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From Initial Value Problems to Volterra Integral Equations: Formulation and Analysis of First-Order Differential Equations

Salha Wanis Saad Bahour *
Department of Mathematics, Faculty of Education, Qaser Bin Ghashir, University of Tripoli, Tripoli, Libya

من مسائل القيمة الابتدائية إلى معادلات فولتيرا التكاملية: صياغة وتحليل مسائل المعادلات التفاضلية من الرتبة الأولى

صالحة ونيس سعد باحور *
قسم الرياضيات، كلية التربية، قصر بن غشير، جامعة طرابلس، ليبيا

*Corresponding author: salhawanis395@gmail.com

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Abstract:
Ordinary differential equations (ODEs) are fundamental mathematical tools used to study a wide range of problems describing various phenomena and applications, particularly when accompanied by initial conditions that determine the solution of interest. The initial value problem (IVP) is an important formulation of such problems, as it seeks to determine a function that satisfies a given differential equation together with a prescribed initial condition. Under suitable conditions, an initial value problem can be reformulated as a Volterra integral equation, thereby highlighting the mathematical relationship between differential and integral formulations. This study aims to analyze the transformation between initial value problems for first-order ordinary differential equations and their corresponding Volterra integral equations. It clarifies the mathematical foundations and conditions underlying this transformation and explains how to move from the differential formulation to the integral formulation, and conversely, when appropriate. The study also examines the relationship between the two formulations in terms of the existence of solutions and presents selected examples to illustrate the transformation procedures. In addition, it investigates the transformation of a Volterra integral equation of the first kind into a Volterra integral equation of the second kind, clarifying the conditions under which this transformation can be performed and the mathematical foundations on which it is based. This study contributes to clarifying the mathematical relationship between ordinary differential equations and integral equations and highlights the integral formulation as an alternative representation of initial value problems that can be useful in analyzing such problems under suitable conditions.

Keywords: Ordinary differential equations, Initial value problem, Volterra integral equation, Transformation between differential and integral formulations, Existence of solutions.

الملخص:
تُعدّ المعادلات التفاضلية العادية من الأدوات الرياضية الأساسية المستخدمة في دراسة مجموعة واسعة من المسائل التي تصف ظواهر وتطبيقات متنوعة، ولا سيما عندما تكون مصحوبة بشروط ابتدائية تحدد الحل المطلوب. وتُعدّ مسألة القيمة الابتدائية إحدى الصياغات المهمة لهذه المسائل، إذ تهدف إلى تحديد دالة تحقق معادلة تفاضلية مع شرط ابتدائي محدد. وتحت شروط مناسبة، يمكن إعادة صياغة مسألة القيمة الابتدائية على هيئة معادلة تكاملية من نوع فولتيرا، مما يبرز العلاقة الرياضية بين الصياغتين التفاضلية والتكاملية. تهدف هذه الدراسة إلى تحليل التحويل بين مسائل القيمة الابتدائية للمعادلات التفاضلية العادية من الرتبة الأولى ومعادلات فولتيرا التكاملية المناظرة لها. كما توضح الأسس الرياضية والشروط التي يستند إليها هذا التحويل، وتبيّن كيفية الانتقال من الصياغة التفاضلية إلى الصياغة التكاملية، والعكس، متى كان ذلك مناسبًا. وتتناول الدراسة أيضًا العلاقة بين الصياغتين من حيث وجود الحلول، وتعرض أمثلة مختارة لتوضيح إجراءات التحويل، بالإضافة إلى دراسة التحويل بين معادلة فولتيرا التكاملية من النوع الأول ومعادلة فولتيرا التكاملية من

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النوع الثاني، مع توضيح الشروط التي تسمح بإجراء هذا التحويل والأسس الرياضية التي يستند إليها. وتسهم هذه الدراسة في توضيح العلاقة الرياضية بين المعادلات التفاضلية العادية والمعادلات التكاملية، وتسلط الضوء على الصياغة التكاملية بوصفها تمثيلًا بديلًا لمسائل القيمة الابتدائية، يمكن الاستفادة منه في تحليل هذه المسائل تحت شروط مناسبة.

الكلمات المفتاحية: المعادلات التفاضلية العادية، مسألة القيمة الابتدائية، معادلة فولتيرا التكاملية، التحويل بين الصياغتين التفاضلية والتكاملية، وجود الحلول.

