African Journal of Advanced Pure and Applied Sciences (AJAPAS)
Online ISSN: 2957-644X
Volume 2, Issue 3, July-September 2023, Page No: 97-108
Website: *https://aaasjournals.com/index.php/ajapas/index*
||Arab Impact factor 2022: 0.87|| SJIFactor 2023: 5.689|| ISI 2022-2023: 0.557

Ball and Hoop System Control Position Using LQG Technique

Ahmed Mohamed Altnazfti [1*], Adel Abobker Ehmied [2], Ali S. Zayed [3], Najeh Ibrahim Allafi [4], Ibrahim Mohammed Al Tanazfti [5]

[1] Higher Institute of Science and Technology, Zliten, Libya
[2] Almansoori petroleum services, Libya
[3] Department of Electrical and Electronic Engineering, Sabratha University, Libya
[4] Higher Institute of Science and Technology, Lazizia, Libya
[5] Higher Institute of Science and Technology-Zliten-Libya

*Corresponding author: *ahmedaltnazfti@yahoo.com*

Received: June 02, 2023	Accepted: July 17, 2023	Published: July 28, 2023

Abstract:
The aim of this paper is to compare the performance between two types of control systems (Ball and Hoop system). It can be used to analyze the dynamic of fluid problems. As soon as the goal from that to identify a control strategy that offers the best performance in controlling the position of the ball and hoop system.
However, the LQG controller designed and compared their performance with the Linear Quadratic LQR controller using MATLAB and Simulink.
The results showed that the performance response of LQG controller better than the LQR control strategy response. Therefore, the performance response of the proposed system was tested and compared with the results of PID as a reference tracking rejection and disturbance behavior.
The simulation results showed that the proposed controller has better performance and lower stability time compared to the traditional PID controller. we hope in the future using high technology to this work.

Keywords: PID controller, LQR Controller, LQG Controller, ball and hoop system.

Cite this article as: A. M. Altnazfti, A. A. Ehmied, A. S. Zayed, N. I. Allafi, I. M. Al Tanazfti, “Ball and Hoop system control Position using LQG technique”, *African Journal of Advanced Pure and Applied Sciences (AJAPAS)*, vol. 2, no. 3, pp. 97 –108, July-September 2023.

الملخص
الغرض من هذه الورقة هو مقارنة الأداء بين نوعين من أجهزة التحكم لنظام الكرة والطوق، وهو نظام يستخدم لتحليل مشاكل ديناميكية السوائل، وهي أنظمة غير خطية وغير مستقرة.
الهدف من المقارنة هو تحديد إستراتيجية عرض أفضل أداء تحكمي لنظام الكرة والطوق. في هذه الورقة تمت المقارنة بين أداء المتحكم (LQG) مع المتحكم (LQR) باستخدام برنامج الماتلاب والمحاكاة، والتي أظهرت نتائجها أن المتحكم (LQG) أنتج أفضل استجابة مقارنة باستراتيجية المتحكم (LQR).
تم اختيار أداء النظام المقترح ومقارنته بنتائج (PID) كمرجع تتبع الرفض والسلوك المضطرب.
محصلة المحاكاة تبين أن المتحكم المقترح (LQG) يملك أفضل أداء وزمن استقرار أقل مقارنة بالمتحكم التقليدي (PID)، كما نأمل مستقبلاً استخدام تكنولوجيا عالية لهذا العمل.

الكلمات المفتاحية: المتحكم PID، المتحكم LQR، المتحكم LQG، نظام الكرة والطوق.

1- Introduction

Based on a previous article titled "Online PID Controller Tuning Using Genetic Algorithms" [1], in which controllers were used, the controllers in this paper were changed to LQG and LQR controllers in a ball-and-hoop system.

The origins can be traced back to Wellstead (1980, 1983), who introduced it as a simple model with qualitatively the same dynamics as the liquid fouling problem; and Fabregas. (2011) also describe how this model can be used to teach linear control theory.

More specifically, the model is used for regions where the ball approaches a stable position on the rim and a linear approximation of motion dynamics is valid.

Despite significant developments in some control theory, proportional-integral-derivative (PID) controllers are widely used in process control systems.

A PID controller is a simple control strategy that can be understood by both plant operators and maintenance groups and is relatively easy to tune.

Linear quadratic Gaussian optimal control (LQG) is used in many areas of control engineering and has proven to be a powerful method for efficiently achieving suboptimal performance. The idea of suboptimality stems from the fact that control techniques minimize a given performance metric in order to reshape the system state and the trajectory of the control surfaces.

However, the LQG technique requires careful tuning of the Q and R parameters that affect the state shape or control signal traces.

Many researchers have tried to find a way to tune LQG controllers and have come up with complex or trial-and-error methods to correct these parameters. These papers introduce new methods for linear quadratic Gaussian (LQG) controller design and Simulink. [1,2].

