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Numerous Chirped and Optical Solitons for the Cubic Nonlinear Schrödinger Equation with Kerr Nonlinearity and Beta Derivatives

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طيف متنوع من الحلول السوليتونية الضوئية ذات الانحراف الطوري لمعادلة شرودنغر اللاخطية التكعيبية مع لاخطية كير ومشتقات بيتا

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Abstract:
This work explores new optical and chirped optical solitons for the space-time fractional cubic nonlinear Schrödinger equation (NLFSEs) in the presence of Kerr law nonlinearity, utilizing the new extended auxiliary equation approach. The solutions show a wide range of behaviors within the system and are stated in terms of trigonometric and hyperbolic functions. A broad range of phenomena is displayed by the several varieties of optical and chirped optical solitons that have been found, including dark, bright, kink, and periodic. A better knowledge of solution creation and properties may be attained by representing 2D and 3D graphs with different values of parameters.

Keywords: the fractional cubic nonlinear Schrödinger equation; Kerr law nonlinearity; beta derivative; Chirped optical solitons; the new extended auxiliary equation method.

الملخص:
يتناول هذا البحث استكشاف حلول جديدة للسوليتونات الضوئية والسوليتونات الضوئية ذات التغريد الترددي لمعادلة شرودنغر اللاخطية التكعيبية ذات المشتقات الكسرية الزمانية والمكانية (NLFSEs)، في ظل وجود لاخطية من نوع كير، وذلك باستخدام منهجية المعادلة المساعدة الموسَّعة الجديدة. وتُظهر الحلول المستحصلة طيفًا واسعًا من السلوكيات الديناميكية للنظام، وقد صيغت بدلالة الدوال المثلثية والزائدية. كما كُشف عن طائفة متنوعة من الظواهر الفيزيائية من خلال الأنماط المختلفة للسوليتونات الضوئية والسوليتونات ذات التغريد الترددي التي جرى التوصل إليها، بما في ذلك السوليتونات المظلمة والمضيئة، وسوليتونات الجبهة الانتقالية (Kink Solitons)، والحلول الدورية. ويمكن تعزيز فهم آليات تولُّد هذه الحلول وخصائصها من خلال تمثيلها بيانيًا في بعدين وثلاثة أبعاد، مع اعتماد قيم مختلفة للمعلمات

الكلمات المفتاحية: معادلة شرودنغر اللاخطية الكسرية، لاخطية قانون كير، مشتقة بيتا، السوليتونات الضوئية ذات التغريد الترددي، طريقة المعادلة المساعدة الموسعة الجديدة.

1. Introduction

In the process of understanding a wide range of real-world occurrences, nonlinear models have proved useful in offering insightful explanations. Fractional nonlinear evolution equations are advanced classes of differential equations that produce better results, and as they are essential for explaining intricate physical phenomena, a large number of scholars are drawn to this area of study. A key element of fractional nonlinear evolution equations, the nonlinear Schrödinger equation has applications in fractal dynamics, biology, optics, plasma physics, fluid mechanics, electricity, shallow water wave phenomena, heat flow, and quantum mechanics [1-8].
The complex interaction between the dispersive and nonlinear properties of solitons in a medium has shown that their wave-like structure is preserved throughout the propagation process. Fractional Nonlinear Evolution

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Equations (FNLEEs) provide a wealth of commercial and practical advantages for solutions in a variety of sectors. These soliton technologies help to improve data transmission speeds and reliability in the field of fiber optic technology [1-3], which is important for fulfilling the growing needs of high-speed communication networks. FNLEE solitons are essential to the telecommunications industry [4] because they guarantee reliable connection and effective signal transmission. These methods support accurate data signal analysis and manipulation in signal processing applications [5], advancing information processing and digital communications. Furthermore, image processing [6], FNLEE solitons enable advanced image enhancement techniques for clearer and more detailed visual representation.

The use of soliton solutions facilitates the identification of systems by offering precise modeling tools for intricate systems, which support control and predictive analysis. Wastewater treatment and desalination procedures are made more efficient by using FNLEE solitons in water treatment processes for better purification techniques. These answers aid in the comprehension and manipulation of plasma behavior in plasma physics, which is crucial for applications ranging from materials processing to fusion research. Soliton products are advantageous for medical device sterilization operations since they provide dependable and efficient sterilization procedures in healthcare environments. FNLEE solitons provide valuable insights into complex chemical processes in the world of chemistry, which can facilitate the creation of new materials and medications. These many uses highlight the adaptability and significance of soliton solutions made from FNLEEs in resolving issues and encouraging creativity in a range of sectors.

Various dynamic approaches, such as the fractional differential transform method and the fractional modified Kudryashov method, have been developed to solve nonlinear fractional differential equations and produce analytical traveling wave solutions [7-25].

In this paper, we will utilize for the first time the fractional cubic NLFSE with Kerr law nonlinearity by using the new extended auxiliary equation method. The fractional cubic NLFSE with the Kerr law nonlinearity is stated as [26, 27].

$$iD_t^{\alpha}u + a_1 D_t^{\alpha} D_x^{\alpha} u + a_2 D_x^{2\alpha} u + a_3 |u|^2 u = 0. \tag{1}$$

The wave profile with complex values is represented by $u(x,t)$. Here a_1, a_2, and a_3 are real coefficients, and $(0 \le \alpha \le 1)$.

