adjusted accordingly to meet the modified angle requirements. Based on this process, all possible applied loads and the corresponding forces in each element can be calculated using the provisions of each code. The applied load is determined by the capacity of the weakest element in the system.

The calculations were conducted across five distinct categories. For the ACI cases, two types of struts were considered based on their shape: uniform cross-section struts and bottle-shaped struts, each having different efficiency factors (β_s). Euro Option-2 follows a procedure similar to that in the Eurocode but modifies the ultimate shear force prediction by applying a factor of $\beta = a/2d$ this factor is applied in accordance with Eurocode guidelines. Additionally, the ultimate shear forces were calculated using the Canadian Code and Eurocode Option-1 without any adjustments.

Figure 4 illustrates a comparison between the nominal and factored shear strength capacities calculated using the provisions of the three codes (V_{calc}). The comparison was conducted by analyzing the percentage of specimens for which the ratio of the predicted nominal and factored capacities (V_{calc}) to the experimental shear strength (V_{exp}) was determined. This analysis included only specimens with a shear span-to-depth ratio of less than 2.5.

The findings indicate that the factored capacities calculated using the Canadian code yield the most consistent results, with 90.99% of the test specimens exhibiting higher capacities than those predicted by the code. In contrast, only 77.48% of the specimens exceeded the predicted capacities when nominal values were used. The difference between these percentages represents the material resistance factor (ϕ), which the Canadian code assigns as 0.65 for concrete and 0.85 for reinforcing bars.

The percentage of the prediction of the nominal capacity predicts 13.51% of the samples predicted by the factored capacity to be less than the actual results. And this result is approximately equal to the value of reduction that applied for concrete as resistance factor for concrete. Although the ACI code uses a constant reduction factor for factored forces across all STM elements (struts, ties, and nodal zones), the improvement in prediction accuracy when using a uniform strut in the ACI code was greater than that observed for the bottle-shaped strut. Specifically, reducing the nominal force by approximately 25% increased the accuracy of the ACI provisions for the uniform strut from 69.67% to 84.09%. In comparison, the prediction accuracy for the bottle-shaped strut improved by only 9.61%, reaching 89.19% of the test specimens exhibiting higher capacities than those provided by the code.

The results shown in Figure 4 highlight a noticeable improvement when Eurocode Option-2 is used for nominal strength, with the percentage increasing to 78.38% compared to Eurocode Option-1. While both Eurocode options apply a similar reduction factor for concrete to calculate the factored force, Eurocode Option-1 shows a greater improvement of 12.91%, whereas Eurocode Option-2 improves by only 4.50%, resulting in 82.88% of the test specimens having higher capacities than those predicted by the code. Among all methods, Eurocode Option-2 demonstrates the least improvement in predicting shear strength after factoring in strength.

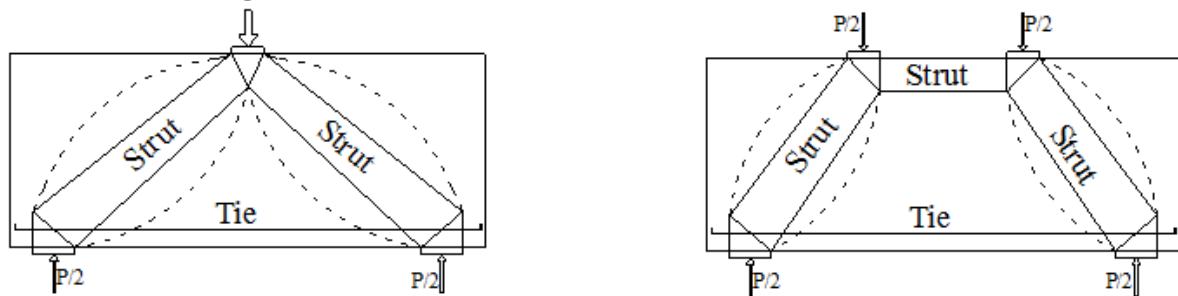


Figure 3: Strut and Tie Models: (a) beam specimen loaded by a single point load, (b) beam specimen loaded by two-point loads

