AJAPAS

African Journal of Advanced Pure and Applied Sciences (AJAPAS)

Online ISSN: 2957-644X

Volume 2, Issue 3, July-September 2023, Page No: 68-76

Website: *https://aaasjournals.com/index.php/ajapas/index*

||Arab Impact factor 2022: 0.87|| SJIFactor 2023: 5.689|| ISI 2022-2023: 0.557

Exploring the Fuzzy Frontier: Unraveling the Mysteries of Fuzzy Functions

Omalkhear Salem Blebalu*

Department of Mathematics, Faculty of science /Aleajilat, University of Zawia, Libya

*Corresponding author: *blabulo1985@gmail.com*

Received: May 29, 2023	Accepted: July 17, 2023	Published: July 28, 2023

Abstract:

In numerous real-world applications, such as sensor data, the data values are vaguely specified. Fuzzy set theory has been proposed to address such ambiguity by generalizing the concept of set membership. In a Fuzzy Set (FS), each element is essentially associated with a point-value chosen from the unit interval, which is known as the grade of membership in the set. A Vague Set (VS) and an Intuitive Fuzzy Set (IFS) are additional generalizations of a Fuzzy Set (FS). In lieu of point-based membership, interval-based membership is utilized in VSs. Membership in VSs based on intervals is more expressive in capturing imprecise data. In the literature, IFSSs and VSs are considered equivalent in the sense that an IFS is isomorphic with a VS. In addition, as a result of this equivalence and the fact that IFSs were formerly known as a tradition, the unique and fascinating features of VSs for handling imprecise data are largely disregarded. In this paper, we compare VSs and IFSs based on their algebraic properties, graphical representations, and practical applications. Here, we present the idea of a fuzzy function. Crisp functions with fuzzy constraints and fuzzifying functions make up fuzzy functions. To locate the highest possible value within the fuzzy domain of the crisp function, we also present and use the techniques of maximizing and reducing sets. Here, we present the idea of a fuzzy function. Crisp functions with a fuzzy constraint are the building blocks of fuzzy functions, together with the fuzzifying function. In order to locate the highest possible value within the fuzzy domain of the crisp function, we additionally introduce and use the concepts of a maximizing set and a minimizing set.

Key words: fuzzy sets, Vague Set, Fuzzy Function, Fuzzy components

Cite this article as: O. S. Blebalu, “Exploring the Fuzzy Frontier: Unraveling the Mysteries of Fuzzy Functions”, *African Journal of Advanced Pure and Applied Sciences (AJAPAS)*, vol. 2, no. 3, pp. 68 –76, July-September 2023.

الملخص:

في العديد من تطبيقات العالم الحقيقي، مثل بيانات المستشعر، يتم تحديد قيم البيانات بشكل غامض. تم اقتراح نظرية المجموعة الضبابية لمعالجة هذا الغموض من خلال تعميم مفهوم عضوية المجموعة. في مجموعة ضبابية (FS)، يرتبط كل عنصر بشكل أساسي بقيمة النقطة المختارة من فاصل الوحدة، والذي يُعرف باسم درجة العضوية في المجموعة. مجموعة غامضة (VS) ومجموعة ضبابية بديهية (IFS) هي تعميمات إضافية لمجموعة ضبابية (FS). بدلاً من العضوية القائمة على النقاط، يتم استخدام العضوية القائمة على الفاصل الزمني في VSs. تكون العضوية في VS على أساس الفواصل أكثر تعبيرًا في التقاط بيانات غير دقيقة. في الأدبيات، تعتبر IFSSs وVSs متكافئة بمعنى أن IFS متماثل مع VS. بالإضافة إلى ذلك، نتيجة لهذا التكافؤ وحقيقة أن IFSs كانت تُعرف سابقًا باسم التقليد، يتم تجاهل الميزات الفريدة والرائعة لـ VSs للتعامل مع البيانات غير الدقيقة إلى حد كبير. في هذه الورقة، قمنا بمقارنة VSs وIFSs بناءً على الخصائص الجبرية والتمثيلات الرسومية والتطبيقات العملية. هنا، نقدم فكرة الدالة الغامضة. تشكل الوظائف الواضحة ذات القيود غير الواضحة

والوظائف الغامضة وظائف غامضة. لتحديد أعلى قيمة ممكنة داخل المجال الضبابي للوظيفة الهشة، نقدم أيضًا ونستخدم تقنيات تعظيم المجموعات وتقليلها. هنا، نقدم فكرة الدالة الغامضة. الدوال الواضحة ذات القيد الغامض هي اللبنات الأساسية للوظائف الغامضة، جنبًا إلى جنب مع الوظيفة الغامضة. من أجل تحديد أعلى قيمة ممكنة داخل المجال الضبابي للوظيفة الهشة، نقدم بالإضافة إلى ذلك مفاهيم مجموعة التكبير ومجموعة التصغير مع استخدامها.