This article is organized as follows: The properties of the beta derivative are covered in Section 2. Section 3 provides specifics on the suggested approach's technique. In Section 4, a mathematical analysis is provided. Section 5 provides a full explanation of the physical significance. Section 6 concludes the article.

2. Definition of Beta Derivative and Its Properties

In recent years, academics have presented numerous definitions of fractional derivatives, including the Riemann-Liouville, modified Riemann-Liouville, Caputo, Caputo-Fabrizio, conformable fractional derivative, and Atangana-Baleanu derivatives. The well-known characteristics of classical calculus, such as the chain rule, the Leibniz rule, and the derivative of a constant being zero, are frequently broken by these fractional derivatives. Atangana and associates put out a noteworthy and novel definition known as the beta derivative to address these variances. This beta derivative is an important development in fractional calculus as it preserves the fundamentals of classical calculus.

Definition: Given $\alpha \in \mathbb{R}$ and a function $h = h(x)$ defined on the interval $[\alpha, \infty) \to \mathbb{R}$, the beta derivative of order α with respect to x is formally defined as follows [28]:

$$D_x^{\alpha}(h(x)) = \mathrm{Lim}_{\varepsilon \to 0} \frac{h\left(x + \varepsilon\left(x + \frac{1}{\Gamma(\alpha)}\right)^{1-\alpha}\right) - h(x)}{\varepsilon}, \tag{2}$$

where Γ is the gamma function and $D_x^{\alpha} h(x) = \frac{d}{dx} h(x)$ for $\alpha = 1$. Given that $h(x)$ and $g(x)$ are α-order differentiable for $x > 0$, and a and b are real constants, the beta derivative exhibits the following properties:

1 - $D_x^{\alpha}(ah(x) + bg(x)) = aD_x^{\alpha} h(x) + b_x^{\alpha} g(x)$.

2 - $D_x^{\alpha}(k) = 0$, where k is a constant.

3 - $D_x^{\alpha}(h(x)g(x)) = h(x)D_x^{\alpha} g(x) + g(x)D_x^{\alpha} h(x)$.

4 - $D_x^{\alpha}\left(\frac{h(x)}{g(x)}\right) = \frac{g(x)D_x^{\alpha}(h(x)) - h(x)D_x^{\alpha}(g(x))}{g^2(x)}$.

5 $- D_x^{\alpha}(h^{-1}(x)) = -\frac{D_x^{\alpha}(h(x))}{h^2(x)}$.

6 $- D_x^{\alpha} h(x) = \left(x + \frac{1}{\Gamma(\alpha)}\right)^{1-\alpha} \frac{dh(x)}{dx}$.

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3. Description of new extended auxiliary equation method

Assume that we are given a nonlinear partial differential equation of the form;

$$H(u, u_x, u_t, D_t^{\alpha}u, D_x^{\alpha}u, D_{tt}^{2\alpha}u, D_{xt}^{2\alpha}u, D_{xx}^{2\alpha}u, \dots) = 0, \tag{3}$$

where H is a polynomial function. The main steps for solving equation Eq. (1) using the new extended auxiliary equation method are summarized as follows:

Step 1: To obtain chirped solitons, the initial assumption is

$$u(x,t) = \varphi(\xi)e^{i(\emptyset(\xi)-\theta t)}, \xi = \frac{1}{\alpha}\left(x + \frac{1}{\Gamma(\alpha)}\right)^{\alpha} - \frac{v}{\alpha}\left(t + \frac{1}{\Gamma(\alpha)}\right)^{\alpha}, \tag{4}$$

where v, θ are the velocity and wave number.

Step 2: Substituting into Eq. (3) yields an ordinary differential equation in ζ of the form;

$$Q\left(U, U^{'}(\zeta), , U^{''}(\zeta), U^{'''}(\zeta), \dots\right) = 0, \tag{5}$$

where Q is a general polynomial.

Step 3: Assume that Eq. (5) has the formal solution;

$$U(\zeta) = \sum_{i=0}^{2N} B_i F^i(\zeta), \tag{6}$$

where A_i are constants to be determined, such that $B_{2N} \neq 0$, and $F(\zeta)$ satisfies the first-order ODE:

$$\left[F^{'}(\zeta)\right]^2 = c_0 + c_2F^2 + c_4F^4 + c_6F^6, \tag{7}$$

where $c_i, (i = 0,2,4,6)$ are arbitrary constants to be determined.

Step 4: Determining the positive integer N by balancing the highest-order derivatives and the nonlinear terms in Eq. (5).

Step 5: Substituting Eq. (6) along with Eq. (7) into Eq. (5) and collecting all the coefficients of $F^j\left(F^{'}\right)^l$, $(j = 0,1,2, \dots)$ and $(l = 0,1)$ then setting each coefficient to zero, a set of algebraic equations is obtained for c_i, $(i = 0,2,4,6)$ and $B_j, (j = 0,1,2, \dots, 2N)$. Solving the system, we find c_i and B_j.