Table 1 Description of collected specimens

Reference	Number of beams	f_c (Mpa)	a/d		ACI	Bottled ACI	CSA	EURO	Mod.EURO
Zhang and Tan [10]	12	25.9-32.4	1.1	Mean	0.91	0.71	0.51	0.86	0.47
				SD	0.09	0.29	0.05	0.12	0.07
				C.O.V	0.10	0.40	0.10	0.14	0.14
Nathan and Sergio [14]	12	27.0-35.6	1.0-2.0	Mean	0.56	0.56	0.44	0.54	0.40
				SD	0.08	0.08	0.09	0.13	0.21
				C.O.V	0.14	0.14	0.19	0.23	0.53
Tan and Lu [15]	12	30.8-49.1	0.56-1.13	Mean	0.46	0.44	0.32	0.42	0.19
				SD	0.10	0.11	0.07	0.10	0.09
				C.O.V	0.22	0.24	0.23	0.23	0.49
Oh and Shin [16]	53	23.72-73.6	0.50-2.0	Mean	0.86	0.80	0.62	0.83	0.58
				SD	0.20	0.19	0.15	0.20	0.23
				C.O.V	0.23	0.23	0.24	0.24	0.41
Smith and Vantsiotis [17]	52	16.1-22.1	0.77-2.01	Mean	0.91	0.70	0.47	0.91	0.50
				SD	0.12	0.08	0.12	0.11	0.15
				C.O.V	0.13	0.11	0.26	0.12	0.30
Walraven & Lehwalter [18]	12	17-21.3	1.0	Mean	0.42	0.41	0.29	0.36	0.18
				SD	0.12	0.11	0.08	0.10	0.05
				C.O.V	0.28	0.28	0.27	0.27	0.27

Reference	Number of beams	f _c (Mpa)	a/d		ACI	Bottled ACI	CSA	EURO	Mod.EURO
Tan et al., [19]	19	56.2-86.3	0.85-1.69	Mean	1.12	1.05	0.59	1.12	0.75
				SD	0.48	0.41	0.22	0.50	0.47
				C.O.V	0.42	0.39	0.37	0.44	0.62
Tan et al., [20]	22	54.8-74.1	0.28-3.14	Mean	1.35	1.17	0.64	1.31	1.06
				SD	0.70	0.54	0.19	0.72	0.79
				C.O.V	0.52	0.46	0.30	0.55	0.74
Foster & Gilbert [21]	16	77-120	0.76-1.88	Mean	0.59	0.63	0.51	0.52	0.37
				SD	0.17	0.15	0.14	0.22	0.25
				C.O.V	0.29	0.23	0.27	0.42	0.68
Shin et al., [22]	30	52 -73	1.5-2.5	Mean	1.13	0.85	0.62	0.86	0.79
				SD	0.43	0.23	0.25	0.32	0.33
				C.O.V	0.38	0.27	0.40	0.37	0.42
Yang et al., [23]	21	31.4-78.5	0.53-1.13	Mean	1.03	0.83	0.68	0.94	0.44
				SD	0.45	0.35	0.29	0.40	0.28
				C.O.V	0.43	0.42	0.43	0.42	0.65
Kong & Rangan [24]	42	63.6-89.4	1.51-3.30	Mean	1.72	1.36	1.10	1.64	1.64
				SD	0.39	0.31	0.23	0.34	0.34
				C.O.V	0.23	0.23	0.21	0.21	0.21
Aguilar et al., [25]	4	28	1.14-1.27	Mean	0.62	0.62	0.50	0.73	0.43
				SD	0.06	0.06	0.04	0.04	0.02
				C.O.V	0.10	0.10	0.08	0.06	0.04
Tan et al., [26]	19	41.06-58.84	0.27-1.62	Mean	0.90	0.75	0.54	0.84	0.55
				SD	0.16	0.15	0.11	0.24	0.29
				C.O.V	0.17	0.20	0.20	0.28	0.54
Rigotti [27]	11	16.5-34.5	1.87-2.33	Mean	0.66	0.45	0.25	0.65	0.63
				SD	0.07	0.04	0.03	0.07	0.07
				C.O.V	0.10	0.09	0.13	0.10	0.11
Garay & Lubell [28]	10	43-48	1.19-2.38	Mean	1.26	1.01	0.64	1.39	1.10
				SD	0.57	0.47	0.19	0.64	0.67
				C.O.V	0.45	0.47	0.30	0.46	0.61
Total	347			Mean	1.00	0.84	0.60	0.95	0.71
				SD	0.48	0.37	0.27	0.47	0.52
				C.O.V	0.48	0.44	0.45	0.49	0.73

* SD: Standard deviation. C.O.V Coeff. of Variation (1Mpa=145.0377psi)