الكلمات المفتاحية: مجموعات ضبابية، مجموعة غامضة، وظيفة ضبابية، حدود ضبابية.

Introduction

Despite their seeming ease for humans, many jobs present a never-ending test for computers. Such methods are often used when one must maneuver in a crowded area, lift a heavy object, or park a car. Humans' natural aptitude for working with imprecise and nebulous information makes these kinds of tasks second nature to us. Therefore, we need to be able to model the activities of the operator, rather than the plant, if we want to imitate the control actions of a human operator. Our model needs to be constructed so that it can process ambiguous data.

This is precisely what fuzzy logic-based systems are good at, and it's why they've found usage in everything from speech and handwriting detection to metro train speed regulation.

Since there is fuzzy information about different applications in the real world, such as in sensor database, we can formalize the measurements from different sensors to a fuzzy set, as was done in Zadeh's landmark study in [1]. The membership degree is a single real number that is allocated to each object U in fuzzy set theory. something between 0 and 1. (U represents the universe of discourse, a set of classical objects.) The evidence for u CU and the evidence against U is muddled when utilizing the single membership value in fuzzy set theory, as pointed out by Gau et al. in [2]. So that we may attack this issue. Instead of employing point-based membership like in FSS, the authors Gau et al. propose the use of interval-based membership via the concept of Vague Sets (VSS). To more expressively capture data fuzziness, VSS employs an interval-based membership generalization. To counter this, however, [7] demonstrates that VSS are identical to Intuitionistic Fuzzy Sets (IFSs). As a result, people rarely take advantage of VSS's special capabilities when it comes to dealing with imprecise data.

Definition of Fuzzy Logic

Remembering the original inspiration for the development of fuzzy logic is crucial to grasping how it works. This inspiration, presented in different ways in Zadeh's (1965, 1973) earliest papers on fuzzy logic, might be summarized as follows. Propositions like "5 is a prime number," "Jan's age is 9," and "if x is a positive integer and y = x+1 then y is a positive integer" are examples of the types of statements that lend themselves well to the formalization of reasoning by classical logic. Collections (of objects) with strong, clear-cut bounds are well-suited to being represented by classical sets, just as "the collection of all prime numbers less than 100" or "the collection of all U.S. Senators as of September 1, 2010" are good examples. An object can be arbitrarily included or excluded from such a set.

As a rule, people don't utilize binary propositions when relaying information about the wider world. These statements hold some degree of truth, rather than simply being true or false. The statement "it is hot outside" is an illustrative example of a weather-dependent statement. Our common sense tells us that the higher the temperature, the more accurate the statement. For a statement to be considered "bivalent," it must allow for two possible outcomes: either the real temperature is more than or equal to t, in which case the statement is true, or the actual temperature is less than 1, in which case the statement is false. Therefore, if the actual temperature is t-0.01, we reject the hypothesis, and if it is r+0.01 we accept it. If the statement "it is hot" is considered to have two possible truth values, then an arbitrary change in the ambient temperature can render it either false or true. The way we normally interpret and use statements like "it is hot outside" is obviously challenged by this.

The borders of most groups to which people make reference when exchanging information are similarly fuzzy. Rather than just being included or excluded, things can have varying degrees of involvement in such collections. An excellent example of this is a passage from Zadeh's seminal article (1965):

“More often than not, the classes of objects encountered in the real physical world do not have precisely defined criteria of membership. For example, the class of animals clearly includes dogs, horses, birds, etc. as its members, and clearly excludes objects as rocks. fluids, plants, etc. However, such objects as starfish, bacteria, etc. have an ambiguous status with respect to the class of animals. The same kind of ambiguity arises in the case of a number such as 10 in relation to the "class" of all real numbers which are much greater than 1.”

“Clearly, the "class of all real numbers that are much greater than 1," or "the class of beautiful women," or "the class of tall men" do not constitute classes or sets in the usual mathematical sense of these terms. Yet, the fact remains that such imprecisely defined "classes" play an important role in human thinking...”

