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The Fuzzy Future: Embracing the Potential of Fuzzy Functions

Omalkhear Salem Blebalu*

College Of Health Sciences /Aleajilat, University of Zawia, Libya

*Corresponding author: *blabulo1985@gmail.com*

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Abstract:

Most of the tools we've used in the past for formal modeling, reasoning, and computing are clear, predictable, and precise. By crisp, we mean that the answer is either yes or no, not more or less. In traditional dual logic, for example, a statement can either be true or false but not both. In set theory, an element either belongs to a set or it doesn't. In optimization, a solution is either possible or it isn't. Precision means that the parameters of a model exactly represent either how we see the phenomenon being modeled or how the real system being modeled works. Most of the time, precision also means that the model is clear or that it has no ambiguities.

This study mostly talks about: 1) a generalized treatment of fuzzy sets of type n, where n is an integer greater than or equal to 1, with an example, mathematical discussions, and real-life interpretations of the given mathematical concepts; 2) the potentials and links between fuzzy logic and probability logic that haven't been talked about in one document; and 3) the representation of random and fuzzy uncertainties and ambiguities that come up in data-driven systems.

Keywords: Fuzzy Sets, Fuzzy Logic, Probability, Members of Elements

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Introduction

In the literature, fuzzy sets are often described as a "primitive notion" that is often, but not always, linked to uncertainty and is intuitively explained by the fact that it is similar to "classical sets." Usually, a textbook on the subject will start by talking about how real-valued generalizations of classical characteristic functions work, and then it will talk about how fuzzy set modeling can be used to make sense of everyday knowledge.

From an epistemological point of view, this attitude seems to assume that the idea itself will be built up step by step. One problem is that, because the premises aren't very strong, more formal and informal elements have to be added at each stage of theoretical development. For example, a good mathematical definition for the basic set-theory operators, such as conjunction, disjunction, and complement, is usually found by starting with a group of natural axioms (e.g., continuity, commutativity, monotonicity, etc.). Even though these axioms make sense, they only apply to each group of things to be defined and are based on common sense. Not surprisingly, method works less and less well as the formal apparatus grows.

This is especially clear in the field of fuzzy logic, which is closely related. For example, Paris (1994) gives a set of natural axioms for negation, conjunction, and disjunction, but he doesn't go into detail about an implication-like connective because "it seems much less clear what axioms should hold for this function." Clearly, the situation doesn't get better when a fuzzy-logical con-sequence relation is put in place.

The main point of this paper is that fuzzy sets could be described in a different way, as a derivative idea in a suitable logical framework. More specifically, fuzzy sets can be made to emerge from a logical "deep structure" that controls how they behave "on the surface." Even though the modal probabilistic framework that has to be used is a bit complicated, the result is a clearer analysis of how we know what we know. Also, the way of doing things seems to keep the quality of the original setting. At the very least, it can be shown that the proposed framework can be used to encode some well-known fuzzy techniques.

Fuzzy logic

Fuzzy logic is a superset of conventional (Boolean) logic that has been extended to handle the concept of partial truth - truth values between "completely true" and "completely false". It was introduced by Dr. Lotfi Zadeh of U.C. Berkeley in the 1960's.

Sets and propositions are the two basic ideas that mathematical logic deals with. Zadeh developed fuzzy logic in the 1960s to expand the analysis of set theory and propositional calculus, which at the time could only be done using classical logic. A continuous multi-valued logic system is fuzzy logic. One could think of fuzzy logic as a generalization of classical logic. Fuzzy logic's linguistic variable, a special idea that it uses only, enables computations based on verbal data to be supported by this logic. Furthermore, FL maps an imprecise concept into one with a better degree of clarity by taking inspiration from the way humans think about uncertain information.

A set is a clear idea in classical logic: a mathematical object (such as a number, partition, matrix, variable, etc.) either "belongs to" the set, in which case its degree of membership to the set is 1, or "does not belong to" the set, in which case its degree of membership to the set is 0. Therefore, a crisp set X may be expressed as a collection of mathematical objects, e.g.,

$$X= \{X_1, X_2, \ldots.., X_n\}$$

Similarly, a proposition in classical logic is either "true" (may be quantified by a crisp value 1) or "false" (may be quantified by a crisp value 0).

In fuzzy logic, sets are fuzzy concepts. Our main focus is on the general case of type-n fuzzy sets, with n=1,2,3,.... To motivate this, we start with an example.

Let's say that, based on the fact that Felix is 27 years old, we would like to know whether or not Felix is a young person. Given that 27 is a quantitative concept and that young is a qualitative one, it is necessary to define young in order to relate the quantitative data on Felix's age to the quantified definition of young.

The graph in Figure 1 can be used to define young; it corresponds to a definition that divides people into two clear categories, young and not young, i.e., people under the age of 40 belong to the young set, and everyone else is not young. So, in terms of quantity, "Felix has a membership degree of 1 in the set young," while in terms of quality, "Felix is unquestionably young."