2- Ball and Hoop System

he Ball and Hoop system demonstrates the dynamics of a steel ball rolling freely inside a rotating ring. There are grooves on the inner edge of the tire to allow steel balls to roll freely in the tire [3]. This results in a complicated rolling radius (r) of the ball, which differs from the actual radius (rb) of the ball, as shown in Figure (1).

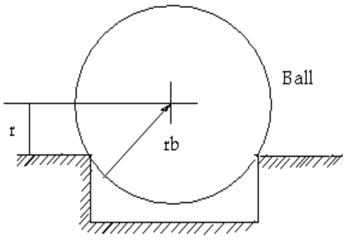

Figure (1) Illustration of the Ball Rolling in its Groove

The overall system is shown in Figure (2). where the ball position is based on the assumption that the hoop is rotating anti-clockwise. The hoop is mounted vertically on the shaft of an electric motor so that it can be rotated about its axis. When the hoop is rotated, the ball will tend to move in the direction of hoop rotation. At some point, gravity will overcome the frictional forces and the ball will fall back. This process will repeat, causing oscillatory motion to the ball. The motor drives the hoop so that its angular position can be placed under control. In Figure (2).

The hoop angular position and the position of the ball is given by:

- y, the position of the ball on the hoop periphery with respect to a datum point.
- Ψ, the slosh angle which measures the deviation of the ball to the rest position.

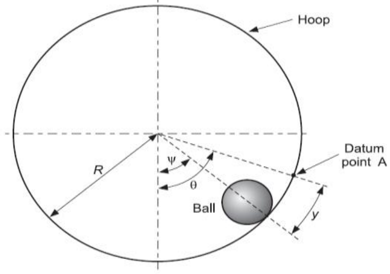

Figure (2) Ball and Hoop System

Figure (3). shows how the ball and hoop system is used to illustrate the complex dynamic of the oscillations in liquid container when the container is moving and undergoing changes in velocity and direction. Oscillation in liquids is called 'slosh' or 'slop' and that is important because the movement of large quantities of liquid can strongly influence the movement of the container itself, this is usually undesirable and often dangerous [3].

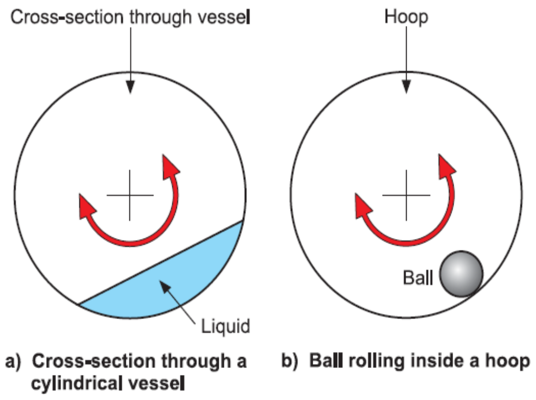

Figure (3) illustrating the analogy [3]

There are many other examples where liquid oscillation must be considered; the important practical examples are [4]:

1. The movement of the liquid cargo in an oil tanker as it changes direction can alter the handling and stability of a truck.

2. The slosh of aviation fuel in an aircraft as it turns may affect its overall stability

3. The liquid load in a railway wagon tanker can rock from side to side on an uneven railway track, causing undue wear to the wagon suspension and railway track.

4. The liquid cargo of a ship may slosh when the ship is in heavy seas, which may subsequently reduce the stability of the ship.

2-1 Measurement of the Hoop Transfer Function

The hoop is powered by an electric motor, for this reason the general form of the transfer function of the hoop follows the general transfer function of a motor and this illustrated as in equation (1).

The following figure shows the general transfer function for a hoop. (2.1).

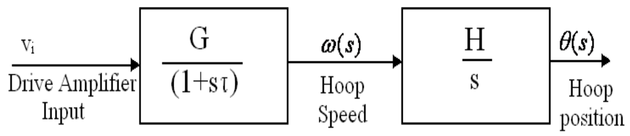

Figure (4) Transfer Function Relating Input Voltage and Hoop Position.

Where:

G - The gain of the system.

τ - The time constant.

$\omega(s)$ - The Hoop speed.

H - The Hoop gain.

$\theta(s)$ - The Hoop position.

To obtain G and τ , constant voltage (v_i) of four volts is applied to the CE109 with the result that the hoop rotates at a steady 1000 rpm [6]. This steady voltage is coupled with a function generator that produces a square wave with a frequency of 0.05 Hz and has amplitude of one volt. This produces a graph that steps up a volt and down a volt every alternate ten seconds.

Zooming in one of the "step ups" produces the graph in Figure (5).

