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Matrixes and their Applications in Electrical Circuits

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Abstract

In this paper, we defined the matrix as an important mathematical tool that has a prominent role in various applications. In order to be able to use matrices, we defined algebraic operations on it, mentioned its most important properties and some types, and showed the importance of matrices for mathematical and physical sciences, which lies in the fact that it allows facilitating calculations in various fields. It also helps in studying physical phenomena. The paper aims to study matrices and their applications in electrical circuits. Their use is very useful in solving electrical circuits because matrices are a way to deal with equations with more than one variable and give a good way to solve them. It helps us a lot to find demands in the electrical circuit, such as calculating and knowing the current in force, knowing the voltage, or another physical variable of the electrical circuit, by converting these circuits into equations and then into matrices, and using the inverse of the matrix to solve them for the purpose of facilitating calculations, avoiding errors, and obtaining results in the least possible time. And by using the methods of analysis of the electric circuit, we reached linear equations in the form of different currents and voltages, and we were able to solve these equations to find the value of the variables, were they efforts or currents that enable us to solve the entire electrical circuit, as matrices were a greatly useful aid when dealing with different electrical circuits.

Keywords: Matrix, square matrix, electrical circuits, importance of matrices, linear equation.

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1- Introduction

Particular electric circuit systems are commonly encountered with uncertainties due to unavoidable parameter variations, component failures, disturbance. Theories and techniques of robust control are capable of dealing with such uncertainties, and much attention has been paid in the field of electronic circuits in recent years, for issues of analysis [1]. On the other hand, it is well known that robustness in control theory means a certain performance is guaranteed against various parametric uncertainties and/or external disturbances and noise [2]. In the same way, in real-life industrial applications, strong nonlinearities of plants may lead to severe difficulties for the analysis and synthesis of control systems [3]. Otherwise, because of the increasing demand for reliability and safety in industrial processes, fault detection (FD) has been the subject of intensive research, and a lot of detection approaches have been presented in the literature [4]. Therefore, in recent years, descriptor systems have been one of the major research fields of control theory due to the comprehensive applications in the Leontief dynamic model and in electrical [5]. According to this study, one issue of fault detection and isolation is increasingly required in various kinds of practical complex systems for guaranteeing reliability and pursuing performance [6]. During the past decades, Markovian jump systems have received a great deal of research attention because they can be employed to model some plants whose structure is subject to random abrupt changes such as random failures or repairs of the components, sudden environmental changes, changing subsystem interconnections, and changes of the operating point of a linearized model of a nonlinear system [7].

1. Matrix definition

A matrix is a rectangular arrangement of entries, each of which is called an element. The elements of the matrix are arranged in the form of rows and columns. The size of the matrix is the number of rows and the number of columns, so any matrix is made up of m rows, n columns, and it is of m × n size [8].

$$A = \begin{bmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} \cdots & a_{mn} \end{bmatrix} = [a_{ij}]$$

where the element in row i ($1 \le i \le m$and column j ($1 \le j \le n$ of the A matrix of degree $m \times n$ is written a_{ij}.

Matrix equals

The matrices $A = [a_{ij}]_{m\times n}$and $B = [b_{ij}]_{m\times n}$are said to be equal if:

1-A, B of the same degree.

2- Each element of matrix A is equal to its corresponding element in matrix B.

Matrix types

- If n = 1, the matrix becomes $m \times 1$and is called the column matrix.
- If m = 1, the matrix becomes of the type $1 \times n$ and is called row matrix
- If m = n, the matrix becomes of $n \times n$ type and is called the n-square matrix, and the set of elements is called $a_{11}, a_{22}, \dots, a_{nn}$in the square matrix the main diagonal of the matrix while the set of elements is called $a_{1n}, a_{2n-1}, \dots, \dots$ The secondary dimension of the matrix.
- A rectangular matrix has the number of rows different from the number of its columns.
- The unit matrix is a square matrix with all the elements of its main diagonal equal to 1 and the rest of its elements being zero.
- The diagonal matrix is a square matrix whose elements outside its main diagonal is zeros.
- A zero matrix is a matrix with all the elements zero.
- The proper matrix, which is diagonal matrix whose main diagonal elements are equal.
- The conjugate matrix of any matrix is a matrix that we get after replacing each element with its conjugate.