Introduction

Ordinary differential equations are fundamental mathematical tools for studying a wide range of problems that describe various phenomena and applications, particularly when they are accompanied by initial conditions that determine the solution of interest. Among the important formulations of such problems is the initial value problem (IVP), which seeks to determine a function satisfying a given differential equation together with a prescribed initial condition. However, in some cases, obtaining the solution directly using conventional methods for solving differential equations may be difficult.

One approach to treating such problems is to transform the differential equation into an equivalent integral equation. This transformation establishes an important relationship between differential and integral equations. Under appropriate conditions, an initial value problem can be reformulated as a Volterra integral equation. Conversely, the differential equation can be recovered from its integral formulation when the required regularity and differentiability conditions are satisfied [1].

Several studies have investigated the relationship between differential and integral equations and the transformations between their respective formulations. Joshi et al. (2019), in their study entitled Conversion of Ordinary Differential Equation with Initial Condition (Initial Value Problem) into Volterra Integral Equation by Using Two Methods, Both the Methods Give Different and Perfect Volterra Integral Equation for Some Problems, investigated two methods for transforming ordinary differential equations with initial conditions into Volterra integral equations. The authors applied the two methods to different problems and compared the resulting integral equations. Their study indicated that the methods could produce identical integral equations in some cases and different formulations in others. This study is relevant to the present research because it directly addresses the transformation of initial value problems into Volterra integral equations [2].

Nadir and Guechi (2016), in their article entitled Integral Equations and Their Relationship to Differential Equations with Initial Conditions, published in General Letters in Mathematics, examined the relationship between differential equations with initial conditions and their corresponding integral equations. The authors presented an analytical approach for transforming differential equations with initial conditions into Volterra integral equations of the second kind. They also addressed the existence of solutions and discussed approximate numerical methods, including Euler's method, the finite difference method, and the trapezoidal method, as well as the convergence of the approximation error. This study provides relevant background for understanding the transformation from the differential formulation to the integral formulation. However, its scope includes aspects of numerical solution and error convergence that are not the primary focus of the present research [3].

Furthermore, Jagtap and Khambayat (2024), in their study entitled Conversion of the Volterra Integral Equation of the First Kind to the Volterra Integral of the Second Kind, investigated the transformation of Volterra integral equations of the first kind into equations of the second kind. Their approach involved differentiating both sides of the integral equation with respect to the independent variable by applying Leibniz's rule for differentiation under the integral sign. The authors also discussed conditions required for the transformation and presented examples to illustrate the procedure. This study is particularly relevant to the present research because it addresses the transformation between two types of Volterra integral equations, which constitutes an additional aspect of the investigation undertaken here [4]

Although these studies address related aspects of the relationship between differential and integral equations, their primary emphases differ. Joshi et al. (2019) focused on methods for transforming initial value problems into Volterra integral equations, whereas Nadir and Guechi (2016) also considered the existence of solutions and numerical approximation methods. In contrast, Jagtap and Khambayat (2024) concentrated on transforming Volterra integral equations of the first kind into equations of the second kind under appropriate conditions.

In light of these studies, the present research aims to provide a focused mathematical analysis of the transformation between initial value problems for first-order ordinary differential equations and their corresponding Volterra integral equations. It examines the mathematical conditions underlying the transformation in both directions and clarifies the relationship between the differential and integral formulations. In addition, the study considers the transformation of Volterra integral equations of the first kind into equations of the second kind, emphasizing the conditions under which this transformation is valid. Selected examples are presented to illustrate the transformation procedures and clarify the relationship between the corresponding formulations.

The significance of this study lies in bringing these related transformation problems together within a unified framework, with particular attention to first-order initial value problems, the mathematical conditions required for the transformations, and the relationship between the resulting formulations. Rather than proposing an

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entirely new transformation method, the study seeks to organize and analyze established mathematical procedures in a coherent manner that facilitates their understanding and application.