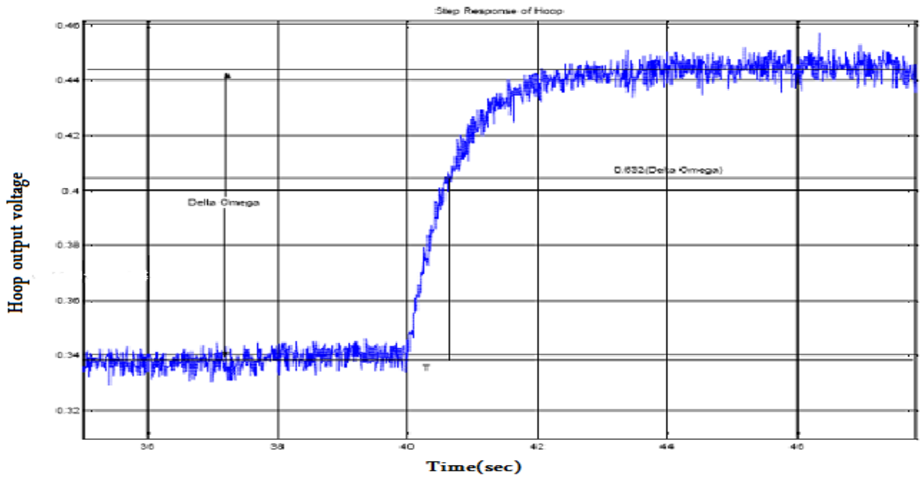

Figure (5) Response of hoop to step input

In a first-order system such as the motor, there is no overshoot. Due to this, the rise time of the system is calculated as the time at which the system reaches 63.2% of its final value.
The time taken for the speed to change by 0.632 $\Delta\omega$ is equal to the time constant of the system τ.
From Figure (5), this value is approximately 0.7sec.
The change in steady state speed $\Delta\omega$ is given by relation (2).

$$\Delta\omega = G\Delta v_i \tag{2}$$

And it can be calculated that:
$\Delta\omega$= 1.06 Volts = Final Value
Δv_i = 1 Volt = Input Value

Then

$$G = \frac{\Delta\omega}{\Delta v_i} = \frac{1.06}{1} = 1.06 \tag{3}$$

Table (1) Data Used to Calculate Hoop gain [5]

Tachometer Output (V)	Frequency (Hz)	Period (Sec)	Amplitude (V)	Frequency X Amplitude
1	5	0.2	13.5	67.5
2	10	0.01	13.5	135
3	15	0.065	13.5	202.5
4	20	0.05	13.5	270

Substituting these values into equation (1) will illustrate the general transfer function of the Hoop.

$$Gh(s) = \frac{1.06}{1+0.7s} \tag{4}$$

2-2 Measurement of Hoop Gain. (H)

The measurement of Hoop Gain H is the relationship between the tachometer output and the rate of change of hoop position (frequency x amplitude) [5].
Where H is given by the following equation.

$$\theta = H\Omega \tag{5}$$

Where:
Ω is the tachometer output (volts).
θ is a Rate of change of position (frequency x amplitude).
To obtain the data in table (1), an initial voltage is applied to the CE109 B

all and Hoop so that the tachometer output reads 1Volt. The period, frequency and amplitude are measured and this process is repeated for unit steps of the tachometer output. [5].
Table (1) illustrates the data which will be used to
Calculate the Hoop gain, and figure (6) shows the plot of data in table (1).

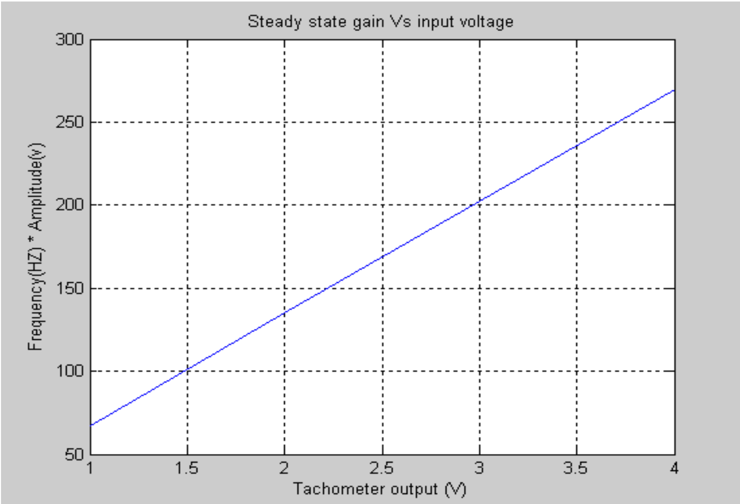

Figure (6) Plot of Data in Table (1)

By calculating the slope of the line in Figure (6) and hence the value of H gives:

$$H = \frac{\theta}{\Omega} = \frac{135}{2} = 67.5\, sec^{-1} \tag{6}$$

The coefficients G, H and τ are known, then the transfer function for the hoop is as depicted in Figure (7).