Step 6: It is well known that Equation Eq. (7) has the following solutions:

$$F(\zeta) = \frac{1}{2}\sqrt{\frac{-c_4}{c_6}\left(1 \pm f_i(\zeta)\right)}, \tag{8}$$

where the function $f_i, (i = 1,2,3, \dots, 12)$ can be expressed through the Jacobi elliptic function $\mathrm{sn}(\zeta, m), \mathrm{cn}(\zeta, m), \mathrm{dn}(\zeta, m)$, where $0 < m < 1$ is the modulus of the Jacobi elliptic functions. When $m \to 0$ or $m \to 1$, the Jacobi elliptic function degenerates to hyperbolic functions and trigonometric functions, respectively.

The functions $f_i(\zeta), (i = 1,2,3, \dots, 12)$ given by Eq. (8) have 12 forms as follows:

If $c_0 = \frac{c_4^3(m^2-1)}{32c_6^2m^2}, c_2 = \frac{c_4^2(5m^2-1)}{16c_6m^2}, c_6 > 0,$ then

$$f_1(\zeta) = \mathrm{sn}(\rho\zeta), f_2 = \frac{1}{m\mathrm{sn}(\rho\zeta)}, \rho = \frac{c_4}{2m}\sqrt{\frac{1}{c_6}} \tag{9}$$

If $c_0 = \frac{c_4^3(1-m^2)}{32_6^2}, c_2 = \frac{c_4^2(5-m^2)}{16c_6}, c_6 > 0,$ then

$$f_3(\zeta) = m\mathrm{sn}(\rho\zeta), f_4 = \frac{1}{\mathrm{sn}(\rho\zeta)}, \rho = \frac{c_4}{2}\sqrt{\frac{1}{c_6}} \tag{10}$$

If $c_0 = \frac{c_4^3}{32c_6^2m^2}, c_2 = \frac{c_4^2(4m^2+1)}{16c_6m^2}, c_6 < 0,$ then

$$f_5(\zeta) = cn(\rho\zeta), f_6 = \frac{sn(\rho\zeta)}{dn(\rho\zeta)}\sqrt{1-m^2}, \rho = \frac{c_4}{2m}\sqrt{\frac{-1}{c_6}} \tag{11}$$

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If $c_0 = \frac{c_4^3 m^2}{32\frac{2}{6}(m^2-1)}, c_2 = \frac{c_4^2(5m^2-4)}{16c_6(m^2-1)}, c_6 < 0,$ then

$$f_7(\zeta) = \frac{dn(\rho\zeta)}{\sqrt{1-m^2}}, f_8 = \frac{1}{dn(\rho\zeta)}, \rho = \frac{c_4}{2}\sqrt{\frac{-1}{c_6(1-m^2)}} \tag{12}$$

If $c_0 = \frac{c_4^3}{32c_6^2(1-m^2)}, c_2 = \frac{c_4^2(4m^2-5)}{16c_6(m^2-1)}, c_6 > 0,$ then

$$f_9(\zeta) = \frac{1}{\mathrm{cn}(\rho\zeta)}, f_{10} = \frac{dn(\rho\zeta)}{\sqrt{1-m^2}\sin(\rho\zeta)}, \rho = \frac{c_4}{2}\sqrt{\frac{1}{c_6(1-m^2)}} \tag{13}$$

If $c_0 = \frac{m^2 c_4^3}{32\frac{2}{6}}, c_2 = \frac{c_4^2(m^2+4)}{16c_6}, c_6 < 0,$ then

$$f_{11}(\zeta) = dn(\rho\zeta), f_{12} = \frac{\sqrt{1-m^2}}{dn(\rho\zeta)}, \rho = \frac{c_4}{2}\sqrt{\frac{-1}{c_6}} \tag{14}$$

Step 7: Substituting $c_i, (i = 0,2,4,6), B_j, (j = 0,1,2,\ldots,2N)$, and Eqs. $[9-14]$ into Eq. (6), a numerous new kinds of Jacobi elliptic function solutions of equations Eq. (1) will be obtained.

4. Mathematical Analysis

In this section, we examined the space-time fractional cubic Nonlinear Fractional Schrödinger Equations (NLFSEs) in order to get more comprehensive and traditional precise wave solutions. The new extended auxiliary equation approach, which simplifies the solution of fractional partial differential equations by turning them into ordinary differential equations, was utilized to accomplish this aim. In order to gain a deeper understanding of these wave solutions' behaviors and properties, we also conducted a comprehensive mathematical investigation. Equation (1) was transformed into an ordinary differential equation that includes the real and imaginary parts of the solution by applying a fractional transformation. Analytical solutions that provide insights into the dynamics and characteristics of the system under study were made easier by this transition.

The real part:

$$(\mathrm{v}+\mathrm{a}_1\theta)\varphi\emptyset^{'} + \theta\varphi + (\mathrm{a}_1\mathrm{v}-\mathrm{a}_2)\varphi(\emptyset^{'})^2 + (\mathrm{a}_2-\mathrm{a}_1\mathrm{v})\varphi^{''} + \mathrm{a}_3\varphi^3 = 0, \tag{15}$$

The imaginary part:

$$(\mathrm{v}-\mathrm{a}_1\theta)\varphi^{'} + 2(\mathrm{a}_2-\mathrm{a}_1\mathrm{v})\varphi^{'}\emptyset^{'} + (\mathrm{a}_2-\mathrm{a}_1\mathrm{v})\varphi\varphi^{''} = 0. \tag{16}$$