“The purpose of this note is to explore in a preliminary way some of the basic properties and implications of a concept which may be of use in dealing with "classes" of the type cited above. The concept in question is that of a fuzzy set, that is a "class" with a continuum of grades of membership.”

Categories of Fuzzy Function

According to which aspect of the crisp function the fuzzy concept was applied, fuzzy functions can be categorized into one of three categories.

(1) A crisp function constrained by fuzziness.
(2) Crisp function that propagates the fuzzy nature of the independent variable to the dependent.
(3) Function that is itself an ambiguous function. This function fuzzes the image of a distinct independent variable.

Clear Logic and Crisp Sets

A collection of unique items, or "crisp set," is the foundation of classical logic. For instance, the notation "red, white" refers to a collection, even though the individual objects white and red are colors are distinct. The aforementioned example can be described as F = {red, white} because crisp sets are typically given capital letters.

From a larger set, we can define a compact subset whose members satisfy some criterion for inclusion. Set A, for instance, can be described as all the numbers from 4 to 12 inclusive. The following notation can be used to describe this statement:

Given that A = {i | i is an integer and 4<= i <= 12}

With the introduction of the concept of the characteristic or indicator function of a set—here, the function defined over the set of integers, which we'll name X, that shows the membership of elements in subset A in X—it is feasible to create a graphical depiction of the aforementioned subset. To do this, we give the elements of X in A the value 1, and the components of X outside of A the value 0.

$$1_A(x) = \begin{cases} 1 & \text{if } 4 \le x \le 12 \\ 0, & \text{otherwise} \end{cases}$$

Graphically this can be displayed as follows:

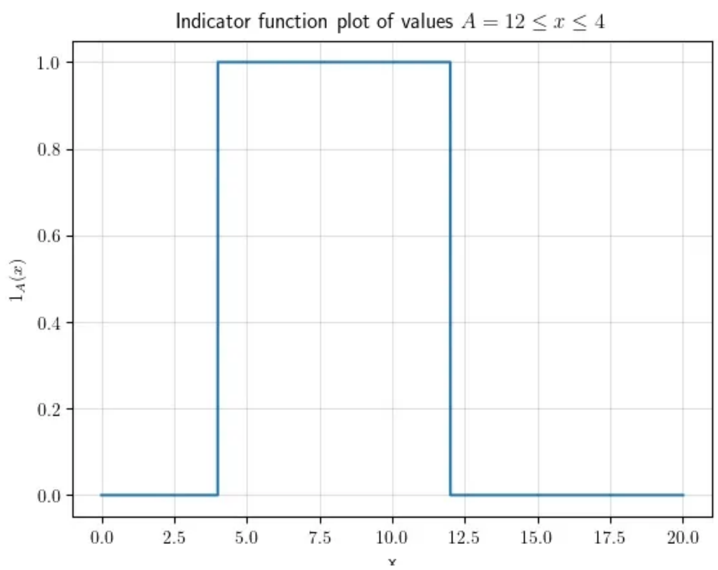

Figure 1: indicator function plot of $A=12<x<4$

Fuzzy sets

Lotfi Zadeh (1921-2017), the inventor of the fuzzy set, first described them in 1965.

An element's degree of membership, denoted by, can range from 0 (element does not belong at all in the set) to 1 (element belongs entirely in the set) in a fuzzy set, whereas in a crisp set, full membership is required.

If we take the fuzzy set given in the preceding section and eliminate all the values of belonging except for 0 and 1, we are left with the crisp set mentioned there.

The set's membership function describes the connection between individual elements and their level of membership in the set. Below is an example of the use of membership functions in the context of temperature.

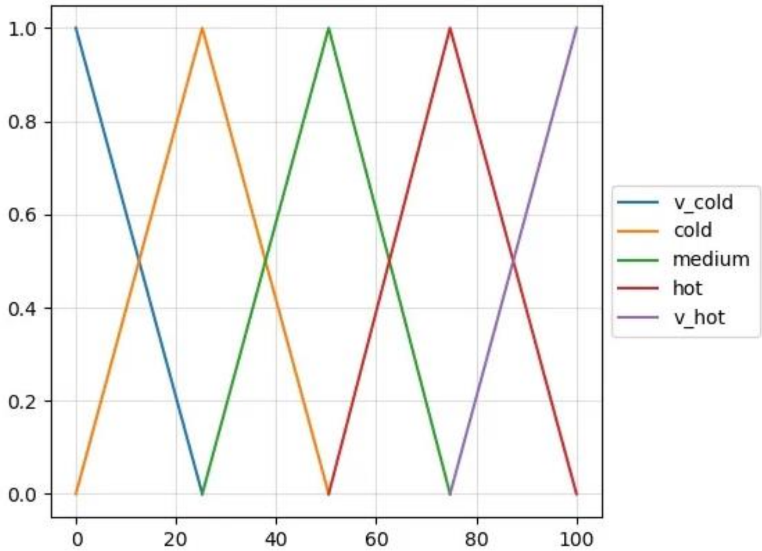

Figure 2: membership functions in the context of temperature.