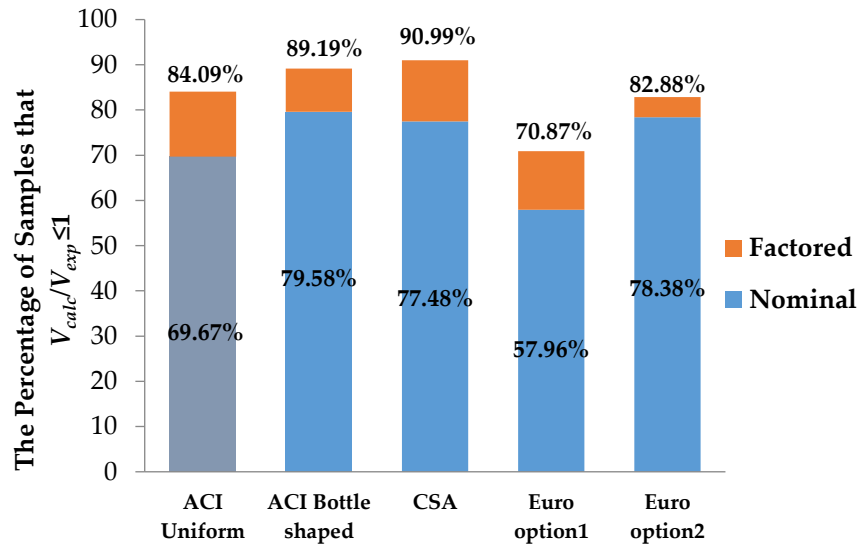


Figure 4: The percentage of samples that $V_{calc}/V_{exp} \leq 1$ for all specimens

2.4. Effect of Web Reinforcements

The three codes reviewed in this paper propose specific requirements for web reinforcement to improve beam stiffness and meet serviceability criteria. However, when calculating the shear strength capacity of concrete deep beams, the strut-and-tie models in these codes generally disregard the influence of web reinforcements. The only exception is the ACI code, where the efficiency factor for bottle-shaped struts is adjusted to account for web reinforcements (as outlined in Appendix A of the code). Numerous studies in literature explore the impact of web reinforcements on factors such as mid-span deflection, crack width, failure modes, ultimate strengths, and the overall behavior of reinforced concrete deep beams. Drawing from the results of the experimental studies compiled here, it has been observed that web reinforcements significantly enhance beam capacity, stiffness, and other characteristics. These effects, as identified in the experimental findings, are briefly discussed below

Effect on the beam stiffness:

Tan et al. [19] observed that web reinforcements enhance beam stiffness, with their effect becoming more significant at shear span-to-depth ratios (a/d) of ≥ 1.13 , as indicated by mid-span deflection. Smith and Vantsiotis [17] highlighted the importance of applying a minimum amount of vertical and horizontal web reinforcement to improve beam stiffness and control cracking. Similarly, Kong et al. [29] noted that the influence of web reinforcement on beam stiffness depends on its arrangement and quantity, which vary with L/d and a/d ratios. They found vertical web reinforcement to be particularly effective in improving stiffness when $a/d \geq 0.7$.

Effect on crack control:

Smith and Vantsiotis [17] observed that beams with web reinforcement exhibited smaller and more uniform crack widths at the same load level compared to beams without web reinforcement. However, they, along with Shin et al. [22], noted that web reinforcement had no significant effect on crack propagation, which remained consistent across all beams. In contrast, Tan et al. [19] found that web reinforcements play a crucial role in controlling diagonal cracks, leading to higher serviceability loads in beams with web reinforcement. The effectiveness of diagonal crack control depends on the placement of shear reinforcements, with web reinforcement being the most effective. Kong et al. [29] concluded that the impact of web reinforcement on crack width and length is closely tied to the stiffness of the beam.

Effect on the modes of failure:

Most researchers have consistently demonstrated that beams with web reinforcements exhibit similar modes of failure to those without web reinforcements. However, Rogowsky et al. [30] have observed that a substantial amount of vertical web reinforcement can shift the failure mode to a more ductile one, indicating that the amount of web reinforcement plays a crucial role in influencing the overall failure behavior of beams.

Effect on the ultimate shear strength:

Smith and Vantsiotis [17] found in their experiments that vertical web reinforcements ranging from 0.18% to 1.25% can enhance ultimate shear strength, whereas horizontal web reinforcements between 0.23% and 0.91% have minimal or no impact. Their study observed that web reinforcements can increase the ultimate shear strength by 0–30%, though the maximum does not exceed $4bd\sqrt{f'_c}$. Similarly, Rogowsky et al. [30] demonstrated that only

vertical web reinforcements significantly influence ultimate shear strength. Tan et al. [19] also confirmed that vertical web reinforcements are more effective than horizontal ones in improving shear strength, particularly for beams made with high-strength concrete. According to Rogowsky et al [30] test results, web reinforcements contributed to ultimate shear strength improvements of 0–50%, with a maximum contribution not exceeding $2bd\sqrt{f_c}$. Oh and Shin [16] noted that vertical web reinforcements slightly increase ultimate shear strength, and their effectiveness depends on the shear-span to depth ratio (a/d). They also observed that horizontal web reinforcements have little impact on ultimate shear strength.