Integrating Eq. (16) yields

$$\phi^{'} = \frac{a_1\theta - v}{a_2 - a_1 v} + C\varphi^{-2}. \tag{17}$$

Where C is the integration constant. The corresponding chirp defined by

$$\delta\mathrm{w} = -\frac{\partial(\emptyset(\xi)-\theta\mathrm{t})}{\partial\mathrm{x}} = \emptyset(\xi)',$$

concurrently, the chirp is shown by

$$\delta\omega = -\left[\frac{a_1\theta - v}{a_2 - a_1 v} + C\varphi^{-2}\right]. \tag{18}$$

Substituting into the real part we get

$$A_0\varphi^4 + A_1\varphi^2 + A_2 + A_3\varphi^3\varphi^{''} + a_3\varphi^6 = 0. \tag{19}$$

Where $A_0 = \frac{(v+a_1\theta)^2-2}{4(a_2-a_1 v)} + \theta, A_1 = 2Cv, A_2 = C^2(a_1 v - a_2), A_3 = a_2 - a_1 v,$ and C is the integration constant. Now balancing between $\varphi^3\varphi^{''}$ and φ^6 the highest order derivatives and highest power of the nonlinear term in Eq. (19), we obtain $\mathrm{N} = 1$. Therefore, the solution of Eq. (19) is of the form

$$\varphi(\xi) = \sum_{i=0}^{2} B_i F^i(\zeta) = B_0 + B_1 F + B_2 F^2. \tag{20}$$

Upon substituting Eq. (20) together with Eq. (7), we derive a polynomial expression in $\mathrm{F}^{\mathrm{i}}(\xi)$. Setting this polynomial to zero, representing the coefficient power of $\mathrm{F}^{\mathrm{i}}(\xi)$, we formulate a system of algebraic equations that incorporate the variables B_0, B_1, B_2, and c_4. Solving this system of algebraic equations using MATLAB provides the parameter values as follows:

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Case 1:

$$A_3 = -\frac{3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3}{4B_0^4c_2}, B_0 = B_0 \neq 0, , c_0 = c_0 \neq 0, c_2 = c_2,$$

$$c_4 = -\frac{4c_2^2(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)},\ c_6 = c_6\ ,$$

$$B_1 = 0, \text{and } B_2 = -\frac{2B_0c_2(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{c_0(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)}$$

Substituting into Eq. (20) along with Eq. (8), we obtain the following solutions of equation Eq. (19):

$$\varphi(\xi) = \mathrm{B}_0 + \frac{2\ \mathrm{B}_0\mathrm{c}_2^3(6\ \mathrm{A}_2 + \mathrm{A}_1\ \mathrm{B}_0^2 + 3\ \mathrm{B}_0^6\mathrm{a}_3)(\mathrm{A}_2 + \mathrm{A}_1\ \mathrm{B}_0^2 + \mathrm{A}_0\ \mathrm{B}_0^4 + \mathrm{B}_0^6\mathrm{a}_3)}{3\mathrm{c}_0^2\mathrm{c}_6(3\ \mathrm{A}_2 + \mathrm{A}_1\ \mathrm{B}_0^2 - \mathrm{A}_0\ \mathrm{B}_0^4 - 3\ \mathrm{B}_0^6\mathrm{a}_3)}(1f_i(\zeta)).$$

Provide that $\mathrm{c}_6 \neq 0$. Now, we obtain the following Jacobi elliptic function solutions for equation Eq. (1). From Eqs. [9 - 14] and Eq. (4), we get

$$u(x,t) = \left(\mathrm{B}_0 + \frac{2\ \mathrm{B}_0\mathrm{c}_2^3(6\ \mathrm{A}_2 + \mathrm{A}_1\ \mathrm{B}_0^2 + 3\ \mathrm{B}_0^6\mathrm{a}_3)(\mathrm{A}_2 + \mathrm{A}_1\ \mathrm{B}_0^2 + \mathrm{A}_0\ \mathrm{B}_0^4 + \mathrm{B}_0^6\mathrm{a}_3)}{3\mathrm{c}_0^2\mathrm{c}_6(3\ \mathrm{A}_2 + \mathrm{A}_1\ \mathrm{B}_0^2 - \mathrm{A}_0\ \mathrm{B}_0^4 - 3\ \mathrm{B}_0^6\mathrm{a}_3)}(1 \pm f_i(\zeta))\right) \cdot e^{i\left(\frac{a_1\theta - v}{a_2 - a_1v} + C\varphi^{-2} - \theta t\right)},$$

$$\delta\omega = -\left[\frac{a_1\theta - v}{a_2 - a_1v} + C\left(\mathrm{B}_0 + \frac{2\ \mathrm{B}_0\mathrm{c}_2^3(6\ \mathrm{A}_2 + \mathrm{A}_1\ \mathrm{B}_0^2 + 3\ \mathrm{B}_0^6\mathrm{a}_3)(\mathrm{A}_2 + \mathrm{A}_1\ \mathrm{B}_0^2 + \mathrm{A}_0\ \mathrm{B}_0^4 + \mathrm{B}_0^6\mathrm{a}_3)}{3\mathrm{c}_0^2\mathrm{c}_6(3\ \mathrm{A}_2 + \mathrm{A}_1\ \mathrm{B}_0^2 - \mathrm{A}_0\ \mathrm{B}_0^4 - 3\ \mathrm{B}_0^6\mathrm{a}_3)}(1 \pm f_i(\zeta))\right)^{-2}\right].$$