Fuzzy sets are used to describe a wide range of values, such as the extreme cold and extreme heat experienced by an engine. The quantity of set membership is represented by the symbol. At 80 degrees Fahrenheit, for instance, the engine is hot to within a factor of 0.8 and very hot within a factor of 0.2.

Because conjunction and disjunction (and/or) are so fundamental to reasoning, we explored the union, intersection, and negation operators of crisp sets in the preceding section.

The maximal operator is the most often used method for determining the union of two fuzzy sets. The product operator can also be used on the two sets, thus other options do exist. Similarly, the minimal operator is typically used to compute the intersection of two fuzzy sets. Subtracting the set membership function from 1 yields the complement of a fuzzy set.

The components of Sets

A universal set X is defined in the universe of discourse, and it contains all conceivable elements associated with the problem at hand. When defining set, A within the universal set X, the following relationships are revealed:

$$A \subseteq X.$$

A set A is included in the universal set X in this instance. If A is omitted from X, this relationship is depicted as follows.

$$A \not\subseteq X .$$

If an element x is included in the set A, this element is referred to as a member of the set, and the notation is as follows.

$$x \in A.$$

If the element x is not included in the set A, the following notation is utilized.

$$x \notin A.$$

Typically, we symbolize a set by enumerating its constituent elements. For instance, the constituents a_1, a_2,..., a_n comprise set A, which is represented as follows.

$$A = \{a_1, a_2..., a_n,\}.$$

By defining the conditions of elements, another method of representing sets is provided. For instance, if the elements of set B must satisfy the conditions P_1, P_2,..., P_n, then the following defines set B.

$B = \{b|$ b satisfies $P_1. P_2... P_n\}$

In this instance, the symbol “|” represents the phrase "such that."

The number of elements is used to symbolize the size of an N-dimensional Euclidean set; this number is known as cardinality. The cardinality of the set A is denoted by the symbol |A|. If the cardinality of the set A is finite, then the set A is finite. A set is infinite if |A| is infinite. In general, all points in N-dimensional Euclidean vector space constitute the universal set X.

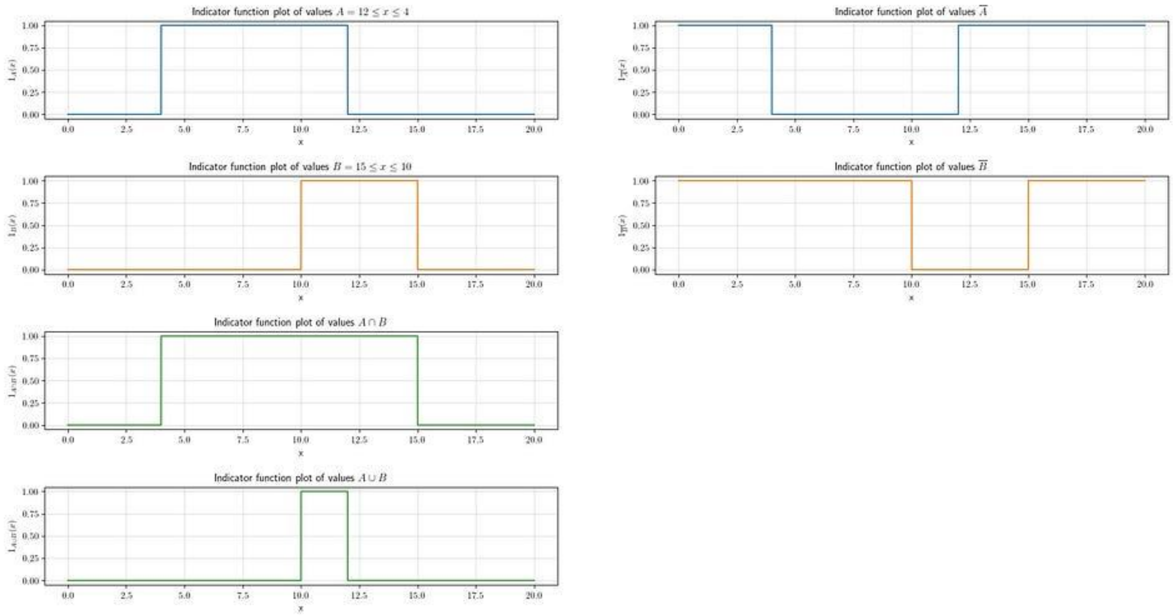

Figure 3: different indicator function plots.