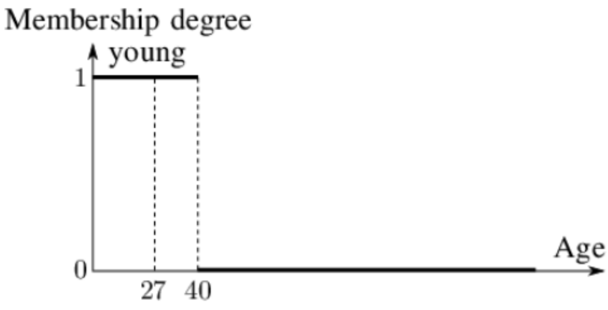

Figure 1: Using crisp sets for quantifying young.

Instead, young can be characterized using the graph in Figure 2, where ages fluctuate along a spectrum rather than being categorically young or obviously old, i.e., the degree of belonging to the set young varies in [0,1] rather than [0,1]. Thus, "Felix has a membership degree of 0.9 and is a member of the set young." In terms of quality, "Felix is largely young." After that, Young is quantified with a type 1 fuzzy set.

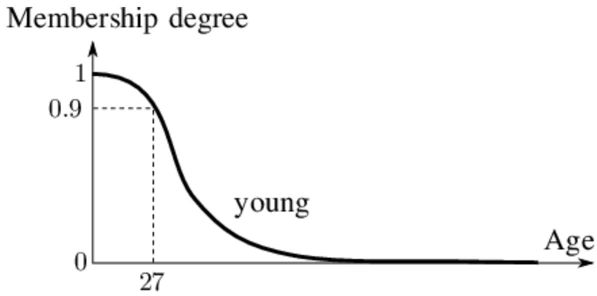

Figure 2: Using type-1 fuzzy sets for quantifying young.

Secondly, assume that it is not precisely understood where the curve's border defines young. In Figure 3, for instance, the intensity of black represents how certain we are that any point on the separating curve will be within the boundaries of the provided 2-dimensional plane. One can answer "Felix belongs to the set young with a primary membership degree in the interval [0.57,0.98]" or "Felix may to a great extent be young" by simply glancing at the vertical axis (also known as the primary membership degree). Consider a third dimension that measures the black color's intensity in real numbers between 0 and 1, where black is equal to 1 and white is equal to 0. (See figure 4). The secondary membership degree is represented by this dimension. Felix is young, with a primary membership degree ranging from [0.57,0.98] to [0,1], and a secondary membership degree ranging from [0,1]. "Felix's age equates to a secondary membership degree of 0.83 for the primary membership degree of 0.88," for example. In this scenario, young have been quantified using a type-2 fuzzy set.

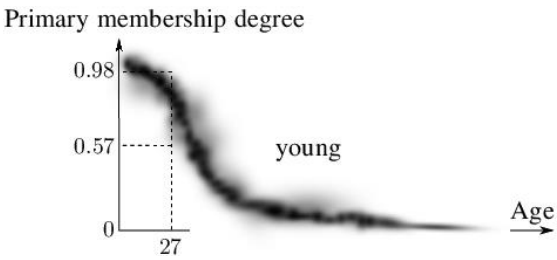

Figure 3: Using type-2 fuzzy sets for quantifying young.

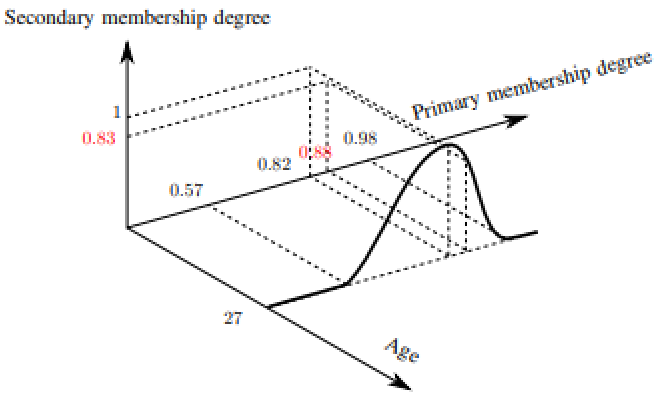

Figure 4: 3D representation of the type-2 membership function for quantifying young, represented for the specific age of 27.

Where is Fuzzy Logic Used?

Directly, very few places. The only pure fuzzy logic application I know of is the Sony PalmTop, which apparently used a fuzzy logic decision tree algorithm to perform handwritten (well, computer lightpen) Kanji character recognition.

The only common use of fuzzy logic, to my knowledge, is as the underlying logic system for fuzzy expert systems.

Logic Operations
Ok, we now know what a statement like

X is LOW

means in fuzzy logic. The question now arises, how do we interpret a statement like

X is LOW and Y is HIGH or (not Z is MEDIUM)

The standard definitions in fuzzy logic are:

truth (not x) = 1.0 - truth (x)
truth (x and y) = minimum (truth(x), truth(y))
truth (x or y) = maximum (truth(x), truth(y))

which are simple enough. Some researchers in fuzzy logic have explored the use of other interpretations of the AND and OR operations, but the definition for the NOT operation seems to be safe. Note that if you plug just the values zero and one into these definitions, you get the same truth tables as you would expect from conventional Boolean logic.