2. Operations on arrays

1- Addition and subtraction of matrices: the two matrices $A = [a_{ij}]_{m\times n}$and $B = [b_{ij}]_{m\times n}$are said to be additive and subtractive if they are of the same degree, so if their sum is the matrix $D = [d_{ij}]_{m\times n}$and their subtraction is the matrix $C = [c_{ij}]_{m\times n}$of the same degree, then:

$$A + B = [a_{ij}]_{m\times n} + [b_{ij}]_{m\times n} = [d_{ij}]_{m\times n} = D$$

$$A - B = [a_{ij}]_{m\times n} - [b_{ij}]_{m\times n} = [c_{ij}]_{m\times n} = C$$

2- Matrix multiplication: If $A = [a_{ij}]_{m\times r}$and $B = [b_{ij}]_{r\times n}$then $A \times B$ is defined and possible if the columns of A are equal to the number of rows of B and the resulting matrix is of degree $m \times n$, meaning that $C = [c_{ij}]_{m\times n}$

Matrix inverse

A square matrix A of order n has an inverse. If there is a matrix B of the same order as the matrix A, where $A \times B = B \times A = I_n$, then we say in this case that the matrix A is invertible and its inverse is B, and it is denoted by the symbol A^{-1}.

a- Algebraic properties of operations on matrices:

(The substitution of addition) A+B = B +A

b- (Coordination for Addition) A+(B+C) = (A+B)+C
c- (Distribute multiplication over addition) A+B) = CA + CB.
d- k (AB) = (KA)B = A(KB)
e- (O zero matrix)A+O = O + A = O e)
f- AO = OA = O
g- A-A = O

The importance of matrices:. The importance of matrices lies in many fields, as they can be used in many mathematical applications or in the field of science, including physics.

- Arithmetic: it allows facilitating mathematical operations in various fields.

- Physics: It helps in the study of physical phenomena such as the movement of bodies, energy conversion, mechanics and electric current.

3. Solving a set of linear equations

Let's have the following system of equations:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_2$$

$$\begin{matrix} . & . & . \\ . & . & . \\ . & . & . \end{matrix}$$

$$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots + a_{nn}x_n = b_n$$

It can be represented using matrices as follows:

We write A and = B where $\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_3 \end{bmatrix}$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \mathrm{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \mathrm{B} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_3 \end{bmatrix}$$

To solve this system, we multiply both sides of the equation by A^{-1}.

$$A^{-1}AX = A^{-1}B \rightarrow IX = A^{-1}B \rightarrow X = A^{-1}B$$

4. Matrix applications

Arithmetic matrices play a major role in life as they are used in many applied fields in order to facilitate calculations and avoid errors and inaccurate outputs. Linear equations are used in many fields, and the solution of these equations is considered one of the basic things in finding variables, as they are used as a mathematical model to represent many applications such as electrical circuits.

2- Analyze electrical circuits using loop analysis to calculate current:

Electrical circuits consist of a mixture of active and passive elements connected in series, parallel, or mixed. All electrical circuits can be solved using basic circuit laws such as Ohm's Law, Kirchhoff's Laws, and others. But with a group of theories that facilitate the writing of linear circuit equations, we get a quick solution to it. We can use arrays in electrical and electronic networks widely.

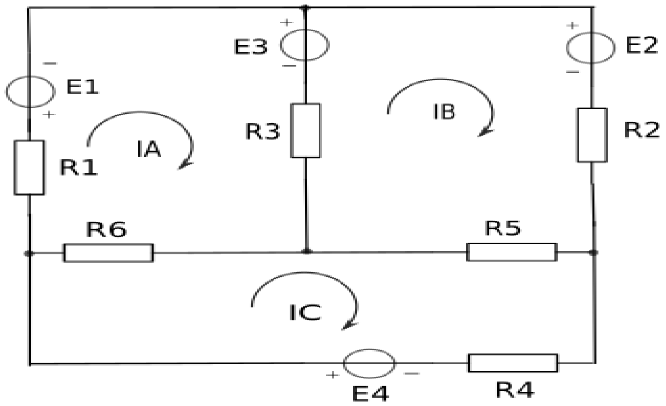
Loop analysis is one way to solve electrical networks. In this method, we use Kirchhoff's voltage law for each loop.......

