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Application of some stability criteria of stability for the Spring equation

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Abstract:
This study aimed to investigate the stability of the equation for energy motion of spring using two criteria, namely Routh_Hurwitz and Mikailove, and compare between them. discuss one of the physical applications of the differential equation, which is the issue of the spiral spring and its waning and non-diminishing vibrations. study its stability by imposing certain conditions so that we get a second-order equation and homogeneous.obtianin stability the diminishing vibratory motion of the spring is conserving energy and stability the differential equation Motion for spring as well as the diminishing vibratory motion of the spring is not conserving energy in the first criterion and instability the diminishing vibratory motion of the spring is conserving energy for tow cases in the second.

Keywords: Mikhailov criteria, Routh_Hurwitz criteria, Stability, Instability, Change sine, Lotka-Volterra, Prey, Predator

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Introduction

In this paper, we will study the stability of one of the applications of physics, which is the motion of the simple pendulum, by applying the stability criteria (Mikhailov and Routh- Hurwitz) and their conditions on the rate of the spring (kinetic energy equation) and knowing if it is an energy conservation by imposing conditions and applying them to the spring equation and converting it into an equation Homogeneous in the second order. Where the scientist (Christian_Hygens) discussed the meaning of the differential equation and noticed that the spring movement is periodic and the time of oscillation does not depend on its amplitude and determined the acceleration of gravity [1] and many physical applications lead to differential equations of the second order [2]. Now Routh-Horwitz stability criteria one of the most important topics in control systems. These criteria which was independently developed by Routh and A. Hurwitz in the late 19th century, is a straightforward but effective method for figuring out how many roots a polynomial has without computing those roots. By using the polynomial's coefficients and a straightforward calculation that is explained, an array is created. the number of roots is determined, however there may be limitations when utilizing the Routh- Hurwitz criterion to determine stability. possess the This criteria used in several areas that were developed, where the researcher Wang and others presented the Analysis and Design of operational Amplifier stability Based on Routh _Hurwitz Stability [3] and used From the researchers Sanchez and others in Stability Analysis of the Lorenz System [4] Ashwell's used in stability analysis of two penda tors and one prey population model in [5] Also use the researcher Teklu in (2022)developed the criterion in Mathematical analysis of the transmission dynamics of covid_19 infection see[6].We will apply it to the (Mikhailov criteria) on the kinetic energy equation. The main idea behind the classical (Mikhailov Criteria) is to Substitute y=iw in P(y) to get P(iw)=u(w)+iv(w) Generalization of Mikhailov's criterion with applications by the scientist KeeL and others (June 2000) [7] Then this scale was modified for the fractional fuentes and Melchor systems by the scientist Mendiola_Fyentes in the year (2018) [8.] and in the year (2022) generalized Scientist Melchor_Aguilar et al. These criteria of fractional order systems with delays in [9] Also, the characteristic roots of differential linear delay equations were calculated by the scientist Niculescu and others in (2023) in [10]

<u>Definition 1.1 [11]:</u>

The zero solution of equation x` = f(x)…… (1.1) x = 0 it is stable if for any integer > 0, we can identify a number > 0 (depending on) such that, if (t, t_0, x_0) is any solution of equation (1.1) having (t, t_0, x_0) = x_0 and || (t, t_0, x_0) |.

Definition 1.2:

The zero-solution x = 0 is unstable if it is not stable.

It is important to notice that there is another types of stability, such as uniform stability, uniform asymptotic stability, absolute stability, stability in the large, ..., etc., but we will not discuss them, since they did not encounter us in the rest of the thesis in [11].

Definition 1.3:

The Routh-Hurwitz stability criteria is Each term in the first column of the Routh array must be exist and positive. In order for the system to be stable if all Each elements characteristic equation be positive if $r_a > 0$ In the absence of this requirement, the system is unstable see [15].