Some examples - assume the same definition of TALL as above, and in addition, assume that we have a fuzzy subset OLD defined by the membership function:

```
old (x) = { 0,                        if age(x) < 18 yr.
            (age(x)-18 yr.)/42 yr.,   if 18 yr. <= age(x) <= 60 yr.
            1,                        if age(x) > 60 yr. }
```

And for compactness, let

a = X is TALL and X is OLD
b = X is TALL or X is OLD
c = not X is TALL

Then we can compute the following values.

height	age	X is TALL	X is OLD	a	b	c
3' 2"	65?	0.00	1.00	0.00	1.00	1.00
5' 5"	30	0.21	0.29	0.21	0.29	0.79
5' 9"	27	0.38	0.21	0.21	0.38	0.62
5' 10"	32	0.42	0.33	0.33	0.42	0.58
6' 1"	31	0.54	0.31	0.31	0.54	0.46
7' 2"	45?	1.00	0.64	0.64	1.00	0.00
3' 4"	4	0.00	0.00	0.00	0.00	1.00

PROBABILISTIC LOGIC

This section contrasts fuzzy logic and probabilistic logic. A proposition in mathematical logic may be analogous to a mathematical object that can belong to either (crisp or fuzzy) sets of true or false statements, using the previously described analogy. The explanations in this part are kept brief by solely utilizing the concept of sets. The notations T (corresponding to true) and F (referring to false) are used for the sets that make up the potential world[1], W, of an event E in order to indicate the relationship between sets and propositions.

The fact that F and T are fuzzy sets in fuzzy logic but crisp sets in probabilistic logic is a key distinction between the two. As a result, there are uncertainty surrounding the fuzzy logic's F and T bounds. As a result,

[1] Possible world of the event E is a world set that embraces all possible realizations of E

there may be overlap between these two sets; for example, we may have $T \sqcap F \neq \Phi$ (see Figure 5), but according to probabilistic logic, we must also have $T \sqcap F = \Phi$ (see figure 6).

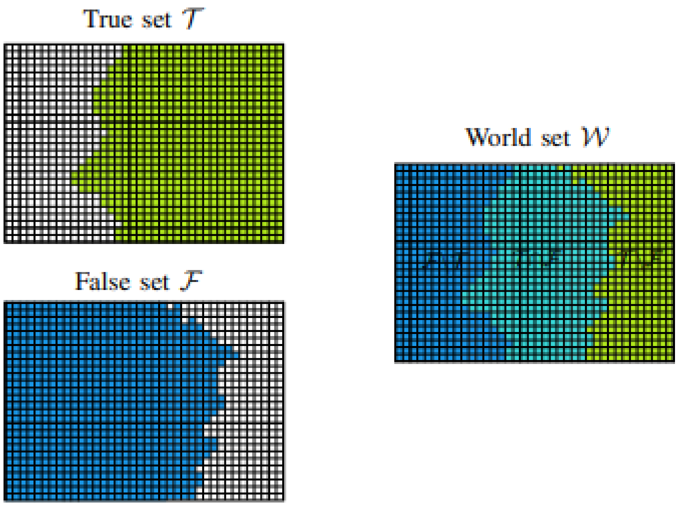

Figure 5: The sets of true T and false F propositions in fuzzy logic may have an overlap, i.e., $T \cap F \neq \emptyset$. Moreover, some mathematical objects that belong to F\T or T \F may not necessarily have a membership degree of 1.

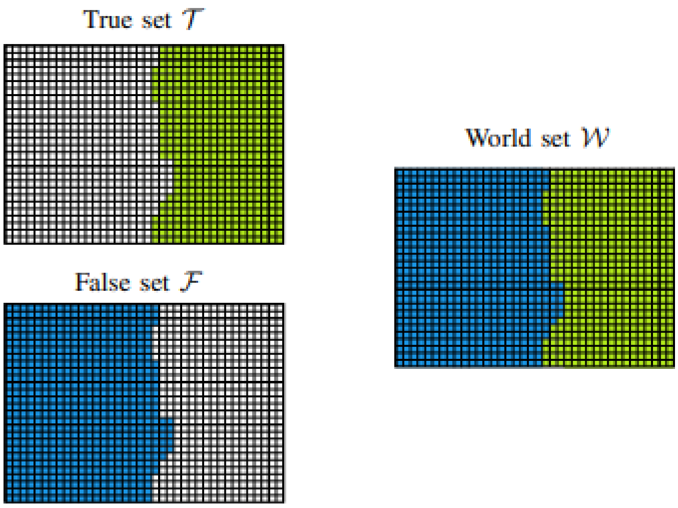

Figure 6: The sets of true T and false F propositions in probabilistic logic do not have any overlap, and any mathematical object that does not belong to either F or T, belongs to the other set with a probability of 1.