The direction of the current is determined for each loop, let it be clockwise $I_A, I_B, I_C, \ldots$..

The poles of the resistors are determined according to the direction of the default current, then the Kirchhoff voltage equations are written for each loop, and from them we calculate the sub-currents I1,I2,I3…..

To study this application in a simple way, we take the following example:

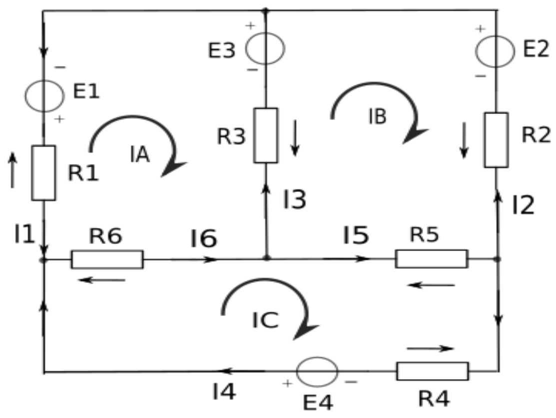
Example: Using the loop analysis method, find the loop currents I_A, I_B, I_C and based on that, select the sub-currents, then make sure that the calculations are correct by the power balancing method?



$$R_1 = 1\Omega R_2 = 2\Omega R_3 = 3\Omega R_4 = 4\Omega R_5 = 5\Omega R_6 = 6\Omega$$

$$E_1 = 2V \quad E_2 = 10V \quad E_3 = 12V \quad E_4 = 29V$$

The solution:



$R_A = R_1 + R_3 + R_6 = 1 + 3 + 6 = 10\Omega$

$R_B = R_2 + R_3 + R_5 = 2 + 3 + 5 = 10\Omega$

$R_C = R_4 + R_5 + R_6 = 4 + 5 + 6 = 15\Omega$

$R_{AB} = R_3 = 3\Omega R_{AC} = R_6 = 6\Omega R_{BC} = R_5 = 5\Omega$

$E_A = -E_1 - E_3 = -2 - 12 = -14V$

$E_B = E_3 - E_2 = 12 - 10 = 2V \qquad E_C = E_4 = 29V$

The toroidal resistor matrix and the toroidal voltage matrix...

$$R_M = \begin{bmatrix} R_A & -R_{AB} & -R_{AC} \\ -R_{AB} & R_B & -R_{BC} \\ -R_{AC} & -R_{BC} & R_C \end{bmatrix} = \begin{bmatrix} 10 & -3 & -6 \\ -3 & 10 & -5 \\ -6 & -5 & 15 \end{bmatrix}$$

$$E_M = \begin{bmatrix} E_A \\ E_B \\ E_C \end{bmatrix} = \begin{bmatrix} -14 \\ 2 \\ 29 \end{bmatrix}$$

The inverse matrix of the matrix of toroidal resistors...

$$R_M^{-1} = \begin{bmatrix} 0.22 & 0.13 & 0.13 \\ 0.13 & 0.2 & 0.12 \\ 0.13 & 0.12 & 0.16 \end{bmatrix}$$

The matrix of the toroidal currents is given by the matrix equation......

$$I_M = R_M^{-1} \times E_M = \begin{bmatrix} 0.22 & 0.13 & 0.13 \\ 0.13 & 0.2 & 0.12 \\ 0.13 & 0.12 & 0.16 \end{bmatrix} \times \begin{bmatrix} -14 \\ 2 \\ 29 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$I_A = 1A \qquad I_B = 2A \qquad I_C = 3A$$

Sub streams...

$I_1 = -I_A = -1A \qquad I_2 = -I_B = -2A \qquad I_3 = I_B - I_A = 2 - 1 = 1A$

$I_4 = I_C = 3A \qquad I_5 = I_C - I_B = 3 - 2 = 1A \qquad I_6 = I_C - I_A = 3 - 1 = 2A$

Balancing ability........