To analyze the effect of web reinforcements on ultimate shear strength and its estimation based on code provisions, the experimental samples from the compiled database were classified into four groups according to the distribution of web reinforcements: (1) samples with both vertical and horizontal web reinforcements, referred to as “web”; (2) samples with only vertical web reinforcements, labeled as “only vertical”; (3) samples with only horizontal web reinforcements, termed “only horizontal”; and (4) samples without any web reinforcements, referred to as “without web”.

Figure 5 illustrates the percentage of samples within each of the four groups where ($V_{calc}/V_{exp} \leq 1$). In generating these results, only the distribution of web reinforcements was considered, irrespective of the shear-span-to-depth ratio or the concrete’s compressive strength. When evaluating the shear strength capacities of samples with web reinforcements using Eurocode Option-1 or ACI (uniform strut), 86.52% and 87.23% of the samples, respectively, were found to satisfy $V_{calc}/V_{exp} \leq 1$, which is deemed acceptable. When the bottle-shaped strut is used instead of the uniform strut section, the results improve significantly, with 96.45% of the samples meeting the condition $V_{calc}/V_{exp} \leq 1$. Similarly, using Eurocode Option-2 leads to further improvement, with $V_{calc}/V_{exp} \leq 1$ 97.87% of the samples having a calculated capacity that does not exceed the experimental capacity. For the Canadian code provisions (CSA), 98.58% of the samples satisfy $V_{calc}/V_{exp} \leq 1$ making it the most conservative in estimating the shear strength capacity of concrete deep beams with web reinforcements. However, as observed in Figure 5, for beams with only vertical or horizontal web reinforcements, or no web reinforcements at all, the Canadian code provisions tend to underestimate shear strength capacity compared to other code provisions. In addition, the other codes provisions overestimate shear strength capacity compared to the Canadian code provisions.

Figures 6 and 7 illustrate the influence of web reinforcements concerning the compressive strength of concrete. The results indicate that all three code provisions provide conservative estimates when the compressive strength is less than or equal to 40 MPa. Using the ACI bottle-shaped strut, fewer than 5% of the samples demonstrate actual capacities lower than those predicted by the code provisions (Figure 6), which contradicts the findings of Michael and Oguzhan [11]. For high-strength concrete samples, where the compressive strength is equal to or greater than 40 MPa, the estimates remain highly conservative. Under these conditions, Eurocode Option-2, the ACI bottle-shaped strut, and the CSA code predict capacities conservatively for 96.92%, 93.85%, and 98.46% of the samples, respectively. In this context, “conservatively” means that the CSA code predicts capacities at lower, more cautious values, ensuring safety by accounting for potential uncertainties or variations in performance. The percentages (96.92%, 93.85%, and 98.46%) reflect how the code predicts the capacities of structures in a way that slightly underestimates their actual capacity, avoiding overestimation that could lead to unsafe designs. It’s not about exceeding 100% but rather being cautious to ensure safety margins (Figure 7).

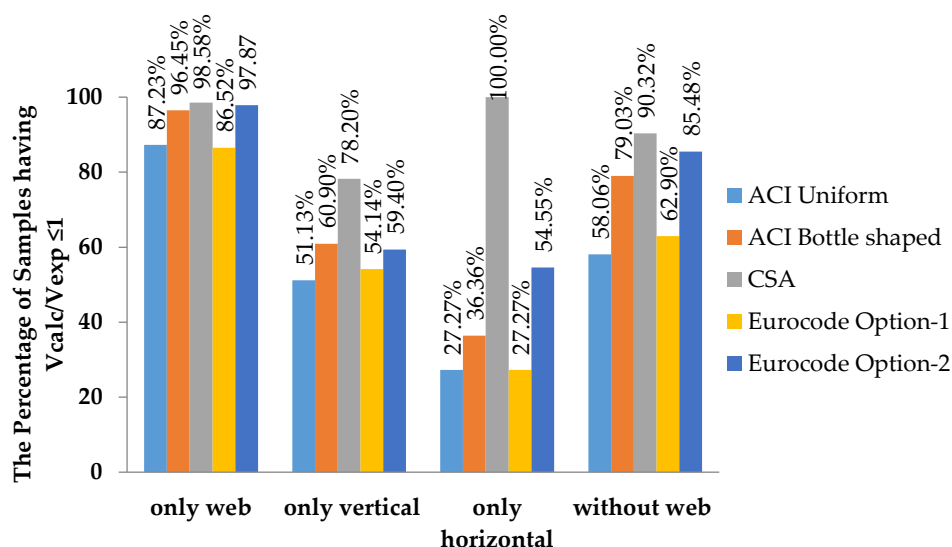


Figure 5: The Percentage of Samples having $V_{calc}/V_{exp} \leq 1$ considering the effect of web reinforcements