A mathematical object z can match any realization of the repeated event E. In probabilistic logic, if E is repeated in II_1 experiments and in the II_2 set of experiments, z goes to F and z goes to T, respectively, then $II_1+II_2=100$. The likelihood of a belonging to F or T is defined as the normalized values π_1 and π_2 of the natural values II_1 and II_2, respectively. If z goes to F in experiment M1 and z gets to T in experiment M2, then M1+M2 is not necessarily 100 experiments of fuzzy logic (may be larger than 100). For some experiments, z may go to the overlap $T \cap F$, meaning that the experiment is double tallied in both M1 and M2. The degrees of membership of a to the fuzzy sets F and T, respectively, are the normalized values μ_1 and μ_2 of the real values M1 and M2.

In general, whereas the sum of the degrees of membership $\sum_{i=1}^{Sc} = \pi i$ of a to se fuzzy sets that make up W may be smaller than, equal to, or bigger than 1, the sum of the probabilities $\sum_{i=1}^{Sc} = \mu i$ of a belonging to Sc crisp sets that build up the potential world W of the event E is necessarily 1.

Theory of Fuzzy Sets

The first papers on fuzzy set theory were written by Zadeh in 1965 and Goguen in 1967 and 1969. These papers show that the authors wanted to expand the traditional idea of a set and a proposition to include fuzziness in the sense described.

Zadeh wrote in 1965 on page 339, "The idea of a fuzzy set makes it easy to build a conceptual framework that is similar to the framework used for ordinary sets in many ways, but is more general and may have a wider range of uses, especially in pattern classification and information processing. Basically, this kind of framework is a natural way to deal with problems where the lack of clearly defined criteria for class membership, rather than random variables, is the source of uncertainty."

"Imprecision" in this case means "vagueness," not "not knowing what the value of a parameter is" (as in tolerance analysis). Fuzzy set theory is a strict mathematical framework (there's nothing fuzzy about it!) that makes it possible to study fuzzy ideas in a clear and precise way. It can also be thought of as a modeling language that works well when there are fuzzy relationships, criteria, or phenomena.

Fuzziness hasn't been defined in a single way from a semantic point of view yet, and it probably never will be. Depending on where it is used and how it is measured, it will mean different things. In the time since then, many authors have added to this theory. In 1984, there were already as many as 4,000 books, and by 2000, there were already more than 30,000.

As these publications get more specialized, it might get harder for people who are new to this field to find a good place to start and to understand and appreciate the philosophy, formalism, and potential applications of this theory. In the last twenty years, fuzzy set theory has mostly grown in two directions:

1. As a formal theory that, as it grew older, got more complicated and specific. It also got bigger by adding new ideas and concepts and "embracing" classical areas of math like algebra, graph theory, topology, and so on by making them more general.

2. As an application-oriented "fuzzy technology," that is, as a tool for modeling, problem-solving, and data mining that has proven to be better than existing methods in many cases and an attractive "add-on" to traditional approaches in other cases.

In this case, it might be helpful to list the main goals of this technology and say a few words about them. This is to correct the common misconception that fuzzy set theory or fuzzy technology is only or mostly useful for modeling uncertainty.

a) Modeling the uncertain

This is the most popular and oldest goal. I'm not sure, though, if it can (still) be thought of as fuzzy set theory's most important goal. Since the Middle Ages, uncertainty has been a very important subject. There are many theories and methods that claim to be the only right way to model uncertainty. In general, though, they either don't define the word "uncertainty" well enough or only do so in a very narrow way. I think that uncertainty, if seen as a subjective thing, can and should be modeled by many different theories, depending on what causes it, what kinds and how much information is available, what the observer wants, etc. In this way, fuzzy set theory is also one of the theories that can be used to model different kinds of uncertainty in different situations. It might then compete with other theories, but it might also be the best way to model this phenomenon in well-defined situations. To talk about this question in detail in this article [Zimmermann 1997] would be way too much.

b) Relaxing

Dual logic is often used as the basis for classical models and methods. So, they can tell if something is possible or not, if it belongs to a cluster or not, if it is optimal or suboptimal, etc. Often, this view doesn't do a good job of describing the real world. Fuzzy set theory has been used a lot to change the dichotomous nature of traditional methods to make them more gradual. Fuzzy mathematical programming [Zimmermann 1996], fuzzy clustering [Bezdek and Pal 1992], fuzzy Petri Nets [Lipp et al. 1989], and fuzzy multi criteria analysis [Zimmermann 1986] are all examples of this.

c) Compactification

Due to the limited size of a person's short-term memory or of technical systems, it is often not possible to store all relevant data or to show a large amount of data to a person in a way that lets him or her understand the information in it. Fuzzy technology has been used to reduce the complexity of data to an acceptable level, usually through linguistic variables or fuzzy data analysis (fuzzy clustering etc.).