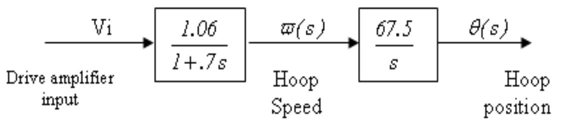

Figure (7) Hoop Transfer function

The transfer function relating the hoop angle $\theta(s)$ to the slop angle $\Psi(s)$ is detailed in Equation (7).[5]

$$\frac{\Psi(s)}{\theta(s)} = \frac{s\left(\frac{2}{5}\left(\frac{rb}{r}\right)^2 s + \frac{bb}{mr^2}\right)}{\left(\frac{2}{5}\left(\frac{rb}{r^2}\right)s^2 + \frac{bb}{mr^2}s + \frac{g}{R}\right)} \tag{7}$$

Where:
rb = Radius of the Ball (= 9.53mm)
r = Rolling Radius of the Ball (= 9.3mm)
g = Gravity (= 9.81m/s)
m = Mass of the Ball (= 0.0282kg)
R = Radius of the Hoop (= 87.5mm)
bb = Coefficient of Rolling Friction = (4.57×10^{06})
The transfer function shown in Equation (7) can be evaluated as:

$$\frac{\Psi(s)}{\theta(s)} = \frac{0.42\, s^2 + 1.873s}{1.42s^2 + 1.873s + 112.11} \tag{8}$$

The experimentally derived Ball and Hoop transfer function is shown in Figure (8):

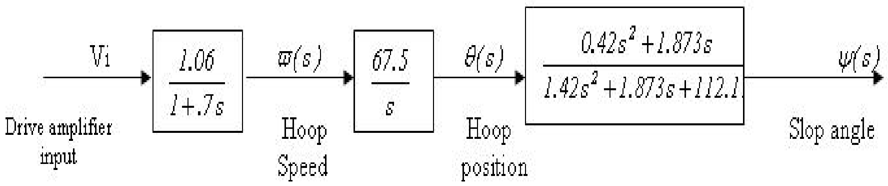

Figure (8) Transfer Functions of the Ball and Hoop System.

Then the Overall Transfer Functions of the Ball and Hoop System can be seen in figure (9)

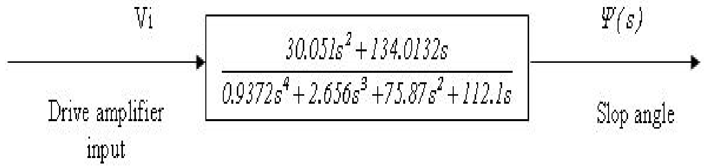

Figure (9) Overall Transfer Functions of the Ball and Hoop System.

The transfer Function of the Ball and Hoop System is given by relation:

$$H(s)=\frac{\Psi(s)}{\theta(s)}=\frac{30.051\,s^2+134.0132s}{0.9372s^4+2.656s^3+75.87s^2+112.1s} \quad (9)$$

3- Basic Structure of a PID Controller.

In this section, only the Ziegler-Nichols method for tuning the PID controller is considered. The control system may result in poor performance and even become unstable, if inappropriate values of the control unit tuning constants are used. Therefore, it is important to adjust the controller parameters to achieve satisfactory control performance. Tuning the controller involves selecting the optimal values of kc, Ti and TD (if using the PID algorithm). This is often a personal procedure and definitely depends on the process. It is a widely accepted method for PID controller tuning. Direct method. First, set the console to P mode only [6]. Next, set the console gain (kc) to a small value. Change a small set point (or load) and note the response of the controlled variable. If kc is low, response should be slow. Increase by a factor of two and make another small change in the set point or load. Keep increasing kc (by a factor of two) until the response becomes oscillatory.

The period of oscillation at this point is called the final period Pu. K u = System profit margin and Pu = (2 * pi)/wcg. Where, wcg file [7]. It is gain via frequency. The profit margin is the opposite of the capacity ratio. Then the control code settings are obtained from the following table 1 as well as wcg. The PID gain values after simulation are given below Table 2.

Table 2. Control law settings

Controller	Kp	Ti	Td
P	$\frac{Ku}{2}$		
PI	$\frac{Ku}{2.2}$	$\frac{Pu}{1.2}$	
PID	$\frac{Ku}{1.7}$	$\frac{Pu}{2}$	$\frac{Pu}{8}$

3- Linear Quadratic Regulator (LQR)

This section of the paper demonstrates the application of a linear quadratic regulator to a ball-and-ring system dynamics of a steel ball (which is free to roll inside a rotating circular hoop) to control the oscillation of fluid in a container as the container moves and undergoes changes in velocity and direction. The primary objective of LQR control is to reduce a specific cost function called a performance indicator [8]. Designing an optimal second-order controller based on this quadratic performance index requires defining the elements of the " K " matrix feedback gain matrix using the following MATLAB command:

$$[K.S.E]=lqr\ (A.B.Q.R) \quad (10)$$

Where:
$Q =$ is the state cost matrix.
$R =$ is the performance index matrix.
$A, B =$ are the state space representations of the system.
Eq. (10) calculates the optimal gain matrix K such that the state-feedback law
$u = -kx$ minimizes the quadratic cost function $J(u)$, which is described by [9]:

$$J(u) = \int_0^{\infty} (x^T Qx + u^T Ru)dt \tag{11}$$

In addition to the state-feedback gain K, lqr returns the solution S of the associated Riccati equation

$$PA + A^T P + Q - PBR^{-1}B^T P = 0 \tag{12}$$

and then solving for the controller gain K

$$K = -R^{-1}B^T P \tag{13}$$

Where P is a positive definite matrix.
The state space representation of the pitch to elevator deflection is obtained by converting the transfer function, relating the pitch angle to the elevator deflection for both the full mode and the short period mode. to their state space form using MATLAB commands.