Vague Sets and Intuitionistic Fuzzy Sets

The ideas of Vague Sets (VSs) and Intuitionistic Fuzzy Sets (IFSs) are introduced here. We show that the VSs's pictorial representation is more natural to comprehend ambiguous values.

Let U be the universe of discourse, a set of classical objects where u represents a single object in U.

(Fuzzy Set) A fuzzy set A= {< u, μA(u)>|u∈ U} in a universe of discourse U is characterized by a membership function, μA, as follows: μA: U→ [0,1].

(Vague Set) A vague set Vin a universe of discourse U is characterized by a true membership function, αV, and a false membership function, βV, as follows: αV: U→[0,1], βV:U→[0,1], and αV(u) + βV(u)≤1,where αV(u) is a lower bound on the grade of membership of u derived from the evidence for u, and βV(u)is a lower bound on the grade of membership of the negation of u derived from the evidence against u.

(Intuitionistic Fuzzy Sets) An intuitionistic fuzzy set A= {< u, μA(u), νA(u) > |u ∈ U} in a universe of discourse U is characterized by a membership function, μA, and a non-membership function, νA, as follows: μA: U→ [0,1], νA: U→ [0,1], and 0 ≤ μA(u) + νA(u) ≤ 1.

Differences between VSs and IFSs can be traced back to how membership intervals are defined. For u in V, we have [αV(u),1−βV(u)], while in A, we have < μA(u), νA(u)>. In this context, both μA and νA have the same semantic value of αV and βV respectively. As indicated in [7], however, the boundary (1 −βV) can provide some indication of the existence of a data value. This nuance allows for a more straightforward, yet meaningful, graphical representation of data sets. Figure 4 shows a VS, while Figure 5 shows an IFS. The likely existence of data is shown by the darkened area generated by the border in a given VS in Fig. 4. In this sense, the "hesitation region" provides a natural way to express uncertain information consistent with our intuition.

In the following paragraphs, we shall explore further advantages of employing vague membership intervals for capturing data semantics. There are intriguing ramifications of the choice of membership limit for modeling relationships between vague data.

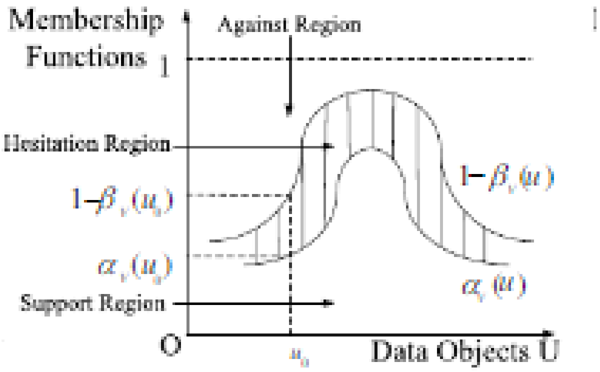

Figure 4: Membership Functions of a VS

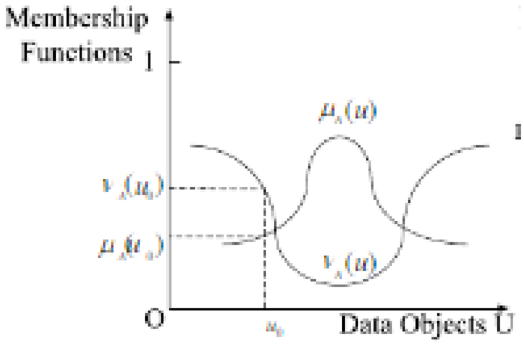

Figure 5: Membership Functions of an IFS

Probability and Fuzzy Logic

The connection between fuzzy logic and probability theory has been the subject of debate once again ever since the publication of (Zadeh 1965). Numerous publications, such as those published in the special issues of Computational Intelligence (Volume 4, Number 2, 1988), IEEE Transactions on Fuzzy Systems (Volume 2, Number 1, 1994), and Technometrics (Volume 37, Number 3, 1995), have explored various features of this relationship. Since both fuzzy logic and probability deal with the phenomena of uncertainty and employ the real unit interval [0, 1], it is not surprising that this problem has been the subject of much discussion. How does fuzzy logic connect to probability theory? is one of the main points of contention.