d) Keeping Meaning and Reasoning

Expert system technology has been around for 20 years, and many times it has let people down. One reason for this could be that expert systems that use dual logic for their inference engines process symbols (truth values like true or false) instead of knowledge. Language variables are used to give words and sentences their meanings in approximate reasoning. Then, inference engines need to be able to process meaningful linguistic expressions instead of just symbols to find membership functions of fuzzy sets, which can then be translated back into words and sentences using linguistic approximation.

e) An easy way to find approximate solutions

In the 1970s, Prof. Zadeh said that he wanted fuzzy set theory to be used as a way to find approximate solutions to real-world problems in a quick or cheap way. This goal has never been reached in a good way. But in recent years, cases have come to light that are great examples of this goal. Bardossy [1996] showed, for example, in the context of modeling water flow, that fuzzy rule-based systems can be much better at solving problems than systems of differential equations. When the results of these two different ways of doing things were compared, it was clear that, for all practical purposes, the accuracy of the results was almost the same. This is especially true when the errors and unknowns in the input data are considered.

Fuzzy sets

This theory, which was first put forth by Lotfi A. Zadeh3 in 1965, has been the subject of a large number of papers in written sources (Zadeh, 1965). A collection of some of L.A. Zadeh's most interesting articles on the subject can be found in (Yager, Ovchinnikov, Tong, & Nguyen, 1987). The most important parts of the theory of fuzzy sets and the theory of possibilities are brought together by Dubois and Prade (1980, 1988) and Zimmerman (1991). (Kruse, Gebhardt, & Klawonn, 1994; Mohammd, Vadiee, & Ross, 1993; Piegat, 2001; Buckley & Eslami, 2002; Nguyen, & Walker, 2005) and especially in (Pedrycz & Gomide, 1998). Ross (2004) discusses some engineering applications, and Sivanandam, Sumathi, and Deepa (2006) give an introduction using MATLAB. There is a full introduction in Spanish in (Galindo, 2001; Escobar, 2003).

The first way that fuzzy sets were understood was as a generalization of the traditional idea of a subset that was expanded to include the description of "vague" and "imperfect" ideas. This generalization is based on the idea that an element's membership in a set becomes "fuzzy" or "vague." Some elements may not be obvious to belong to a set or not. Then, their membership in the set can be measured by a degree, which is usually called the "membership degree" of that element and has a value in the range [0,1].

The term "ecosystem" refers to a group of people who work in the construction industry. In traditional logic, the set 0,1> indicates that an element does not belong to a set and that it does belong if and only if it does. On the other hand, in fuzzy logic, this set is expanded to include the range [0,1]. As a result, it could be said that fuzzy logic is an extension of traditional systems (Zadeh, 1992)[2]. The term "ecosystem" refers to a group of people who work in the construction industry. Its significance lies in the fact that many types of human reasoning, especially common-sense reasoning, are by their very nature approximate. Note that the use of membership degrees has a lot of potential because they can be used to express something qualitative (fuzzy) in a quantitative way. Formally, a fuzzy set is as follows:

Definition 1: A **Fuzzy Set**: A over a universe of discourse X (a finite or infinite interval within which the fuzzy set can take a value) is a set of pairs

[2] L.A. Zadeh is a Professor in the University of California, Berkeley. He has received many awards all over the world and holds honorary doctorates in universities worldwide, one of them being from the University of Granada, Spain, in 1996, in recognition of his important contribution in this scientific field. Website: http://www.cs.berkeley.edu/~zadeh

$$A = \{\mu_A(x)/x : x \in X, \mu_A(x) \in [0,1] \in \Re\} \quad (1)$$

where μ_A (X) is called the membership degree of the element x to the fuzzy set A. This degree ranges between the extremes 0 and 1 of the dominion of the real numbers μ_A (X) = 0 indicates that x in no way belongs to the fuzzy set A, and μ_A (X)= 1 indicates that x completely belongs to the fuzzy set A. Note that μ_A (X) = 0.5 is the greatest uncertainty point.

Sometimes, instead of giving an exhaustive list of all the pairs that make up the set (discreet values), a definition is given for the function μ_A (X), referring to it as characteristic function or membership function.

The universe X may be called underlying universe or underlying domain and in a more generic way, a fuzzy set A can be considered as a function μ_A that matches each element of the universe of discourse X with its membership degree to the set A:

$$\mu_A(x): \quad X \rightarrow [0,1] \quad (2)$$

The universe of discourse X or the set of considered values can be of these two types:

• Finite or discreet universe of discourse X= {X_1, X_2, ….., X_n}, where a fuzzy set A can be represented by:

$$A = \mu_1 / x_1 + \mu_2 / x_2 + ... + \mu_n / x_n \quad (3)$$

where μi with i = 1, 2,..., n represents the membership degree of the element xi. Normally the elements with a zero degree are not listed. Here the + does not have the same significance as in an arithmetical sum but rather it has the meaning of aggregation, and the / does not signify division but rather the association of both values.