Routh Table:

The first two rows are derived from the characteristic of the array equation. The remaining are calculated as:

The characteristic equation of the control system is given as: see [15]

$$r_0 y^n + r_1 y^{n-1} + r_2 y^{n-2} + \dots + r_{n-1} y + r_n = 0 \tag{1}$$

shows the coefficients of Routh-Hurwitz's equation in table 1 see [15].

Table 1: Coefficients Routh-Hurwitz.

y^4	r_0	r_2	r_4
y^3	r_1	r_3	r_5
y^2	S_1	S_2	S_3
y^1	G_1	G_2	G_3
y^0	C_1	C_2	C_3

$r_0, r_1, \dots, r_n$, $S_1, S_2, \dots, S_n$, and $C_1, C_2, \dots, C_n$, and $G_1, \dots, G_n$ are coefficients, $S_1, S_2, s_3, C_1, C_2, G_1$, and G_2, are coefficients calculated from the operation as:

$$S_1 = \frac{(r_1 r_2 - r_0 r_3)}{r_1},$$

$$S_2 = \frac{(r_1 r_4 - r_5 r_0)}{r_1},$$

$$S_3 = \frac{(r_1 r_6 - r_7 r_0)}{r_1},$$

$$G_1 = \frac{S_1 r_3 - S_2 r_1}{S_1},$$

$$G_1 = \frac{S_1 r_5 - S_3 r_1}{S_1},$$

$$C_1 = \frac{G_1 S_2 - S_1 G_2}{G_1},$$

$C_2 = \frac{G_1S_3 - S_1G_3}{G_1}$,

In this operation, the missing terms are considered zero and elements of any row can be divided by positive number to simplify the calculation no.

(Mikhailov criteria):

The main idea behind the classical (Mikhailov criteria) is to substitute:

y = iw in P(y) to get P(iw) = u(w) + iv(w) and then, measure the total variation of the argument of the function P(iw) as w increases from 0 to ∞. The corresponding plot of P(iw) in the complex plane is the – so called Mikhailov curve in the Fig. (1) see [14].

The step of solution follows:

Puty = iw ,

f(iw) = u(w) + iv(w),

$u(w) = r_n - r_{n-2}\, w^2 + r_{n-4}\, {}^{w4} - \dots$,

$v(w) = r_{n-1}\, w - r_{n-3}\, w^3 + \dots$,

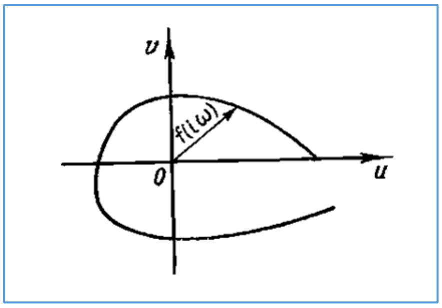

Figure 1: Mikhailov curve

1.Mass Spring System:

The system consists of a spring of length and weight w of mass m attached to one of its ends. Forces will affect the mass, namely gravity of $w = mg$ and the tensile force is its magnitude, and k is the (spring constant) and other forces F(t) may affect the mass and its direction depends on whether F(t) is negative or positive.

Now, take the differential equation describing the waning vibratory motion of the spring of the form:

$$\frac{w}{g} x``(t) + bx`(t) + kx(t) = \frac{w}{g} F(t) \qquad (2)$$

Simplify equation (1) and assuming:

$$m = \frac{w}{g}$$

As well as we assume that there is no oscillating external force, that is $(F(t) \equiv 0)$.

And divides by m obtain a homogeneous differential equation of the second order.

$$x^{``}(t) + \frac{b}{m} x`(b) + \frac{k}{m} x(t) = 0 \qquad (3)$$

$$x``(t) + \propto x`(t) + \beta x(t) = 0 \qquad (4)$$

$\propto, \beta$ *is positive constant and,* $\propto = \frac{b}{m}, \beta = \frac{k}{m}$

Remark (1)**:** general form of the equation for energy motion of spring is:

$$x''(t) + \propto x'(t) + \beta x(t) = 0 \qquad (5)$$

must satisfy the following condition:

1. Assume that there is no oscillating external force such that: $(F(t) \equiv 0)$.
2. A suppression (damped force) is represented by $(bx^{'})$ it is of arc that obstructs the direction of motion of a body.
3. The displacement should be to wards that top.