4- LQG Design
LQG (Linear Quadratic Gaussian) control is one kind of optimal control. It is the combination between LQR (Linear Quadratic Regulator) and Kalman Filter [9]. LQG solves the weakness of LQR control which requires the number of sensors as the number of states and replaces them with an observer, Kalman Filter. This is very useful since not all the states of the system can be measured. Replacing the sensors with an observer also reducing the cost of the system. However, it does not guarantee the robustness of the system against uncertainties in operating conditions [10]. The block diagram of LQG control is shown in Figure (4) Kalman filter used to estimate the state of the system based on system output.
. In this research, the second method of modeling is chosen. The flow diagram of the LQG control design process is depicted in Figure (5).

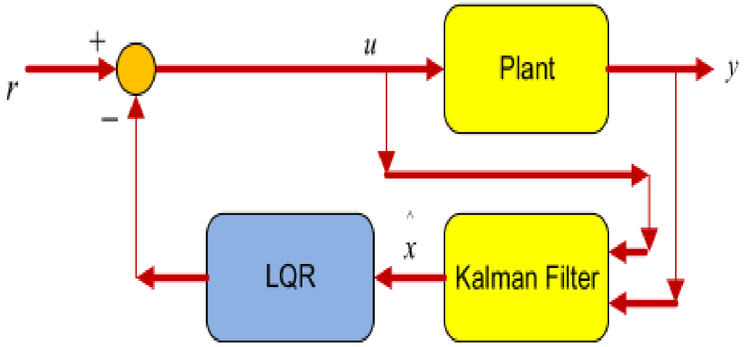

Figure (10) LQG: combination of LQR and Kalman Filter.

5 - Simulation Results
The simulation procedure will be summarizing as follows:

5 -1 Close loop without control
From the plots of the step response Figure (11), it can be said that the behavior is highly oscillatory and long lasting.

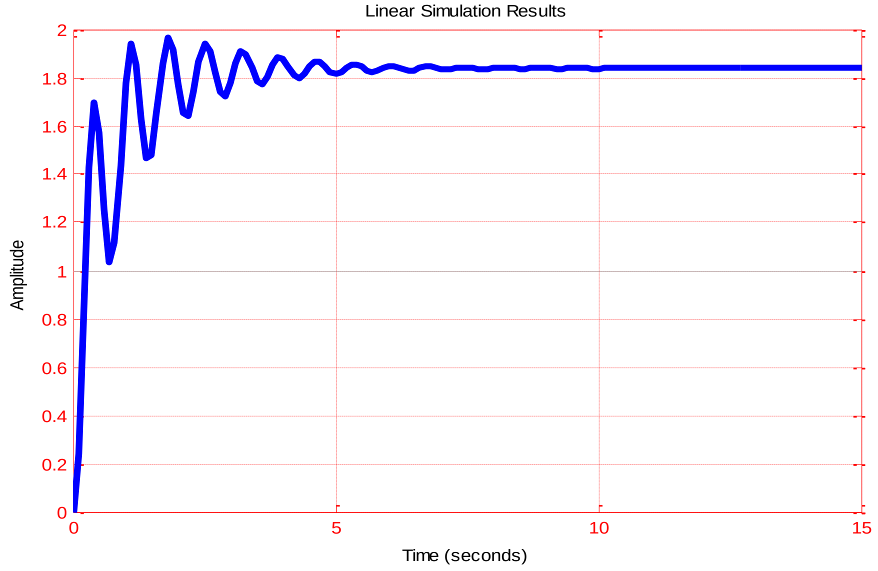

Figure (11) Step response of closed loop system without controller.

Figure (11) shows the response of the closed system without controller, from which it is clearly with a maximum percentage Overshoot = 6.8454, settling time of about settling time = 4.3281 sec, Rise Time = 0.3074sec.

5-2 Tracking Signal of PID Controller

Figure (12) shows the response of the PID controller, from which it is clearly with a maximum percentage Overshoot = 7%, settling time of about settling time=1.8 sec.