• Infinite universe of discourse, where a fuzzy set A over X can be represented by:

$$A = \int \mu_A(x)/x \quad (4)$$

In reality, a fuzzy set's membership function, A(x), A expresses how much x confirms the category that A has specified.

A linguistic label is a word that represents or identifies a vague set that may or may not have a formal definition in natural language. Using this definition, we may be certain that humans frequently employ linguistic labels to represent abstract ideas, such as "young," "old," "cold," "hot," "cheap," "expensive," etc. In the chapter by Xexéo and Braga in this manual, the term "linguistic variable" (Zadeh, 1975) is defined. A linguistic variable is, in essence, a variable whose values may be ambiguous. The name of the variable, the underlying universe, a group of linguistic labels, or the method used to create these names and their definitions, are all characteristics of linguistic variables.

The intuitive definition of labels varies based on the context in which they are used as well as from person to person and moment to moment. A "high" person and a "high" building, for instance, do not measure the same.

Example 1: A linguistic variable is "Temperature." Using the membership functions shown in Figure 7, we may define four linguistic labels, such as "Very Cold," "Cold," "Hot," and "Very Hot."

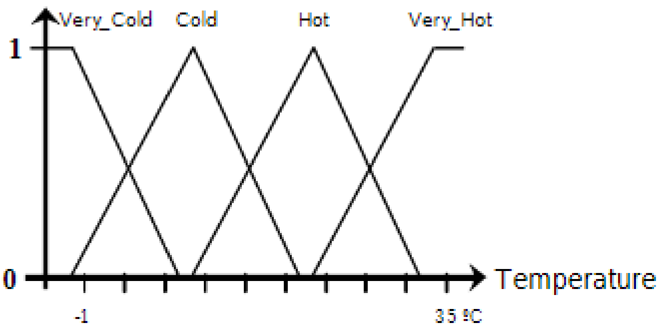

Figure 7: A Frame of Cognition with Four Linguistic Labels for Temperature (Example 1)

Characteristics and Applications

- This logic is multivalued, and according to Zadeh (1992), it has the following key traits:
- Exact reasoning is seen as a particular instance of approximation reasoning in fuzzy logic.
- Fuzzy logic can be used to translate any logical system.
- Knowledge is viewed as a collection of flexible or ambiguous limits over a set of variables in fuzzy logic (e.g., the variable Temperature is Cold).
- A method of spreading those limits is thought to be inference. The process by which a conclusion is made, repercussions are attained, or one fact is inferred from another is known as inference. Everything in fuzzy logic is a matter of degree.

A whole mathematical and computing theory has been created from this straightforward idea, making it easier to solve some issues (see the references in the beginning of this chapter). Control systems, modeling, simulation, prediction, optimization, pattern recognition (such as word recognition), information or knowledge systems (databases, knowledge management systems, case-based reasoning systems, expert systems...), computer vision, biomedicine, picture processing, artificial intelligence, artificial life, etc. are just a few of the many fields to which fuzzy logic has been applied.

In conclusion, fuzzy logic may be a useful tool when other methods have failed in the past, highlighting complex processes, requiring the introduction of expert knowledge from knowledgeable individuals, or when there are unknown magnitudes or these magnitudes are challenging to measure accurately. Fuzzy logic is typically utilized when we need to express and work with ambiguous, hazy, or subjective data.

Several applications combine fuzzy logic with other general or soft computing technologies, such as rule-based systems, evolutionary algorithms, and neural networks.

Membership Functions

Zadeh developed a number of membership functions that might be divided into two categories: "linear" membership functions, which are composed of straight lines, and "curved" membership functions, which are composed of Gaussian forms. We'll now look at a few other membership functions. These fuzzy sets are referred to as convex fuzzy sets in fuzzy set theory, with the exception of extended trapezium, which need not necessarily be convex although this quality is always desirable for semantic reasons.

1. (Figure 8) **Triangular**: defined by its modal value m, lower and upper bounds a and b, and upper bound b, such that amb. When it equals the value m-a, the value is referred to as the b-m margin.

$$A(x) = \begin{cases} 0 & \text{if } x \le a \text{ or } x \ge b \\ (x-a)/(m-a) & \text{if } x \in (a, m] \\ (b-x)/(b-m) & \text{if } x \in (m, b) \end{cases} \tag{5}$$

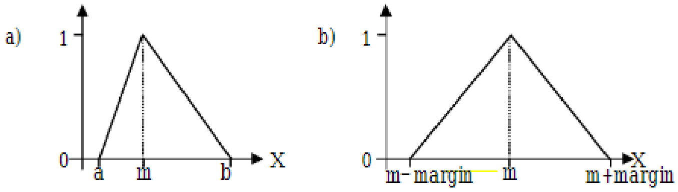

Figure 8: Triangular Fuzzy Sets: a) General. b) Symmetrical.