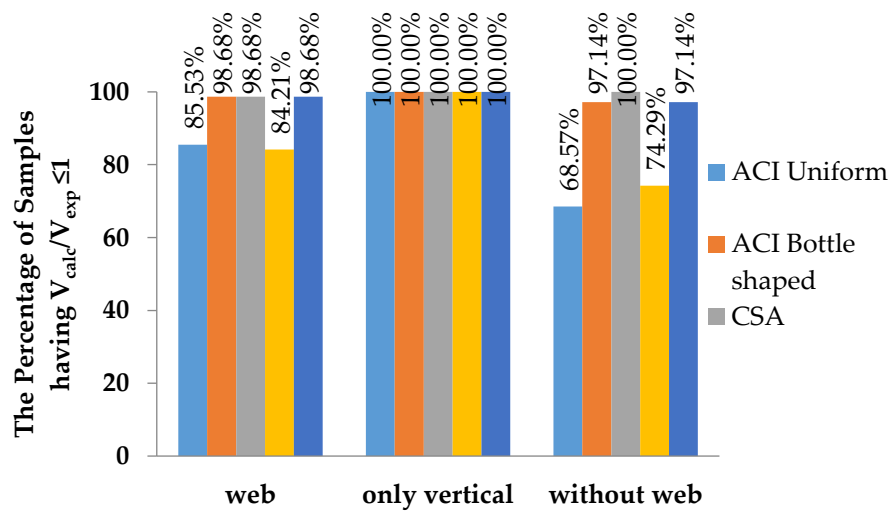


Figure 6: The Percentage of Samples having $V_{calc}/V_{exp} \leq 1$ (for Specimens with $f'_c \leq 40$ MPa)

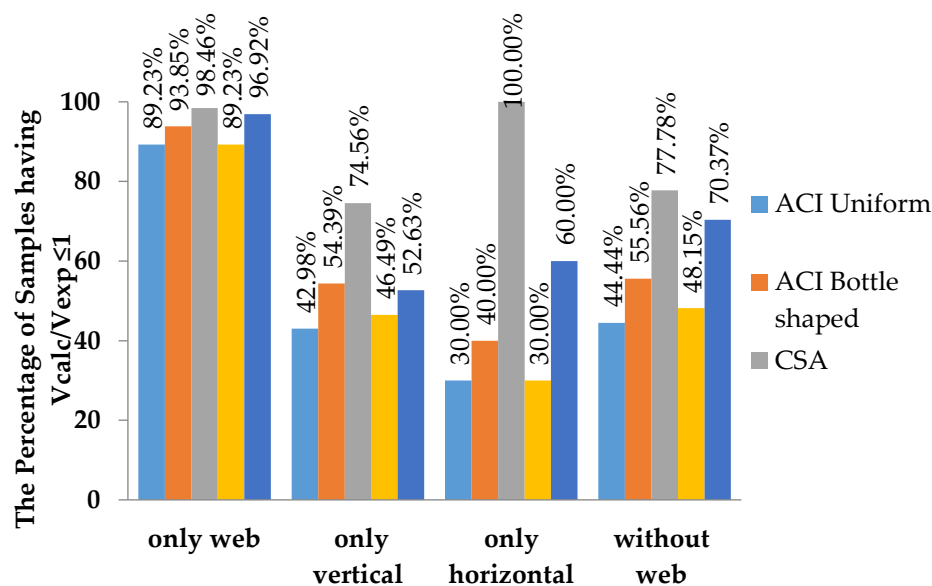


Figure 7: The Percentage of Samples having $V_{calc}/V_{exp} \leq 1$ (for Specimens with $f'_c > 40$ MPa)

Results

3.1. Effect of shear-span to depth ratio

The shear-span-to-depth ratio (a/d) significantly influences the transition of the stress pattern from linear to non-linear behavior. Furthermore, when the ratio of the shear span (a) to the effective depth of the beam (d) is less than 1 ($a/d < 1$), the beam typically behaves more like a truss, as shear forces dominate over bending moments. This causes the beam to transfer loads using diagonal compression and tension elements, similar to a truss mechanism. This ratio is a key parameter in defining a deep beam, which is commonly characterized by having the load applied closer than approximately twice the effective depth ($a/d < 2$). Tan and Lu [15] observed that beams with higher a/d ratios exhibit gentler stress-deflection curves and increased flexibility as the ratio rises. Wight and Macgregor [31] demonstrated that a/d plays a crucial role in determining the failure mechanism. For beams with $a/d < 1$, the behavior shifts from beam action to truss action, while beams with a/d between 1 and 2.5 experience failure at less than their flexural moment capacity. Nathan and Sergio [14] further noted that a/d influences crack patterns. Beams with a/d between 1 and 1.5 show cracking consistent with a tied-arch load

transfer mechanism, whereas beams with $a/d = 2$ exhibit crack patterns indicative of a truss load transfer mechanism.