If $m \to 1$, then $sn(\zeta) \to \tanh(\zeta)$, and hence equation Eq. (1) has the hyperbolic function solutions

$$u_1 = \left(\mathrm{B}_0 + \frac{2\ \mathrm{B}_0\mathrm{c}_2^3(6\ \mathrm{A}_2 + \mathrm{A}_1\ \mathrm{B}_0^2 + 3\ \mathrm{B}_0^6\mathrm{a}_3)(\mathrm{A}_2 + \mathrm{A}_1\ \mathrm{B}_0^2 + \mathrm{A}_0\ \mathrm{B}_0^4 + \mathrm{B}_0^6\mathrm{a}_3)}{3\mathrm{c}_0^2\mathrm{c}_6(3\ \mathrm{A}_2 + \mathrm{A}_1\ \mathrm{B}_0^2 - \mathrm{A}_0\ \mathrm{B}_0^4 - 3\ \mathrm{B}_0^6\mathrm{a}_3)}\left(1 \pm \tanh\left(\frac{c_4}{2}\sqrt{\left(\frac{1}{c_6}\right)}\zeta\right)\right)\right) e^{i\left(\frac{a_1\theta - v}{a_2 - a_1v} + C\varphi^{-2} - \theta\right)}, \quad (21)$$

$$\delta\omega_1 = -\left[\frac{a_1\theta - v}{a_2 - a_1v} + C\left(\mathrm{B}_0 + \frac{2\ \mathrm{B}_0\mathrm{c}_2^3(6\ \mathrm{A}_2 + \mathrm{A}_1\ \mathrm{B}_0^2 + 3\ \mathrm{B}_0^6\mathrm{a}_3)(\mathrm{A}_2 + \mathrm{A}_1\ \mathrm{B}_0^2 + \mathrm{A}_0\ \mathrm{B}_0^4 + \mathrm{B}_0^6\mathrm{a}_3)}{3\mathrm{c}_0^2\mathrm{c}_6(3\ \mathrm{A}_2 + \mathrm{A}_1\ \mathrm{B}_0^2 - \mathrm{A}_0\ \mathrm{B}_0^4 - 3\ \mathrm{B}_0^6\mathrm{a}_3)}\left(1 \pm \tanh\left(\frac{c_4}{2}\sqrt{\left(\frac{1}{c_6}\right)}\zeta\right)\right)\right)^{-2}\right], \quad (22)$$

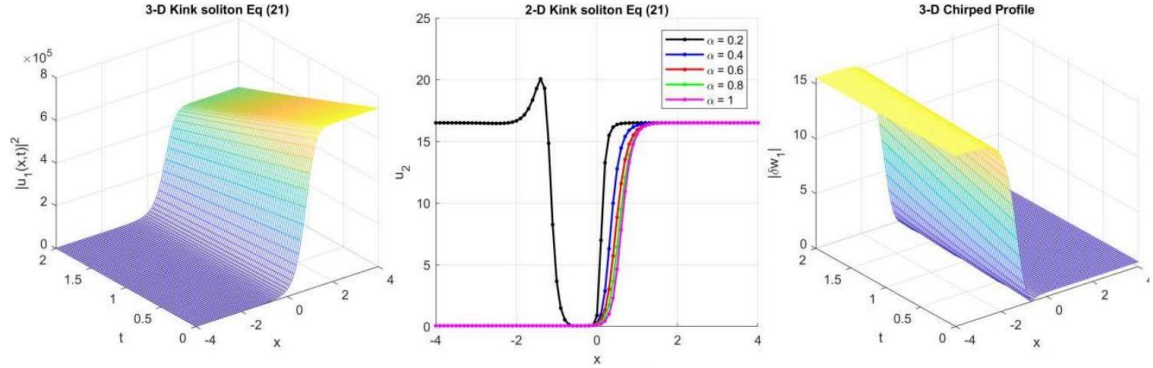


Figure 1: 3D, 2D and 3D chirped Kink soliton solutions $|\mathbf{u_1(x,t)}|^2$, of Equation (21) for $\mathbf{a_1 = 0.1, a_2 = 1.2, a_3 = 0.5,\ A_0 = 0.1,\ A_1 = 1.5,\ A_2 = -0.125, C = 1, c_0 = 0.2, c_2 = 0.4, c_6 = 0.01,\ B_0 = 0.25, \theta = 2.5, v = 0.75}$ and $\boldsymbol{\alpha = 1}$, with $\mathbf{x \in [-4,4]}$ and $\mathbf{t \in [0,2]}$.

$$u_2 = \left(B_0 + \frac{2B_0c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)}\left(1 \pm \coth\left(\frac{c_4}{2}\sqrt{\left(\frac{1}{c_6}\right)}\zeta\right)\right)\right) e^{i\left(\frac{a_1\theta - v}{a_2 - a_1v} + C\varphi^{-2} - \theta t\right)}, \quad (23)$$

$$\delta\omega_2 = -\left[\frac{a_1\theta - v}{a_2 - a_1v} + C\left(B_0 + \frac{2B_0c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)}\left(1 \pm \coth\left(\frac{c_4}{2}\sqrt{\left(\frac{1}{c_6}\right)}\zeta\right)\right)\right)^{-2}\right]. \quad (24)$$