$P_S = I_1E_1 + I_2E_2 + I_3E_3 + I_4E_4$

$P_S = (-1 \times 2) + (-2 \times 10) + (1 \times 12) + (3 \times 29) = 77W$

$P_L = R_1I_1^2 + R_2I_2^2 + R_3I_3^2 + R_4I_4^2 + R_5I_5^2 + R_6I_6^2$

$P_L = (1 \times (-1)^2) + (2 \times (-2)^2) + (3 \times (1)^2) + (4 \times (3)^2) + (5 \times (1)^2) + (6 \times (2)^2) = 77W$

$$\therefore P_S = P_L$$

3- Analyze electrical circuits using nodal analysis to calculate voltages

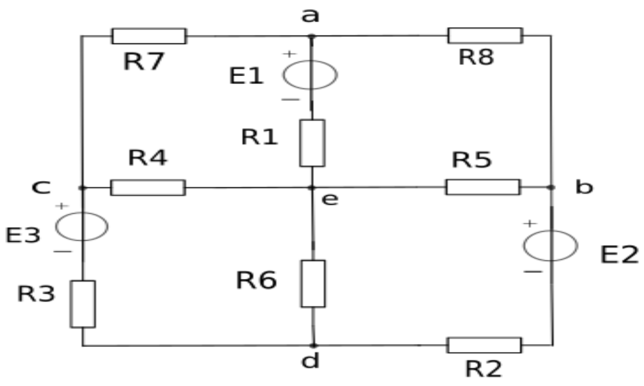
Nodal analysis is one way to solve electrical networks. Kirchhoff's current law (KCL) is used to calculate the nodal voltages.

- Determine the number of nodes in the circuit.
- Choosing a reference node connected to the ground and its voltage is zero, V=0.
- We impose the direction of the currents for each node (according to the source of the voltage or the source of the current).

Write the Kirchhoff equation for each node

To make it clearer, we will give an example.

An example of how Using nodal analysis, find the contract voltages?



$E_1 = 20V \qquad E_2 = 15V \qquad E_3 = 17V \qquad R_1 = 15\Omega R_2 = 15\Omega$

$R_3 = 15\Omega R_4 = 10\Omega R_5 = 12\Omega R_6 = 14\Omega R_7 = 15\Omega$

$R_8 = 20\Omega$

The solution

1- Convert voltage sources into current sources.

2- We focus on the currents that feed the nodes, not the sub-currents.

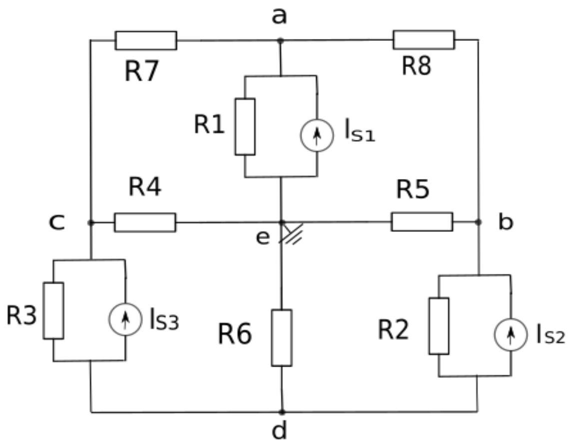
3- Coding the contract.

4- Writing the equations in terms of continuity for each node.

5- Whistling one of the knots, let it be e

$V_e = 0 \qquad G_1 = 0.0667s \qquad G_2 = 0.0667s \quad G_3 = 0.0667s$
$G_4 = 0.1s \quad G_5 = 0.083s \quad G_6 = 0.0714s \quad G_7 = 0.0667s$
$G_8 = 0.05s$
$G_a = G_1 + G_7 + G_8 = 0.0667 + 0.0667 + 0.05 = 0.1834s$

$G_b = G_2 + G_5 + G_8 = 0.0667 + 0.083 + 0.05 = 0.2s$
$G_C = G_3 + G_4 + G_7 = 0.0667 + 0.1 + 0.0667 = 0.233s$
$G_d = G_3 + G_2 + G_6 = 0.0667 + 0.0667 + 0.0714 = 0.2048s$
$G_{ab} = G_8 = 0.05s \qquad G_{ac} = G_7 = 0.0667s \quad G_{ad} = 0$