The plot in Figure 8 compares the Strut-and-Tie Model (STM) with the traditional sectional beam theory in predicting the dimensionless shear stress capacity (v) of beams as a function of the a/d ratio. The dimensionless shear stress (v) is calculated using the formula $v = V_{calc}/bdf_c$, where b and d represent the beam's width and effective depth, respectively, and f_c denotes the compressive strength of the concrete. The results in Figure 8 highlight the significant influence of the a/d ratio on specimens with ($a/d \leq 2$) and underscore the effectiveness of STM in designing deep beams. Moreover, for $a/d < 3$, the sectional model (traditional beam theory) fails to provide accurate capacity predictions, as observed across all three selected code provisions. In this case, the Strut-and-Tie Model (STM), particularly the provisions of the Canadian Code (CSA-A23-3-04), offers a highly accurate estimate of beam capacity when compared to experimental results. The Canadian Code's predictions are more conservative than those provided by the other two codes. It is important to note that the methods for calculating capacity using the traditional thin beam theory differ slightly between the codes, as illustrated in Figure 8.

For experimental specimens with $a/d < 1$, the capacities predicted by the code provisions are generally conservative compared to the actual capacities. As shown in Figure 9, the Eurocode Option-2 and the Canadian Code significantly underestimate the capacity for 100% and 99.08% of the samples, respectively.

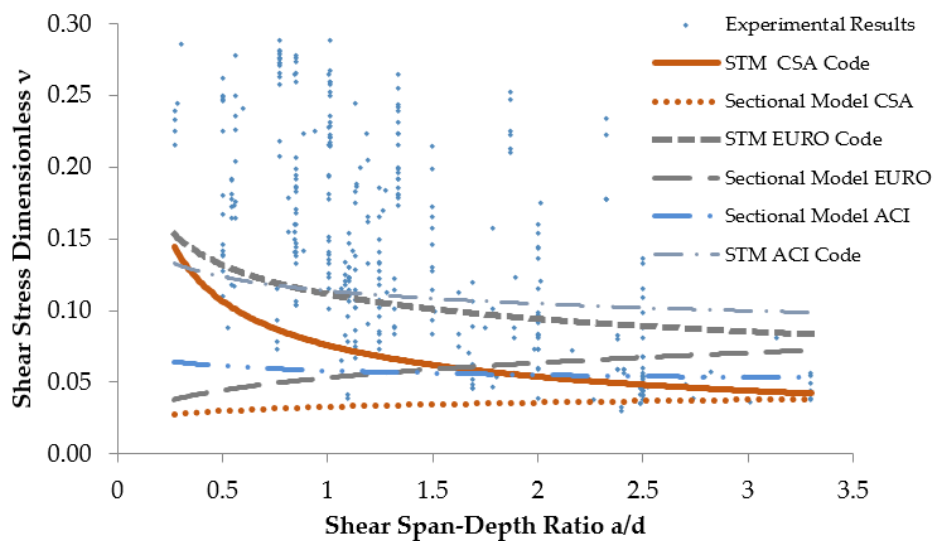


Figure 8: Use of strut and tie model and sectional mode to predict the strengths of a series of beams in different codes

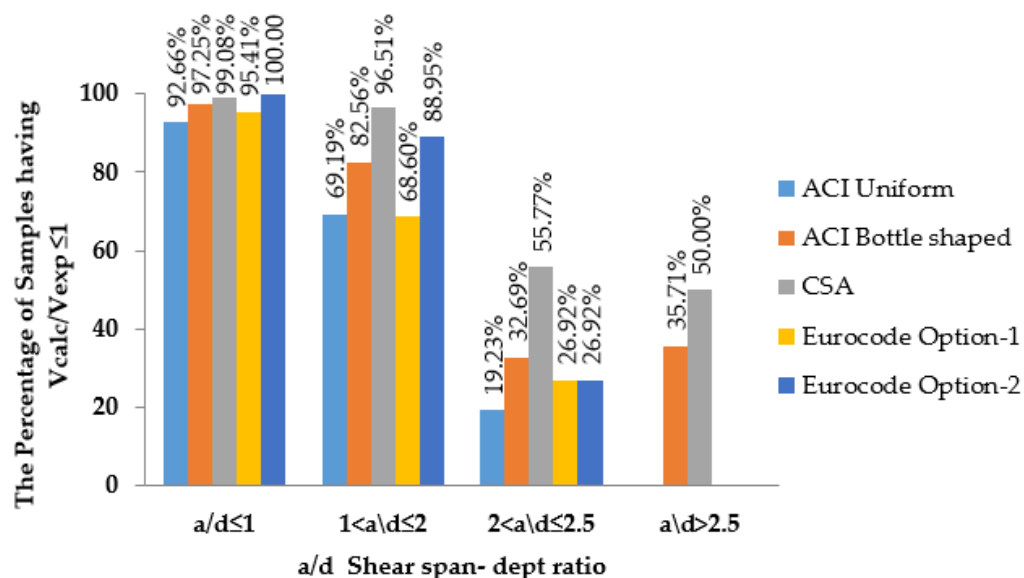


Figure 9: Percentage of Samples having $V_{calc}/V_{exp} \leq 1$ considering a/d ratio.