2. Preliminaries

2.1 Ordinary Differential Equation

Let $J \subseteq \mathrm{R}, \mathrm{U} \subseteq R^n$, and $\Lambda \subseteq R^k$ J be open subsets, and suppose that $f: J \times U \times \Lambda \to R^n$ is a smooth function. Here the term "smooth" means that the function f is continuously differentiable. An ordinary differential equation (ODE) is an equation of the form

$$\dot{X} = f(t, x, \lambda) \qquad (1)$$

Where the dot denotes differentiation with respect to the independent variable t (usually a measure of time), the dependent variable x is a vector of state variables, and , λ is a vector of parameters.[1]

It is equivalent to, the ordinary differential equation of order n be

$$\frac{d^n y}{dx^n} + \alpha_1(x)\frac{d^{n-1}y}{dx^{n-1}} + \alpha_2(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + \alpha_n y = \emptyset(x) \qquad (2)$$

Where $\alpha_1(x), \alpha_2(x), \ldots, \alpha_n(x), \emptyset(x)$ are defined and are continuous in $a \leq x \leq b$. [5]

2.2 Initial Value Problem

If the conditions are specified at a single point $x = x_0$, such as

$$y(x_0) = q_0, y'(x_0) = q_1, \ldots, y^{(n-1)}(x_0) = q_n \qquad (3)$$

They are termed initial conditions.

The differential equation (2) with the initial conditions (3) is known as Initial Value Problem (IVP). [6]

2.3 Integral Equation.

An Integral Equation is one in which function to be determined appears under the integral sign. The most general form of a linear integral equation is

$$h(x)u(x) = f(x) + \int_a^{b(x)} K(x, \mathrm{s})\mathrm{u}(\mathrm{s})\mathrm{ds} \text{ for } all\ x \in [a, b] \qquad (4)$$

In which, $u(x)$ is the function to be determined and $K(x, s)$ is called the Kernel of integral equation.[7]

2.4 Volterra Integral equation

A Volterra integral equation is of the type:

$$h(x)u(x) = f(x) + \int_a^x K(x, s)u(s)ds \text{ for all } x \in [a, b] \qquad (5)$$

That is, in Volterra equation $b(x) = x$.

(i) If $h(x) = 0$, the equation (5) reduces to

$$-f(x) = \int_a^x K(x, s)u(s)ds. \qquad (6)$$

This equation is called Volterra integral equation of first kind.

(ii) If $h(x) = 1$ the equation (5) reduces to

$$u(x) = f(x) + \int_a^x K(x, s)u(s)ds \qquad (7)$$

This equation is called Volterra integral equation of second kind.

In particular, If $f(x) = 0$ for all $x \in [a, b]$, the equation (5) reduces to

$$h(x)u(x) = \int_a^{bx} K(x, s)u(s)ds. \qquad (8)$$

This equation is called homogeneous integral equation. Otherwise, it is called non-homogeneous integral equation [5].

3. Conditions for Converting an Initial Value Problem into a Volterra Integral Equation

The Criteria governing the conversion of an initial value problem (IVP) into an equivalent Volterra integral equation can be summarized as follows:

1. **Type of the differential equation:** The differential equation may be linear or nonlinear, provided that the required assumptions for the conversion are satisfied.

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2. **Properties of the functions involved:** The functions appearing in the differential equation and the resulting integral equation should possess the required continuity and integrability properties. In particular, the kernel of the resulting Volterra integral equation should be sufficiently regular and free from non-integrable singularities.

3. **Initial conditions and integrability:** An initial value problem can be converted into an equivalent Volterra integral equation when the functions involved are integrable over the interval under consideration and the initial conditions are well defined. If the differential equation contains non-integrable singularities, the standard conversion procedure may not be applicable. [1]

5. Conversion of an Initial Value Problem into a Volterra Integral Equation

In this section, we will present the procedure for converting an initial value problem (IVP) into an equivalent volterrat integral equation. For simplicity, consider the following second-order ordinary differential equation:

$$y''(x) + p(x)y'(x) + q(x)y(x) = g(x) \qquad (9)$$

subject to the initial conditions $y(0) = \alpha, y'(0) = \beta$, Where $p(x), q(x)$ are analytic known functions, $g(x)$ is a known continuous function and α, β are given real constants.

To obtain the corresponding volterra integral equation, we first set:

$$y''(x) = u(x) \qquad (10)$$

Where $u(x)$ is a continuous function.

Integrating both sides of equation (10) with respect to x from 0 to x, we obtain:

$$y'(x) = y'(0) + \int_0^x u(t)dt \qquad (11)$$

Integrating both sides equation (11) again and by using the initial conditions in equation (9), we obtain:

$$y(x) = y(0) + xy'(0) + \int_0^x u(t)dt \qquad (12)$$

At this point, the initial value problem has been transformed into an equivalent Volterra integral equation

$$y(x) = \alpha + \beta x + \int_0^x (x - t)u(t)dt. \qquad (13)$$

After simplifying the resulting expression, we obtain:

$$u(x) = g(x) - [\beta p(x) + \alpha q(x) + \beta x q(x)] - \int_0^x \big(p(x) + q(x)(x - t)\big)u(t)dt. \qquad (14).$$

This equation can be written in the standard form:

$$u(x) = f(x) - \int_0^x K(x,t)u(t)dt \qquad (15)$$

Where $K(x,t) = p(x) + q(x)(x - t)$ and $f(x) = q(x) - \beta p(x) + \alpha q(x) + \beta x q(x)$.