If $m \to 0$, then $\mathrm{sn}(\zeta) \to \sin(\zeta)$, and hence equation Eq. (1) has the following solutions

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$$u_3 = \left(B_0 + \frac{2B_0c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \right) e^{i\left(\frac{a_1\theta - v}{a_2 - a_1v} + C\varphi^{-2} - \theta t\right)}, \quad (25)$$

$$\delta\omega_3 = -\left[\frac{a_1\theta - v}{a_2 - a_1v} + C\left(B_0 + \frac{2B_0c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \right)^{-2} \right]. \quad (26)$$

And the periodic solution

$$u_4 = \left(B_0 + \frac{2B_0c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \left(1 \pm \csc\left(\frac{c_4}{2}\sqrt{\left(\frac{1}{c_6}\right)}\zeta \right) \right) \right) e^{i\left(\frac{a_1\theta - v}{a_2 - a_1v} + C\varphi^{-2} - \theta t\right)}, \quad (27)$$

$$\delta\omega_4 = -\left[\frac{a_1\theta - v}{a_2 - a_1v} + C\left(B_0 + \frac{2B_0c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \left(1 \pm \csc\left(\frac{c_4}{2}\sqrt{\left(\frac{1}{c_6}\right)}\zeta \right) \right) \right)^{-2} \right]. \quad (28)$$

If $m \to 1$, then $\mathrm{cn}(\zeta) \to \mathrm{sech}(\zeta)$, and hence equation Eq. (1) has the following solutions

$$u_5 = \left(B_0 + \frac{2B_0c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \left(1 \pm \mathrm{sech}\left(\frac{c_4}{2}\sqrt{\left(\frac{-1}{c_6}\right)}\zeta \right) \right) \right) e^{i\left(\frac{a_1\theta - v}{a_2 - a_1v} + C\varphi^{-2} - \theta t\right)}, \quad (29)$$

$$\delta\omega_5 = -\left[\frac{a_1\theta - v}{a_2 - a_1v} + C\left(B_0 + \frac{2B_0c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \left(1 \pm \mathrm{sech}\left(\frac{c_4}{2}\sqrt{\left(\frac{-1}{c_6}\right)}\zeta \right) \right)^{-2} \right) \right], \quad (30)$$

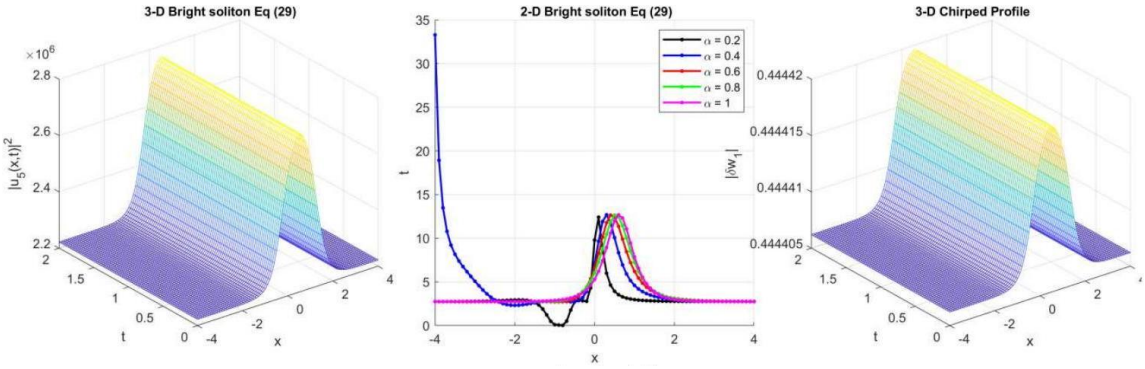


Figure 2: 3D, 2D and 3D chirped Bright soliton solutions $|u_5(x,t)|^2$, of Equation (29) for $a_1 = 0.1, a_2 = 1.2, a_3 = 0.5$, $A_0 = 0.1$, $A_1 = 1.5$, $A_2 = -0.125, C = 1, c_0 = 0.2, c_2 = 0.4, c_6 = -0.01$, $B_0 = 0.25, \theta = 2.5, v = 0.75$ and $\alpha = 1$, with x $\in [-4, 4]$ and t $\in [0, 2]$.

$$u_6 = \left(\mathrm{B_0} + \frac{2\,\mathrm{B_0c_2^3}(6\,\mathrm{A_2} + \mathrm{A_1}\,\mathrm{B_0^2} + 3\,\mathrm{B_0^6a_3})(\mathrm{A_2} + \mathrm{A_1}\,\mathrm{B_0^2} + \mathrm{A_0}\,\mathrm{B_0^4} + \mathrm{B_0^6a_3})}{3\mathrm{c_0^2c_6}(3\,\mathrm{A_2} + \mathrm{A_1}\,\mathrm{B_0^2} - \mathrm{A_0}\,\mathrm{B_0^4} - 3\,\mathrm{B_0^6a_3})} \right) e^{i\left(\frac{a_1\theta - v}{a_2 - a_1v} + C\varphi^{-2} - \theta t\right)}, \quad (31)$$