The Canadian Code provides the most accurate predictions for specimens with a/d values between 1 and 2, with 96.51% of these specimens exhibiting higher actual capacities than those predicted. However, for specimens with $a/d > 2$, the capacities predicted by the code provisions are often unconservative, as demonstrated in Figure 9. For specimens with $a/d > 2.5$, the Canadian Code overestimates the capacity for 50% of the samples, while the other codes yield even less accurate results, as shown in Figure 9. These findings suggest that the Strut-and-Tie Models (STMs) provided in the selected codes are not reliable for specimens with $a/d > 2$.

3.2. Failure modes in Deep beams

Michael and Oguzhan [11], Carlos et al. [12], and Tan et al. [19] identified three primary failure modes in their experiments: (i) shear failure, (ii) flexural failure, and (iii) anchorage failure. Shear failure, the most common type, is brittle in nature. Flexural failure, or tensile failure, occurs at the bottom mid-span of a beam at the location of the lower longitudinal reinforcement when the reinforcement is insufficient. Anchorage failure takes place at the bottom end of the beam's main reinforcements when the anchorage or development length is inadequate or when no mechanical anchorage is provided. Shear failure can be further categorized into three specific types.

Diagonal Splitting Failure: This failure occurs along the middle depth of the beam, parallel to the concrete strut. Cracks propagate in both directions, extending toward the loading plate and the bearing plate. In the absence of adequate reinforcements, this type of failure can occur suddenly due to the splitting of the concrete strut. Notably, this failure mode cannot be accurately predicted by the Strut-and-Tie Model (STM) provisions of any of the three selected codes;

Diagonal Compression (Strut Crushing Failure): This type of failure occurs at the mid-depth of a beam, along the length between the end of the loading plate and the start of a strut. It is typically preceded by the development of multiple diagonal cracks, which eventually lead to the crushing of the concrete structure;

Shear Compression Failure (Node Failure): This failure occurs near the loading or bearing plate, where the shear forces cause the concrete to compress excessively, leading to failure at the node of the strut-and-tie system

Figure 10 illustrates the accuracy in predicting the failure mode and its location for 310 specimens based on the code provisions. Among these specimens, 213 (68.71%) failed due to diagonal compression, a subcategory of shear failure, resulting in strut crushing, as documented in experimental studies [32-34]. However, when the ACI code provisions, considering the uniform strut section, were applied, only 53 specimens (24.88%) were accurately predicted to fail by strut crushing (Figure 10). When the bottle-shaped strut was applied within the ACI provisions, the accuracy in predicting the failure mode significantly improved, with 155 specimens (72.77%) being correctly identified as failing due to strut-crushing. This improvement highlights the impact of reducing the efficiency factor β_s by approximately 25% for bottle-shaped struts compared to the uniform strut section, enhancing the precision in predicting strut-crushing failure. In comparison, the CSA code provisions predicted 138 specimens (64.79%) to fail in the strut-crushing mode, while the Eurocode (Option-1) provisions accurately predicted this failure mode for only 57 specimens (26.76%). When Eurocode Option-2 was applied, no significant improvement was observed, with only 58 specimens (27.23%) correctly predicted to fail due to strut-crushing. Figure 10 also presents the accuracy of predicting failure at the bearing plate, regardless of its location (loading or support). Using the provisions of the Canadian Code, only 22 out of 44 specimens (50.00%) were accurately predicted to fail at the bearing plate. In contrast, the ACI provisions (uniform strut) achieved higher accuracy, correctly predicting the failure for 35 specimens (79.55%). However, the use of the bottle-shaped strut in the ACI provisions reduced the accuracy, with only 12 specimens (27.27%) correctly predicted to fail at the bearing plate. The use of the bottle-shaped strut in the ACI provisions significantly underperformed compared to the uniform strut, yielding much worse results for predicting failure at the bearing plate. Under the Eurocode provisions, even after adjustment (Option-2), failure at the bearing plate was correctly predicted for only 26 specimens (59.09%). For the tension failure mode, the ACI provisions with uniform strut sections correctly predicted the failure for 37 specimens (69.81%), while the Eurocode provisions (Option-1) correctly predicted the failure for only 24 specimens (45.28%). The Canadian Code (CSA A23.3-04) performed the worst for tension failure, correctly predicting it in just 14 specimens (26.42%). Thus, while the CSA code provisions are conservative in estimating the shear strength capacity of deep beams, they fall short in effectively predicting the failure modes in such beams. In fact, none of the codes provides a robust or reliable method for accurately predicting the failure modes in concrete deep beams. The bottle-shaped strut in the ACI provisions demonstrates improved accuracy in predicting the strut-crushing mode of failure but fails to effectively predict failure at the bearing plates or ties (tension failure), where the uniform strut option performs better. On the other hand, the Eurocode provisions prove to be largely ineffective in accurately predicting any of the failure modes, as illustrated in Figure 10.