2. **Singleton** (Figure 9): It takes the value zero in all the universe of discourse except in the point m where it takes the value 1. It is the representation of a non-fuzzy (crisp) value.

$$SG(x) = \begin{cases} 0 & \text{if } x \ne m \\ 1 & \text{if } x = m \end{cases}$$

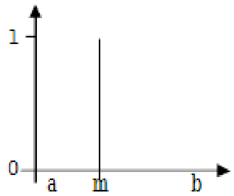

Figure 9: Singleton Fuzzy Set.

3. **L Function** (Figure 10): This function is defined by two parameters a, and b, in the following way, using linear shape:

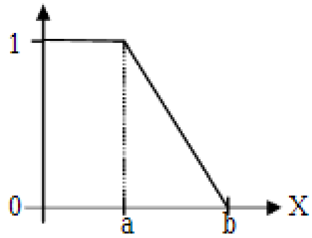

Figure 10: L Fuzzy Set.

4. **Gamma Function** (Figure 11): It is defined by its lower limit a, and the value k > 0. Two definitions:

$$\Gamma(x) = \begin{cases} 0 & \text{if } x \le a \\ 1 - e^{-k(x-a)^2} & \text{if } x > a \end{cases} \tag{8}$$

$$\Gamma(x) = \begin{cases} 0 & \text{if } x \le a \\ \dfrac{k(x-a)^2}{1 + k(x-a)^2} & \text{if } x > a \end{cases} \tag{9}$$

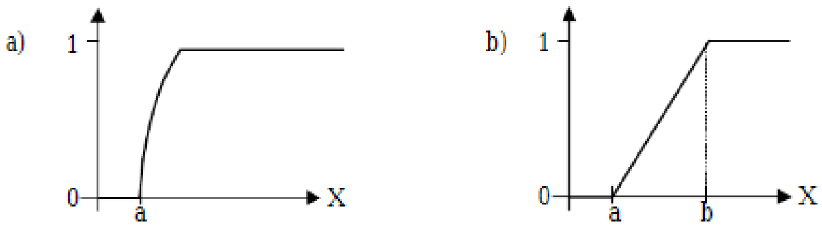

Figure 11: Gamma Fuzzy Sets. a) General. b) Linear.

- This function is characterized by rapid growth starting from a.
- The greater the value of k, the greater the rate of growth.
- The growth rate is greater in the first definition than in the second.
- Horizontal asymptote in 1.
- The gamma function is also expressed in a linear way (Figure 11 b):

$$\Gamma(x) = \begin{cases} 0 & \text{if } x \le a \\ \dfrac{x-a}{b-a} & \text{if } a < x < b \\ 1 & \text{if } x \ge b \end{cases} \tag{10}$$

5. **S Function** (Figure 12): Defined by its lower limit a, its upper limit b, and the value m or point of inflection so that a<m<b. A typical value is: m = (a+b) / 2. Growth is slower when the distance a−b increases.

$$S(x) = \begin{cases} 0 & \text{if } x \le a \\ 2\{(x-a)/(b-a)\}^2 & \text{if } x \in (a, m] \\ 1 - 2\{x-b)/(b-a)\}^2 & \text{if } x \in (m, b) \\ 1 & \text{if } x \ge b \end{cases} \tag{12}$$

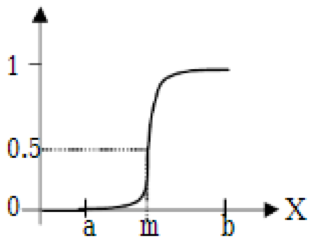

Figure 12: S Fuzzy Set.

Quantifiers with Fuzzy Results

Several applications, including database applications, have successfully used fuzzy or linguistic quantifiers (Yager, 1983; Zadeh, 1983; Liu and Kerre, 1998 and 1998b). The number of items in a subset satisfying a given criterion or the proportion of this number in respect to the total number of potential elements can be approximated using fuzzy quantifiers, which allow us to describe fuzzy numbers or proportions. Absolute or relative fuzzy quantifiers are possible:

- Absolute quantifiers describe quantities over the whole number of elements in a specific set, indicating, for instance, whether this number is "far more than 10," "near to 100," or "a great number of... In order to employ statements like "roughly between 5 and 10," "about -8," etc., we can generalize this idea and think of fuzzy numbers as absolute fuzzy quantifiers. Remember that the expressed value may be positive or negative. We can observe in this instance that a single quantity determines whether the quantifier is true. As a result, as we shall see, fuzzy numbers and absolute fuzzy quantifiers have very similar definitions.
- According to the total number of potential elements, relative quantifiers represent measurements across the total number of elements that satisfy a certain criterion (the proportion of elements). As a result, two factors determine whether the quantifier is true. Expressions like "the majority" or "most," "the minority," "little of," and "approximately half of..." all employ this kind of quantifier. Finding the total number of components meeting the requirement and comparing that number to the entire number of elements that may meet it are necessary in this scenario to assess the veracity of the quantifier (including those which fulfil it and those which do not fulfil it).