$$\delta\omega_6 = -\left[\frac{\mathrm{a_1}\theta - \mathrm{v}}{\mathrm{a_2} - \mathrm{a_1v}} + \mathrm{C}\left(\mathrm{B_0} + \frac{2\,\mathrm{B_0c_2^3}(6\,\mathrm{A_2} + \mathrm{A_1}\,\mathrm{B_0^2} + 3\,\mathrm{B_0^6a_3})(\mathrm{A_2} + \mathrm{A_1}\,\mathrm{B_0^2} + \mathrm{A_0}\,\mathrm{B_0^4} + \mathrm{B_0^6a_3})}{3\mathrm{c_0^2c_6}(3\,\mathrm{A_2} + \mathrm{A_1}\,\mathrm{B_0^2} - \mathrm{A_0}\,\mathrm{B_0^4} - 3\,\mathrm{B_0^6a_3})} \right)^{-2} \right]. \quad (32)$$

If $m \to 0$, then $dn(\zeta) \to 1$, and hence equation Eq. (1) has the following solutions

$$u_7 = u_8 = \left(B_0 + \frac{4B_0c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \right) e^{i\left(\frac{a_1\theta - v}{a_2 - a_1v} + C\varphi^{-2} - \theta t\right)}, \quad (33)$$

$$\delta\omega_7 = \delta\omega_8 = -\left[\frac{a_1\theta - v}{a_2 - a_1v} + C\left(B_0 + \frac{24c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \right)^{-2} \right]. \quad (34)$$

If $m \to 0$, then $dn(\zeta) \to 1, cn(\zeta) \to \cos(\zeta)$, and $\mathrm{sn}(\zeta) \to \sin(\zeta)$ hence equation Eq. (1) has the following solutions

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$$u_9 = \left(B_0 + \frac{2B_0c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \left(1 \pm \sec\left(\frac{c_4}{2}\sqrt{\left(\frac{1}{c_6}\right)}\zeta \right) \right) \right) e^{i\left(\frac{a_1\theta - v}{a_2 - a_1v} + C\varphi^{-2} - \theta t\right)}, \quad (35)$$

$$\delta\omega_9 = -\left[\frac{a_1\theta - v}{a_2 - a_1v} + C\left(B_0 + \frac{2B_0c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \left(1 \pm \sec\left(\frac{c_4}{2}\sqrt{\left(\frac{1}{c_6}\right)}\zeta \right) \right) \right)^{-2} \right], \quad (36)$$

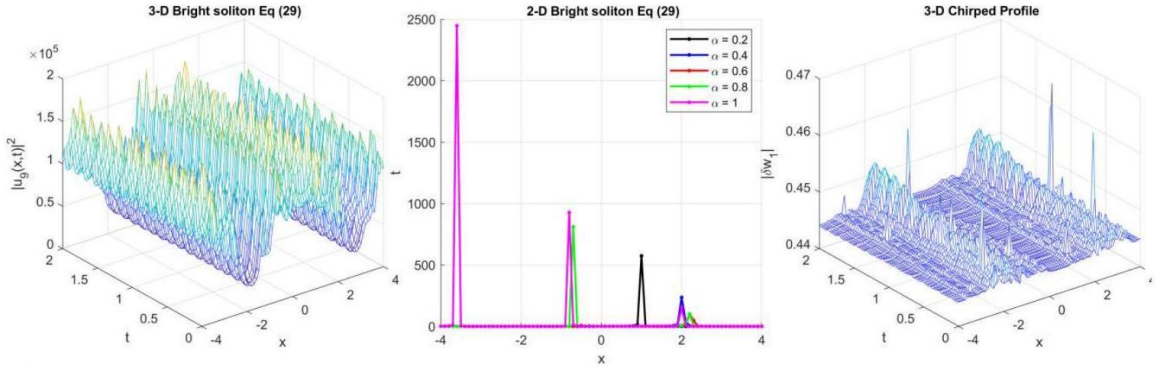


Figure 3: 3D, 2D and 3D chirped interaction multi-periodic Dark - Bright solution $|u_9(x,t)|^2$, of Equation (29) for $a_1 = 0.1, a_2 = 1.2, a_3 = 0.5$, $A_0 = 0.1$, $A_1 = 1.5$, $A_2 = -0.125$, $C = 1, c_0 = 0.2, c_2 = 0.4, c_6 = 0.0648$, $B_0 = 0.25, \theta = 2.5, v = 0.75$ and $\alpha = 1$, with $x \in [-4, 4]$ and $t \in [0, 2]$.

$$u_{10} = \left(B_0 + \frac{2B_0c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \left(1 \pm \csc\left(\frac{c_4}{2}\sqrt{\left(\frac{1}{c_6}\right)}\zeta \right) \right) \right) e^{i\left(\frac{a_1\theta - v}{a_2 - a_1v} + C\varphi^{-2} - \theta t\right)}, \quad (37)$$

$$\delta\omega_{10} = -\left[\frac{a_1\theta - v}{a_2 - a_1v} + C\left(B_0 + \frac{2B_0c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \left(1 \pm \csc\left(\frac{c_4}{2}\sqrt{\left(\frac{1}{c_6}\right)}\zeta \right) \right) \right)^{-2} \right], \quad (38)$$