$I_{S1} = \frac{E_1}{R_1} = \frac{20}{15} = 1.33A \qquad I_{S2} = \frac{E_2}{R_2} = \frac{15}{15} = 1A$

$I_{S3} = \frac{E_3}{R_3} = \frac{17}{15} = 1.133A$

$$G_M = \begin{bmatrix} G_a & -G_{ab} & -G_{ac} & -G_{ad} \\ -G_{ab} & G_b & -G_{bc} & -G_{bd} \\ -G_{ac} & -G_{bc} & G_c & -G_{cd} \\ -G_{ad} & -G_{bd} & -G_{cd} & G_d \end{bmatrix} = \begin{bmatrix} 0.1834 & -0.05 & -0.067 & 0 \\ -0.05 & 0.2 & 0 & -0.067 \\ -0.067 & 0 & 0.233 & -0.067 \\ 0 & -0.067 & -0.067 & 0.2048 \end{bmatrix}$$

$$V_M = \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} I_M = \begin{bmatrix} I_{S1} \\ I_{S2} \\ I_{S3} \\ I_{S2} - I_{S3} \end{bmatrix} = \begin{bmatrix} 1.33 \\ 1 \\ 1.133 \\ -2.133 \end{bmatrix}$$

$\because \boldsymbol{G_M} \times \boldsymbol{V_M} = \boldsymbol{I_M}$

$$\begin{bmatrix} 0.1834 & -0.05 & -0.067 & 0 \\ -0.05 & 0.2 & 0 & -0.067 \\ -0.067 & 0 & 0.233 & -0.067 \\ 0 & -0.067 & -0.067 & 0.2048 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} 1.33 \\ 1 \\ 1.133 \\ -2.133 \end{bmatrix}$$

$\therefore V_M = G_M^{-1} \times I_M$

$$V_M = \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} 11.001 \\ 5.5402 \\ 6.129 \\ -6.597 \end{bmatrix}$$

5. Conclusion

Matrices are one of the branches of linear algebra. They are used in the study of various topics of daily life, including mathematics and physics. Matrices provide the fastest and easiest way to reach values, whether in mathematics or other sciences. They are used in applied sciences as a means in addition to shortening the time and giving the correct results. Therefore, we recommend expanding the study of matrices science and benefiting from it because of its other applications, such as those mentioned in this paper, and other applications that were not mentioned, such as the use of matrices in economics, chemistry, mechanics, and computers, and solving many complex equations. Therefore, we have to employ knowledge in its applications for further development and progress.

References

1. Zhao, X., et al., Robust control of continuous-time systems with state-dependent uncertainties and its application to electronic circuits. IEEE Transactions on Industrial Electronics, 2013. **61**(8): p. 4161-4170.
2. Yao, J., Z. Jiao, and D. Ma, Adaptive robust control of DC motors with extended state observer. IEEE transactions on industrial electronics, 2013. **61**(7): p. 3630-3637.
3. Wei, Y., et al., Fuzzy-model-based decentralized dynamic-output-feedback H∞ control for large-scale nonlinear systems with time-varying delays. IEEE Trans Ind Electron, 2016. **173**: p. 1054-1065.
4. Li, X.-J. and G.-H. Yang, Fault detection for T–S fuzzy systems with unknown membership functions. IEEE Transactions on Fuzzy Systems, 2013. **22**(1): p. 139-152.
5. Yang, C., et al., Positive realness and absolute stability problem of descriptor systems. IEEE Transactions on Circuits and Systems I: Regular Papers, 2007. **54**(5): p. 1142-1149.
6. Yao, X., L. Wu, and W.X. Zheng, Fault detection filter design for Markovian jump singular systems with intermittent measurements. IEEE Transactions on Signal Processing, 2011. **59**(7): p. 3099-3109.
7. Zhu, Q., Stabilization of stochastically singular nonlinear jump systems with unknown parameters and continuously distributed delays. International Journal of Control, Automation and Systems, 2013. **11**(4): p. 683-691.
8. Fujita, M., P.C. McGeer, and J.-Y. Yang, Multi-terminal binary decision diagrams: An efficient data structure for matrix representation. Formal methods in system design, 1997. **10**: p. 149-169.