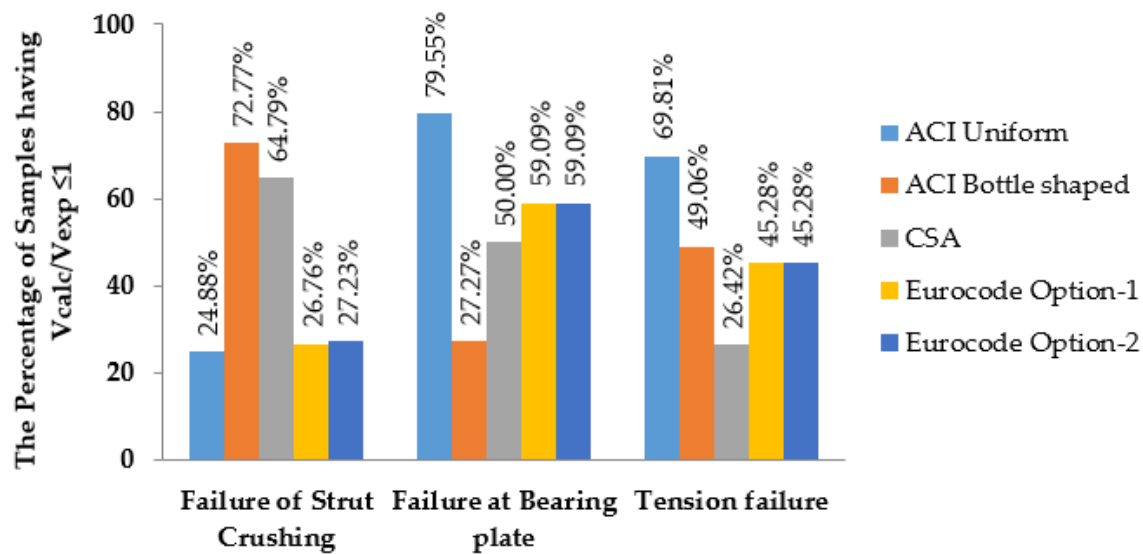


Figure 10: The Prediction of Failure Occurrence by the STM provision of codes.

Conclusion

This paper reviews the Strut-and-Tie Models (STMs) provided in three prominent codes: ACI-318-08, CSA-A23.3-04, and Eurocode EN1992-1-1. The code provisions were evaluated using experimental data from a database containing over 300 test specimens compiled for this study. The paper examines the effectiveness of the STM provisions in predicting the capacity and failure modes of deep beams. Additionally, the study investigates the impact of various factors—such as web reinforcements, shears pan to depth ratio, and concrete strength on the capacity of deep beams, based on the experimental results. The following conclusions are drawn from the findings of the study:

- The Strut-and-Tie Models provided in the three prominent codes—ACI-318-08, CSA-A23.3-04, and Eurocode EN1992-1-1—are generally suitable methods for designing and evaluating the shear strength capacity of concrete deep beams with a shear-span-to-depth ratio of less than or equal to two.
- Although some code provisions do not account for the effect of web reinforcements, experimental studies indicate that these reinforcements enhance the capacity predicted by the STM provisions.
- The code provisions may not accurately predict the mode and location of failure, as observed in experimental studies. This inaccuracy arises from variations in the efficiency factor values assigned to the three elements in the STM of each code.
- When code recommendations, such as the use of web reinforcements and a shear-span-to-depth ratio of less than two, are considered, the provisions of the Canadian Code appear to provide the most conservative estimates for the capacity of deep beams. ▪ When the Eurocode method is adjusted by multiplying the ultimate load by a factor β as specified in the code, it provides a reasonable and conservative capacity estimate comparable to that of the Canadian Code.
- The ACI procedure significantly improves when bottle-shaped struts are used instead of uniform cross-section struts, resulting in conservative estimates.

Compliance with ethical standards

Disclosure of conflict of interest

The authors declare that they have no conflict of interest.

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