If $m \to 0$, then $dn(\zeta) \to 1$, hence equation Eq. (1) has the following solutions

$$u_{11} = u_{12} = \left(B_0 + \frac{4B_0c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \right) e^{i\left(\frac{a_1\theta - v}{a_2 - a_1v} + C\varphi^{-2} - \theta t\right)}, \quad (39)$$

$$\delta\omega_{11} = \delta\omega_{12} = -\left[\frac{a_1\theta - v}{a_2 - a_1v} + C\left(B_0 + \frac{24c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \right)^{-2} \right]. \quad (40)$$

If $m \to 1$, then $dn(\zeta) \to \mathrm{sech}(\zeta)$ hence equation Eq. (1) has the following solutions

$$u_{11} = \left(B_0 + \frac{2B_0c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \left(1 \pm \mathrm{sech}\left(\frac{c_4}{2}\sqrt{\left(\frac{-1}{c_6}\right)}\zeta \right) \right) \right) e^{i\left(\frac{a_1\theta - v}{a_2 - a_1v} + C\varphi^{-2} - \theta t\right)}, \quad (41)$$

$$\delta\omega_{11} = -\left[\frac{a_1\theta - v}{a_2 - a_1v} + C\left(B_0 + \frac{2B_0c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \left(1 \pm \mathrm{sech}\left(\frac{c_4}{2}\sqrt{\left(\frac{-1}{c_6}\right)}\zeta \right) \right) \right)^{-2} \right], \quad (42)$$

$$u_{12} = \left(B_0 + \frac{4B_0c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \right) e^{i\left(\frac{a_1\theta - v}{a_2 - a_1v} + C\varphi^{-2} - \theta t\right)}, \quad (43)$$

$$\delta\omega_{12} = -\left[\frac{a_1\theta - v}{a_2 - a_1v} + C\left(B_0 + \frac{24c_2^3(6A_2 + A_1B_0^2 + 3B_0^6a_3)(A_2 + A_1B_0^2 + A_0B_0^4 + B_0^6a_3)}{3c_0^2c_6(3A_2 + A_1B_0^2 - A_0B_0^4 - 3B_0^6a_3)} \right)^{-2} \right]. \quad (44)$$

The soliton solutions originating from the general solutions in this study are diverse and novel.

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5. Physical Significance and Explanations

The fractional cubic nonlinear Schrödinger equation (NLFSE) with Kerr-law nonlinearity is very important in nonlinear optics and soliton dynamics. Here are some basic physical explanations associated with this equation

1. Modeling nonlinear optical phenomena: The strong mathematical model NLFSE with Kerr law nonlinearity may effectively describe several nonlinear optical processes in optical fibers. Light waves and nonlinear effects can be represented by combining beta derivatives with space and time, which is generally challenging in optical systems.
2. Soliton Dynamics: Solitons are solitary waves propagating across a medium while maintaining their form and velocity. They are self-reinforcing waves. Through the analysis of fractional cubic NLFSE solutions, scientists may learn more about the intricacy and behavior of solitons in optical fibers. These soliton solutions have special qualities and are crucial for the information and signal transfer in optical communication systems.
3. Wave propagation: A comprehensive grasp of wave propagation in nonlinear circumstances is provided by the NLFSE solution. Self-focusing and self-phase modulation are examples of phenomena created by the energy-dependent refractive index of the medium, which is captured by the Kerr nonlinearity factor in the equation. In order to improve optical device performance and communication systems overall, wave propagation in nonlinear media is crucial.
4. Nonlinear Effects: The intricate interaction between linear and nonlinear effects in optical systems is reflected in the existence of nonlinear factors in the NLFSE. Four-wave mixing, frequency conversion, and soliton creation are examples of phenomena that can result from nonlinear processes. Through the examination of NLFSE solutions, scientists may investigate how these nonlinear factors impact the behavior of light waves within optical fibers.
5. Mathematical Analysis: A methodical technique for solving fractional partial differential equations is provided by the mathematical analysis of the NLFSE utilizing the new extended auxiliary equation method. Researchers may obtain precise wave solutions that clarify the dynamics and characteristics of the system with this method, which also streamlines the solution process. The mathematical research conducted here provides valuable insights into the physical ramifications of the NLFSE in nonlinear optics.

In general, nonlinear optical processes, soliton dynamics, wave propagation, and nonlinear effects in optical fibers may all be studied with great benefit from the fractional cubic NLFSE with Kerr law nonlinearity. Scholars may enhance our comprehension of light-matter interactions in nonlinear media and provide avenues for novel applications in optical communication and signal processing by investigating the physical relevance and consequences of this equation.

6.Conclusion

In this work, optical and chirped optical solitons are studied in the context of space-time fractional cubic nonlinear Schrödinger equations (NLFSEs) incorporating Kerr law nonlinearity. The investigation, employing the extended auxiliary equation method, discloses a range of soliton solutions with hyperbolic and trigonometric functions. The solitons exhibit a variety of characteristics and come in dark, bright, kink, and periodic types. By displaying these solitons as 2D and 3D graphs with varying parameters, one may understand the dynamics and characteristics of the system. The results provide fresh insights into processes in condensed matter physics, quantum mechanics, and nonlinear optics and advance our knowledge of nonlinear dynamics. This study opens up new avenues for the investigation of intricate nonlinear phenomena and enhances scientific discourse.

Compliance with ethical standards

Disclosure of conflict of interest

The authors declare that they have no conflict of interest.

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