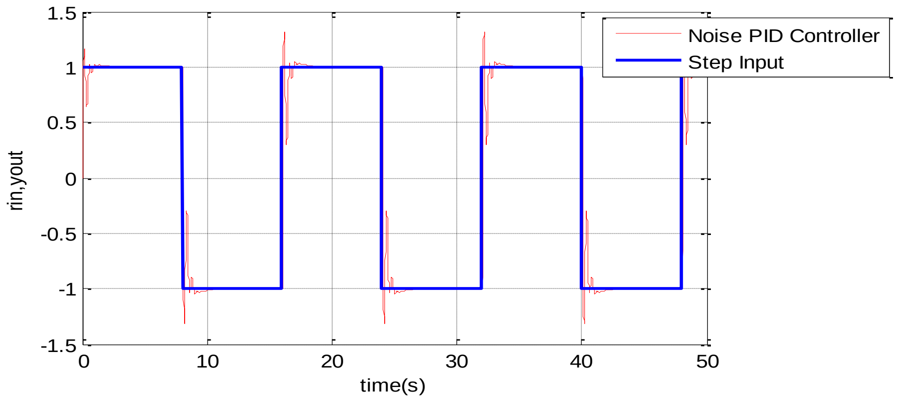

Figure (13) Tracking and Noise PID Controller

5 - 3 Control input for PID Controller

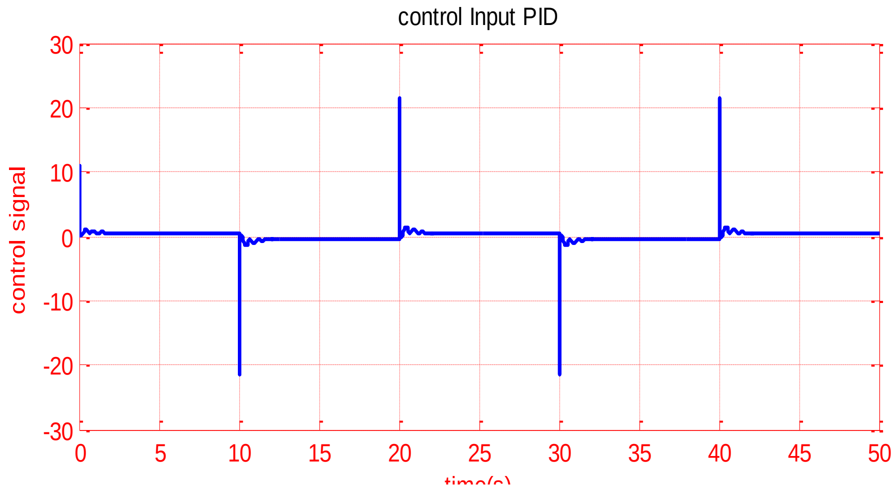

Figure (14) control input for PID Controller

5-4 Tracking Signal of LQR Controller

Figure (15) shows the response of the LQR controller, from which it is clearly noticeable that the LQR controller meets the desired requirements with a maximum percentage overshoot = 4, settling time of about settling time=0.7 sec and a zero steady state error.

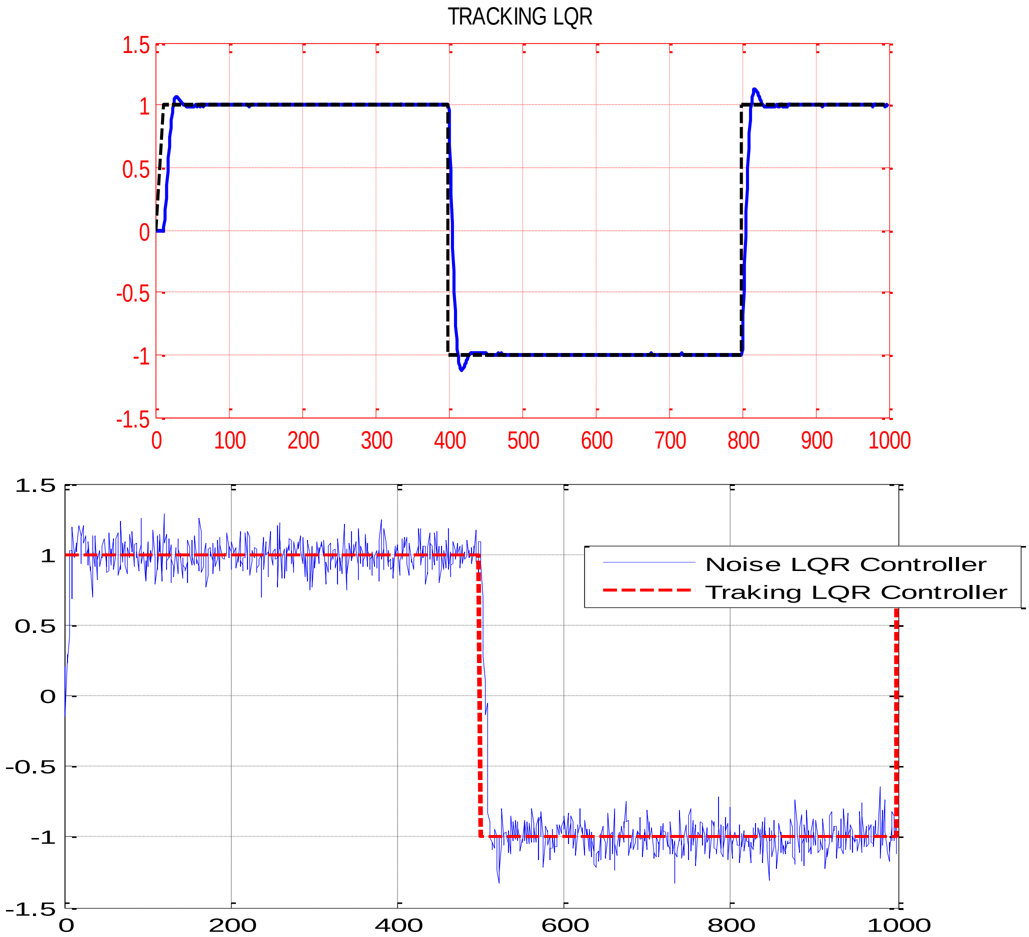

Figure (15) Tracking and Noise LQR Controller.