***Remark (2)*:** general form of the equation for energy motion of spring is:

$$x^{''}(t) - \propto x^{'}(t) + \beta x(t) \tag{6}$$

must satisfy the following condition:

1. Assume that there is no oscillating external force such that: $(F(t) \equiv 0)$.
2. The a suppression (damped force) is represented by $(-bx^{'})$, it is force a hindering in the opposite direction of body's motion.
3. The displacement should be to wards that top.

Studies stability differential equation motion using Routh-Hurwitz criteria:

Case:(1)

$$x^{''}(t) - \propto x^{'}(t) + \beta x(t) = 0 \tag{7}$$

Example:

$$y^2 - 2y + 2 = 0 \tag{8}$$

Solution:

$a_1 = \frac{(-2 \times 2)-(1 \times 0)}{-2} = \frac{4}{2} = 2$

Table 2: Coefficients Routh-Hurwitz:

y^2	1	+2
y^1	-2	0
y^0	+2	0

Since change sign in the first column the equation is in stability.

the diminishing vibratory motion of the spring is conserving energy.

Case: (2)

$$x^{''}(t) + \propto \bar{x}(t) + \beta x(t) = 0 \tag{9}$$

Example:

$$y^2 + 3y + 1 = 0 \tag{10}$$

Solution:

$a_1 = \frac{3 \times 1 - 1 \times 0}{3} = \frac{3}{3} = 1$

$a_2 = \frac{3 \times 0 - 1 \times 0}{3} = 0$

Table 3: Coefficients Routh-Hurwitz:

y^2	1	1	0
y^1	3	0	0
y^0	1	0	

Since there is no change sing in the first column a ***Studies stability general differential equation motion energy for spring using (Mikhailov-criteria) in tow case:***

Case :(1)

$$x''(t) + \propto x'(t) + \beta = 0 \tag{11}$$

Solution:

$f(iw) = (iw)^2 + \propto(iw) + \beta = 0$

$= -w^2 + i\propto(w) + \beta$

$u(w) = \beta - w^2 \Rightarrow w = \pm\sqrt{\beta}$

$v(w) = i\propto w \Rightarrow w = 0$

$u(0) = \beta - (0)^2 = {}^{+}\beta$

$u(+\sqrt{\beta}) = \beta - (\sqrt{\beta})^2 = 0$

$v(0) = \propto(0) = 0$

$v(\sqrt{\beta}) = +\propto\sqrt{\beta}$

f(iw)	0	$\sqrt{\beta}$
u	$+\beta$	0
v	0	$+\propto\sqrt{\beta}$

The table obtains the real root, so the critical point (0,0) is instable. Thus, proved that the optimistic vibrational spring movement is energy conservation

Case: (2)

$$x''(t) - \propto \bar{x}(t) + \beta = 0 \tag{12}$$

Solution:

$f(iw) = (iw)^2 - \propto(iw) + \beta = 0$

$= -w^2 - i\propto(w) + \beta = 0$

$u(w) = \beta - w^2 \Rightarrow w = \pm\sqrt{\beta}$

$u(0) = \beta - (0)^2 = {}^{+}\beta$

$u(\sqrt{\beta}) = \beta - (\sqrt{\beta})^2 = 0$

$v(0) = \propto(0) = 0$

$v(\sqrt{\beta}) = -\propto\sqrt{\beta}$

f(iw)	0	$\sqrt{\beta}$
u	$+\beta$	0
v	0	$-\propto\sqrt{\beta}$

The table obtains the real root, so the critical point (0,0) is instable. Thus, proved that the optimistic vibrational spring movement is energy conservation

Conclusion

The goal of this research was to explore and compare the stability of the equation for energy motion of spring by using two criteria, namely Routh Hurwitz and Mikailove. produced two distinct scenarios. The results showed that the first method is simpler and more effective than the second method for determining the system's stability.

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