Is there a difference between uncertainty and chance? Can we reduce all forms of ambiguity to probability?

Membership degrees of fuzzy sets may be read as conditional probabilities, as proposed in the first study to examine the relationship between fuzzy logic and probability (Loginov 1966). Later on, many others came to agree with this or a related viewpoint. Many experts in the field, such as Cheeseman (1988a, b) and Lindley (1987), have argued that probability approaches, and Bayesian methods in particular, are all that is needed to capture and manage uncertainty. For instance, consider this excerpt from (Lindley 1987):

"The only satisfactory description of uncertainty is probability. By this I mean that every uncertainty statement must be in the form of a probability; that several uncertainties must be combined using the rules of probability; and that the calculus of probabilities is adequate to handle all situations involving uncertainty. ... We speak of "the inevitability of probability."

Fuzzy logic, on the other hand, investigates a form of uncertainty that is fundamentally distinct from that investigated by probability theory, as has been pointed out on numerous occasions (e.g., Klir 1989 and Kosko 1990). One such example is the statement "Peter is a tall man." Fuzzy logicians, as we've seen, view this as a many-valued (fuzzy) statement, where the truth value can be somewhere between 0 and 1 (or some other suitable scale). Higher degrees of accuracy indicate greater certainty. Human conceptions, like the concept of a tall man, have a similar graduated structure, and so do these assertions. Many experiments in the psychology of concepts (Belohlavek and Klir, 2011) revealed the gradated nature of human notions. The statement "Peter is a tall man."

cannot be properly evaluated as a binary (yes/no) statement. As an illustration, the question "Is the proposition true, but answer 'yes' or 'no' only?" is not suitable since it misrepresents the ambiguity of the concept of a tall man. When probability theorists say that a proposition's truth degree is a (conditional) probability, they mean that the propositions are assumed to be bivalent, and that the truth degree measures the (subjective) uncertainty of whether or not the proposition is true. It's easy to see how this perspective differs significantly from that of fuzzy logicians. Fuzzy logicians reject this perspective as fundamentally deficient due to its ambivalent treatment of fuzzy propositions.

Fuzzy Logic Controller

Benefits of a Fuzzy Logic Control System

When compared to classical logic, which is more similar to mathematical reasoning, fuzzy logic is far more like human thought and natural language. The approximate, imprecise quality of the real world is well captured by this method. Therefore, the core of the fuzzy logic controller (FLC) is the translation of expert knowledge into a set of language control strategies.

The FLC is a good strategy since it outperforms more traditional control algorithms in terms of the outcomes it produces. There are two situations where the FLC really shines.

(1) The complexity of the control procedures precludes their analysis with traditional quantitative methods.
(2) There is a lack of precision or confidence in the interpretation of the available data.

The following is a concise explanation of FLC's benefits.

(1) Parallel or distributed control: in a traditional control system, a control action is decided by a single control strategy, such as $P = f(x_1, x_2,.., x_n)$. In contrast, FLC makes it simple to model both complex and nonlinear systems by using several fuzzy rules to represent the control strategy.
(2) Linguistic control: the control strategy is depicted using linguistic terms, making it straightforward to represent human knowledge.
(3) Robust control: there are multiple control rules, so the failure of a single rule is not catastrophic for the entire system.

Setting up a Fuzzy Logic Control System

The development of an FLC lacks a standard methodology. Here, however, we can show you how FLC is set up in its most basic form, as illustrated in (Fig 6). Fuzzification interface, knowledge base, decision-making logic, and defuzzification interface are the four essential parts of the setup.

The fuzzification interface performs the following operations to convert crisp input values into fuzzy values.

- Takes in the parameter values
- The input variable's values are converted into a corresponding linguistic universe.
- Data is translated into meaningful language values (fuzzy sets).

This part of fuzzy inference is essential if the input data are fuzzy sets.