To test the system reliability, a step input signal with varying amplitude was applied to the system input. From and the system behavior was observed. From Figure (15), it is clear that the tracking of the LQR controller acceptable and that the LQR controller responses very well.

5-5 Control input for LQR Controller

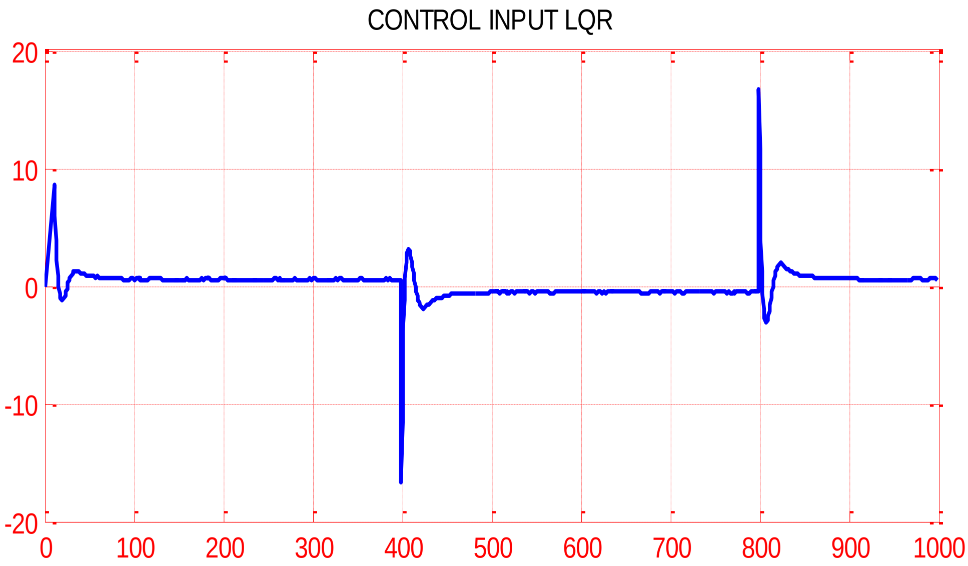

Figure (16) control input for LQR Controller.

5-7 Close Loop control system by using LQG Controller

Figure (17) shows the response of the LQG controller, from which it is clearly noticeable that the LQG controller meets the desired requirements with a maximum percentage Overshoot: 3, settling time of about 0.5 sec and a zero steady state error.

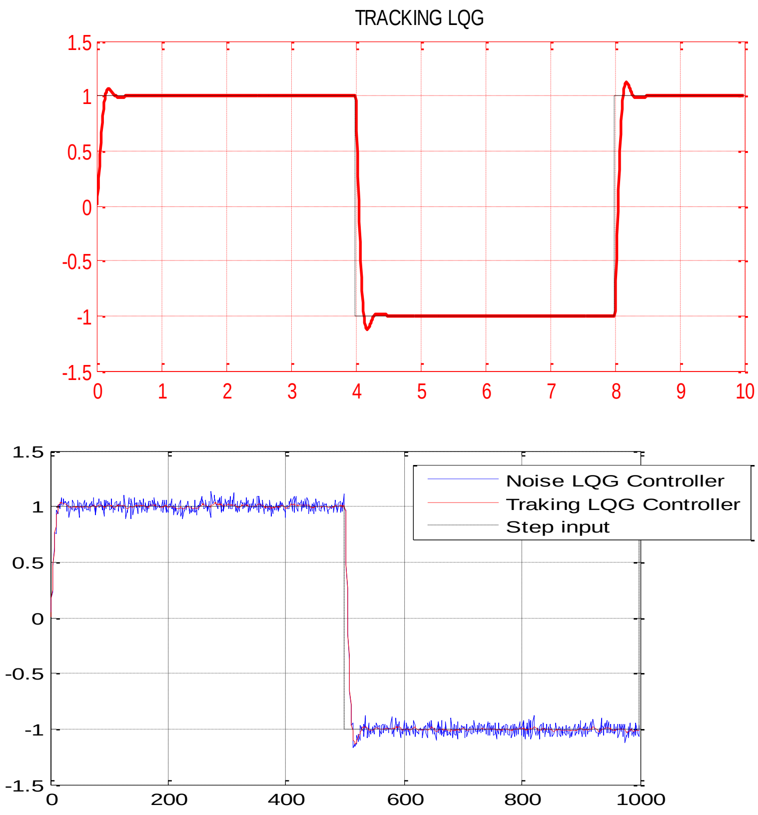

Figure (17) Tracking and Noise LQG Controller.