Depending on the context, certain quantifiers, like "many" and "few," can be used either way (Liu and Kerre, 1998). Absolute fuzzy quantifiers are defined in (Zadeh, 1983) as fuzzy sets in positive real numbers, whereas relative fuzzy quantifiers are defined as fuzzy sets in the range [0, 1]. All real numbers now fall within the category of "absolute fuzzy quantifiers."

Imprecision Without Fuzzy Logic

Without referring to either the fuzzy set theory or the possibility theory, several concepts allowing for the treatment of imprecise knowledge will be outlined in this section. Although some of the concepts presented here have not been implemented in any of the models, these models are discussed in the bibliography as a whole in the section on imprecision in conventional databases.

Codd (1979) introduced NULL values as the first attempt to represent erroneous data in databases, which were later developed upon (Codd, 1986, 1987 and 1990). The fuzzy set theory was not applied in this model. A NULL value in an attribute denotes that the value is any value that falls inside the attribute's domain.

Any comparison with a NULL value results in a result called "maybe" (m), which is neither True (T) nor False (F) (or unknown, in the SQL of Oracle). Table 1 displays the truth tables for the traditional comparators NOT, AND, and OR.

Table 1: Truth Tables for the tri-valued logic: True, False and Maybe.

NOT	
T	F
m	m
F	T

AND	T	m	F
T	T	m	F
m	m	m	F
F	F	F	F

OR	T	m	F
T	T	T	T
M	T	m	m
F	T	m	F

Subsequently, a further distinction was made, dividing the NULL value into two marks: the "A-mark," which represented an absent or unidentified value, even though it was applicable, and the "I-mark," which represented the absence of the value because it was not applicable (undefined). For example, a person without an automobile might have an I-mark on their license plate. This is a tetra-valued logic in which the A value, which has a significance similar to that of the m in the tri-valued logic discussed above, is formed by comparing any value including an A-mark, and a new I value is added as a result of the comparison of any value containing an I-mark. Table 2 displays the tetra-valued logic.

Table 2: Truth Tables for the tetra-valued logic.

NOT	
T	F
A	A
I	I
F	T

AND	T	A	I	F
T	T	A	I	F
A	A	A	I	F
I	I	I	I	F
F	F	F	F	F

OR	T	A	I	F
T	T	T	T	T
A	T	A	A	A
I	T	A	I	F
F	T	A	F	F

Future Perspectives

Recent issues of fuzzy sets and systems have several examples of these kinds of applications, including the efficient solution of differential equation systems (see [Bardossy 1996]). But, in general, fuzzy set theory has not yet demonstrated that it is computationally capable of effectively resolving big and complex issues. This is due to the fact that either traditional computing methods (linear programming, branch and bound, traditional inference) are still required, or the additional information in fuzzy set models makes computations overly complex. Prudent standards (supporting fuzzy logic, etc.) and effective algorithmic mashes-ups of heuristics and fuzzy set theory may show genuine potential in this situation. In other words, there is a pressing need for research into fuzzy algorithms.

Decision analysis has since 1970 been one of the prominent application areas of fuzzy set theory. Only one chapter could be devoted to this subject in this comprehensive textbook. Several books and papers included in the bibliography, as well as my book "Fuzzy Sets, Decision Making and Expert Systems" (1987, third printing 1993), have further information. It is hoped that further research efforts will advance this area and help to close still existing gaps.

Conclusion and Results

One of the best things about fuzzy set theory is that it is very general. This means that it can handle a lot of new developments that are needed to deal with problems and challenges that are already present and those that are yet to come. Some areas, like possibility theory [Dubois and Prade, 1988a], fuzzy clustering, fuzzy control, fuzzy mathematical programming, etc., are already very well developed. However, there is still a lot of room for growth in other places.

Fuzzy control is the area where fuzzy set theory is best known and where many scientists, students, and practitioners are interested. Excellent books like [Babuska 1988] and [Verbruggen et al. 1999] show how things are going in this area right now. Unfortunately, the popularity of this field has made it hard to see what else fuzzy set theory can do. We hope that after reading this book, the reader is aware of all the other, unexplored, ways to apply this theory.

To deal with these problems, there will need to be a lot more research, both formal and empirical. Much of this research will only be possible through interdisciplinary team efforts. Let us indicate some of the research that is needed. Fuzzy set theory can be considered as a modeling language for vague and complex formal and factual structures. Although many other connectives, ideas, and operations have been suggested in the literature, the min-max version of fuzzy set theory has primarily been used and applied thus far. Membership functions are meant to "be given." To use fuzzy set theory effectively as a modeling language, a lot of empirical research and good modeling work on connectives and the measurement of membership functions are required. In the field of artificial intelligence, there are great opportunities that have not yet been taken advantage of. The majority of the methods and approaches that have been suggested so far have been binary. The phenomenon of uncertainty will need to be modeled much more accurately than has been done so far if artificial intelligence is to be useful in capturing human thinking and perception. Here, of course, fuzzy set theory offers many different opportunities.

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