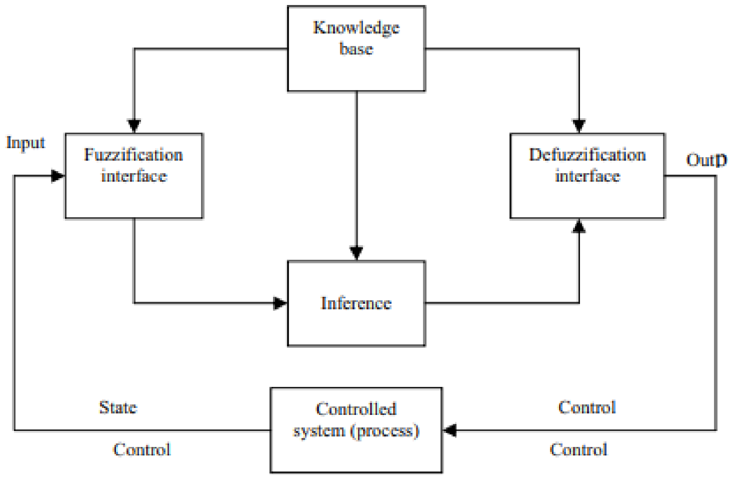

Figure 6: Configuration of FLC

Fuzzy Sets and Fuzzy Relations

The notion of a fuzzy set extends the classical notion of a (characteristic function of) a set. Assigning each element u of U with a degree A: U → L on a scale L of truth degrees, known as the degree of membership of u to A, is the definition of a fuzzy set A in universe U. Standard fuzzy sets are those with L = {0, 1}. Clearly, if L = {0, 1}, we obtain a typical idea of a normal set.

In addition to analogs from classical sets, there are also several novel concepts and procedures associated with fuzzy sets. To illustrate the latter, consider the concept of an ∝-cut, which is defined for a fuzzy set A and a range ∝∈L of arithmetic operations as the ordinary subset an $^{\propto}A$ of U defined by an $^{\propto}A = \{u \in U \mid A(u) \geq \propto\}$. Connecting fuzzy sets with regular sets, the collection an {∝A | ∝ ∈ L} of all of a set's ∝-cuts provides a unique representation for a fuzzy set A. The top half of Fig. 7 compares a fuzzy set and a classical set, with the former expressing the concept of "normal" (temperature). The bottom portion explains the ideas of an ∝-cut and support of a fuzzy set defined as supp(A) = {u ∈ U | A(u) > 0} by displaying three fuzzy sets representing "cold," "normal," and "hot," respectively.

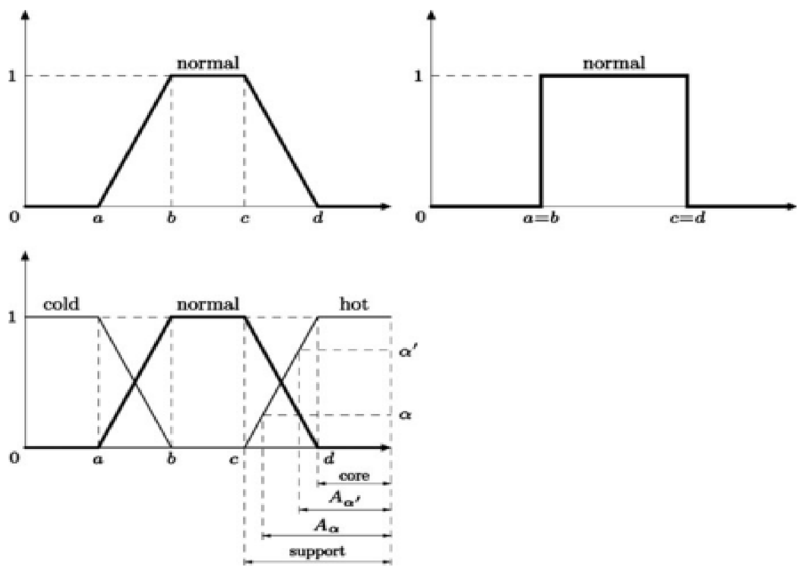

Figure 7: Concept of fuzzy set

A n-ary operation, described in terms of its constituent parts, is induced by every logical n-ary connective c on L. The standard intersection of fuzzy sets (denoted by ∩) is defined as follows, where c is the truth function of the minimal Gödel conjunction.

$$(A \cap B)\,(x) = \min\,(A(x), B(x))$$

Future Perspectives

Recent issues of fuzzy sets and systems have several examples of these kinds of applications, including the efficient solution of differential equation systems (see [Bardossy 1996]). But, in general, fuzzy set theory has not yet demonstrated that it is computationally capable of effectively resolving big and complex issues. This is due to the fact that either traditional computing methods (linear programming, branch and bound, traditional inference) are still required, or the additional information in fuzzy set models makes computations overly complex. Prudent standards (supporting fuzzy logic, etc.) and effective algorithmic mashes-ups of heuristics and fuzzy set theory may show genuine potential in this situation. In other words, there is a pressing need for research into fuzzy algorithms.