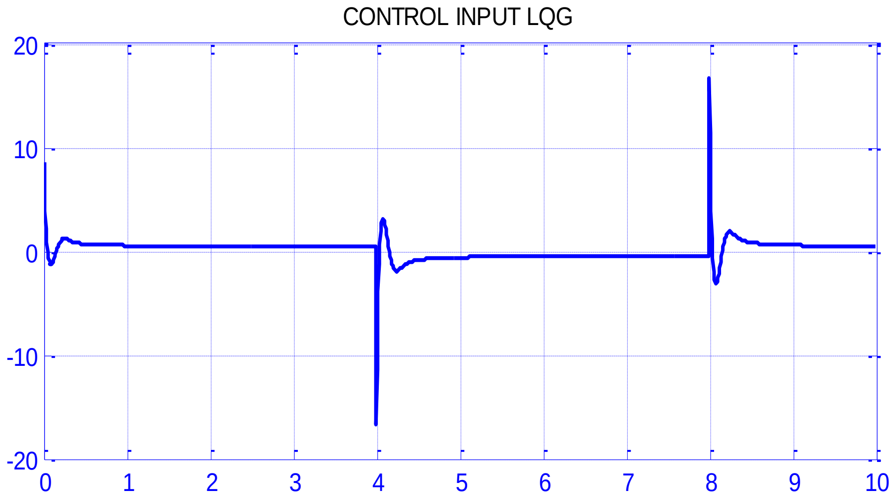

Figure (18) control input for LQG Controller.

Analysis of the Simulation Results. Table (3) indicates the improvement of the LQG controller response in terms of minimizing the overshoot, the rise time and the settling time.

Table (3) Comparison of result

ALL Controllers response	t_S	t_r	Op%
PID	1.8		7
LQR	0.7		4
LQG	0.5		3

6 - Conclusions

for Ball and Hoop system was simulated in the Matlab environment. We used Kalman Filter to reduce noise in the system. Both the LQG, LQR and PID controllers are applied to this model, which did not consider disturbances from noise. The control results demonstrate that the PID, LQG and LQR controller have good performance and are able to balance the in controlling the position of the ball and hoop system. The results of the system control demonstrate that the LQG controller has a better control performance than the tune PID controller and LQR .in terms of minimum rise and settling time values and peak overshoot value as required by the designer.

7 - References

[1]. Salvador Alc´antara Cano, Ramon Vilanova Arb´os, Carles Pedret i Ferr´e "PID Tuning a Modern Approach via the Weighted Sensitivity Problem" Taylor & Francis Group, LLC, (2021).

[2]. *Liuping Wang* RMIT University Australia "PID Control System Design and Automatic Tuning using MATLAB/Simulink ". John Wiley & Sons Ltd, (2020).

[3]. Iván D. Díaz-Rodríguez, Sangjin Han, Shankar P. Bhattacharyya " Analytical Design of PID Controllers " Springer Nature Switzerland AG (2019) pp. 20-21.

[4]. Banks J., Carson J., Nelson B., Nicaol D. "On Line PID Controller Tuning Using Genetic Algorithms. Discrete-Event System Simulation". Prentice Hall, (2001).1-3.

[5]. F., S. L." Numerical Solution of Ordinary Differential Equations". NEWYORK: Chapman & Hall (1994).

[6]."Online, Ball and Hoop" White Paper, URL: http://www.control systemsprinciples.co.uk.

[7]. Ball and Hoop White Paper, URL:http://www.control-systemsprinciples.co.uk

[8].Mr.Jennifer, Bruton,"On-line PID Controller Tuning using Genetic Algorithms" Date: 22/08/03.

[9]. Gustavo Maia de Almeida, Valceres Vieira Rocha e Silva, Erivelton Geraldo Nepomuceno, and Ryuichi Yokoyama " Application of Genetic Programming for Fine Tuning PID Controller Parameters Designed Through Ziegler-Nichols Technique", ICNC (2005), LNCS 3612, pages 313–322, *Springer*-Verlag Berlin Heidelberg.

[10]. Saifudin bin Mohamed Ibrahim,"The PID Controller design using genetic algorithm" Submitted: 27th October, 2005.
[11]. K. Turkoglu, E. M. Jafrov, "Augmented optimal LQR control system design for the longitudinal flight dynamics of an UAV: Inneand outer loop concepts", Proceedings of the 9th WSEAS International Conference on Automatic Control, Modeling & Simulation, Istanbul, Turkey, May 27-29, 2007.
[12]. Dunn, H.J., Realizable." Optimal Control for a Remotely Piloted Research Vehicle", May (1980) NASA.
[13]. Kumar R., Singh B., Das J." European Scientific Journal February" (2018).
[14]. Maravall D., Zhou C., Alonso J." Hybrid fuzzy control of the inverted pendulum via vertical forces". Int. J. Intell.sys (2005).
[15]. Ogata. "Modern control engineering". New Jersey: Pearson (2002).