It is evident from the resulting formulation that the initial condition is explicitly incorporated into the integral equation, where the term x_0 appears outside the integral.[1,5]

6. Converting a Volterra Integral Equation into an

$$f\big(x, b(x)\big) - a'(x)f\big(x, a(x)\big) + \int_{a(x)}^{b(x)} \frac{\partial f(x,t)}{\partial x} dt$$

The initial conditions for the resulting ordinary **Initial Value Problem (IVP).**

In this section, we consider the converse procedure, namely, transformation of a Volterra integral equation into an equivalent initial value problem (IVP).

The conversion is performed by differentiating both sides of the Volterra integral equation with respect to the independent variable as many times as necessary to eliminate the integral term and obtain the corresponding differential equation. This procedure requires the use of Leibniz's rule for differentiating an integral with variable limits: $\frac{d}{dx}\left(\int_{a(x)}^{b(x)} f(x,t)dt = b'(x)\right.$differential equations can be obtained by evaluating the integral equation and its successive derivatives at the initial point. [1,8]

7. conversion of Volterra integral equations.

Volterra integral equations can be classified into first-kind and second-kind equations according to the position of the unknown function and its occurrence in the integral term. Under suitable regularity conditions on the kernel, a Volterra integral equation of the first kind can be transformed into an equivalent Volterra integral equation of the second kind by differentiating both sides and applying Leibniz's rule. The following theorem states the conditions under which this transformation can be performed. [9]

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Theorem

The Volterra integral equation of the first kind

$$f(x) = \lambda \int_0^x k(x,t)u(t)dt \qquad (16)$$

With the kernel $k(x,t)$ continuously differential with respect to x and $k(x,x) \neq 0$ for all $x \in [0,b]$, $f(x)$ is differentiable. Then the first kind equation can be transformed into a Volterra integral equation of the second-kind.

Proof

The equation (15) is differentiating, and using leibntiz rule

$$\frac{df}{dx} = \lambda \mathrm{k(x,x)u(x)} + \lambda \int_0^x \frac{\partial k(x,t)}{\partial x} u(x)dt - \frac{df}{dx}$$

$$-\lambda \mathrm{k(x,x)u(x)} = \lambda \int_0^x \frac{\partial k(x,t)}{\partial x} u(t)dt - \frac{df}{dx}$$

$$u(x) = \frac{1}{\lambda \mathrm{k(x,x)}} \frac{df}{dx} - \frac{1}{k(x,x)} \int_0^x \frac{\partial k}{\partial x} u(t)dt \;\; ; \;\; k(x,x) \neq 0$$

This equation can be written as:

$u(x) = g(x) + \int_0^x H(x,t)u(t)dt \qquad (17)$

Where $g(x) = \frac{1}{\lambda \mathrm{k(x,x)}} \frac{df}{dx}$ and $H(x,x) = \frac{-1}{k(x,x)} \frac{\partial k}{\partial x}$.

Here, equation represents the Volterra integral equation of second kind. [9]

8. Condition for the Equivalence of solutions to differential and integral equations for continuous function.

Theorem

Let f be a continuous function defined a domain D of the xy-plane, φ be a continuous function defined on a real interval $\alpha \leq x \leq \beta$ and such that $[x, \varphi(x)] \in D$ for all $x \in [\alpha, \beta]$ and let x_0 be any real number such that $\alpha \leq x_0 \leq \beta$. Then: φ is a solution of the differential equation

$\frac{dy}{dx} = f(x,y)$ on $[\alpha, \beta] \qquad (18)$

and is such that $\varphi(x_0) = y_0$ if and if only if φ satisfies integral equation

$\varphi(x) = y_0 + \int_{x_0}^x f\big(t, \varphi(t)\big)dt$ for all $x \in [\alpha, \beta]$. $\qquad (19)$

Proof

First, suppose that φ satisfies the differential equation $\frac{dy}{dx} = f(x,y)$ on $[\alpha, \beta]$

Then $\frac{d\varphi}{dx} = f(x,, \varphi(x))$ for all $x \in [\alpha, \beta]$

Since f is continuous, φ' is a continuous function on $[\alpha, \beta]$, therefore $\varphi(x)$ is continuously differentiable on $[\alpha, \beta]$.