Decision analysis has since 1970 been one of the prominent application areas of fuzzy set theory. Only one chapter could be devoted to this subject in this comprehensive textbook. Several books and papers included in the bibliography, as well as my book "Fuzzy Sets, Decision Making and Expert Systems" (1987, third printing 1993), have further information. It is hoped that further research efforts will advance this area and help to close still existing gaps.

Conclusion

This research provided a high-level overview of fuzzy sets and fuzzy inferencing. It is demonstrated how language concepts can be used to express human knowledge, allowing for more precise system control. A fuzzy inference system will be built from the ground up in the following article using Python.

In mathematical modeling, classical crisp functions play a crucial role. Fuzzy functions form the basis of fuzzy modeling. To map fuzzy sets to fuzzy sets, one can obtain a fuzzy function by extending a crisp function. The extension principle and the alpha cuts-based method are two examples of approaches that can be used to characterize fuzzy functions. In this study, we examine the relationship between the differentiability and integrability of fuzzy functions.

References

1. Goguen, J. A. (1973). LA Zadeh. Fuzzy sets. Information and control, vol. 8 (1965), pp. 338–353.-LA Zadeh. Similarity relations and fuzzy orderings. Information sciences, vol. 3 (1971), pp. 177–200. The Journal of Symbolic Logic, 38(4), 656-657.\
2. Gau, W. L., & Buehrer, D. J. (1993). Vague sets. IEEE transactions on systems, man, and cybernetics, 23(2), 610-614.
3. Reyna, V. F., & Brainerd, C. J. (1998). Fuzzy-trace theory and false memory: new frontiers. Journal of experimental child psychology, 71(2), 194–209. https://doi.org/10.1006/jecp.1998.2472
4. Lu, An & Ng, Wee Keong. (2005). Vague Sets or Intuitionistic Fuzzy Sets for Handling Vague Data: Which One Is Better? 401-416. 10.1007/11568322_26.
5. Kenny, J., Heath, A., & Richards, L. (2021). Fuzzy Frontiers? Testing the Fluidity of National, Partisan and Brexit Identities in the Aftermath of the 2016 Referendum. Political Studies, 0(0). https://doi.org/10.1177/00323217211050001
6. HODGSON, G. (2014). On fuzzy frontiers and fragmented foundations: Some reflections on the original and new institutional economics. Journal of Institutional Economics, 10(4), 591-611. doi:10.1017/S1744137414000307
7. Bustince, H., & Burillo, P. (1996). Vague sets are intuitionistic fuzzy sets. Fuzzy sets and systems, 79(3), 403-405.
8. Lee, K. H. (2004). First course on fuzzy theory and applications (Vol. 27). Springer Science & Business Media.
9. Kharal A. (2014). A study of frontier and semifrontier in intuitionistic fuzzy topological spaces. TheScientificWorldJournal, 2014, 674171. https://doi.org/10.1155/2014/674171
10. Sengupta, J. K. (1992). A fuzzy systems approach in data envelopment analysis. Computers & mathematics with applications, 24(8-9), 259-266.
11. Cohen, R. (1995). Fuzzy frontiers of identity: the British case. Social Identities, 1(1), 35-62.
12. Demirci, M. (2000). Fuzzy functions and their applications. Journal of Mathematical Analysis and Applications, 252(1), 495.
13. Francalanza, E., Borg, J. C., & Constantinescu, C. (2016). A fuzzy logic-based approach to explore manufacturing system changeability level decisions. Procedia CIRP, 41, 3-8.
14. Hellmann, M. (2001). Fuzzy logic introduction. Université de Rennes, 1(1).
15. Zadeh, L. A. (1988). Fuzzy logic. Computer, 21(4), 83-93.
16. Trillas, E., & Eciolaza, L. (2015). Fuzzy logic. Springer International Publishing. DOI, 10, 978-3.
17. Perfilieva, I. (2011, August). Fuzzy function: theoretical and practical point of view. In Proceedings of the 7th conference of the European Society for Fuzzy Logic and Technology (pp. 480-486). Atlantis Press.
18. Belohlavek, R., Kruse, R., & Moewes, C. (2011). Fuzzy logic in computer science. Computer Science: The Hardware, Software and Heart of It, 385-419.