Integrating both sides from x_0 to x, we obtain

$\varphi(t) = \int_{x_0}^x f\big(t, \varphi(t)\big)dt.$

Using the initial condition $\varphi(x_0) = y_0$, it follows that:

$\varphi(x) = y_0 + \int_{x_0}^x f\big(t, \varphi(t)\big)dt.$

Thus, φ satisfies the in equation (19).

Conversely, suppose that φ satisfies the integral equation (19).

Since f is continuous and $\varphi(x) = y_0 + \int_{x_0}^x f\big(t, \varphi(t)\big)dt.$

Differentiating both sides with respect to x, and applying the Fundamental Theorem of Calculus, gives: $\varphi'(x) = f\big(x, \varphi(x)\big)$.

Furthermore, by setting $(x = x_0)$ in the integral equation, we obtain

$$\varphi(x_0) = y_0 + \int_{x_0}^{x_0} f\big(t, y(t)\big)dt \; = y_0.$$

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Therefore, φ satisfies the initial value problem.[10]

9. Applications of the Transformation between Differential and Integral Forms.

This section presents illustrative applications of the transformation between initial value problems and Volterra integral equations. The examples demonstrate the practical implementation of the transformations discussed in the preceding sections and show how the initial condition is incorporated into, or recovered from, the corresponding integral or differential formulation.

Example 9.1

Consider the initial value problem $y''(x) - y(x) = \sin x, \quad y(0) = 0,\ y'(0) = 0.$

By leveraging the aforementioned steps and conducting a comparative analysis with equation (9) and with the initial conditions it is evident that, $\alpha = \beta = 0, p(x) = o, q(x) = 1$, and

$g(x) = \sin x.$

Thus the Volterra integral equation is: $u(x) = \sin x + \int_0^x (x - t)u(t)dt.$ [1]

Example 9.2

Consider the following Volterra integral equation $u(x) = e^x + \int_0^x u(t)dt.$

Differentiating both sides and using Leipetiz rule we find:

$$u'(x) = e^x + u(x).$$

It is clear that there is no need for differentiating again because we get rid of the integral sign.

To determine the initial condition, we substitute $x = 0$ into both sides, we get $u(0) = 1$ this in turn gives the initial value problem: [1]

$$u'(x) - u(x) = e^x, u(0) = 1.$$

10. Application of the Transformation of a Volterra Integral Equation of the First Kind into One of the Second Kind

Example 10.1

Reducing a Volterra Integral Equation of the First Kind to the Second Kind

$$\sin x = \lambda \int_0^x e^{x-t} u(t)dt$$

The given integral equation can be transformed into a Volterra integral equation of the second kind by differentiating both sides with respect to the independent variable. Thus, we obtain the following equation

$$\cos x = \lambda e^{x-x} u(x) + \lambda \int_0^x e^{x-t}\, u(t)dt$$

$$\cos x = \lambda \mathrm{u}(\mathrm{x}) + \lambda \int_0^x e^{x-t}\, u(t)dt$$

$$\rightarrow u(t) = \frac{1}{\lambda}\cos x - \int_0^x e^{x-t}\, u(t)dt$$

Which is Volterra integral equation of second kind. [9]

11. Conclusion

This study has clarified the mathematical relationship between first-order ordinary differential equations and Volterra integral equations. Under suitable conditions, an initial value problem for a first-order ordinary differential equation can be transformed into an equivalent Volterra integral equation, and the corresponding differential formulation can be recovered from the integral formulation under appropriate conditions.

The study also shows that such transformations are not possible for every differential or integral equation without imposing suitable assumptions. In particular, the transformation between the two formulations depends on the properties of the functions involved and the conditions imposed on the problem.

Furthermore, under appropriate conditions, a Volterra integral equation of the first kind can be transformed into a Volterra integral equation of the second kind by differentiating the equation and applying Leibniz's rule.

Finally, the relationship between the initial value problem and its corresponding integral formulation was considered from the perspective of solution existence. The selected examples demonstrate how the

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transformation between the differential and integral formulations can provide equivalent representations of the same problem and clarify the mathematical connection between the two formulations.

Compliance with ethical standards

Disclosure of conflict of interest

The authors declare that they have no conflict of interest.

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