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Study of Weak Continuous Functions with Separation Axioms in Bitopological Spaces

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Abstract:

The purpose of this work to set the notion of weak continuity in bitopology. Its characterizations and fundamental properties are introduced. The major conclusions bear on the concept of weakly continuous functions in Bitopological settings. Also, the implication of some separation axioms is investigated. Additionally, the attitude of p-weak continuity with pairwise Urysohn and pairwise connectedness spaces is studied.

Keywords: *p*-Weakly Continuity, Pairwise Subweak Continuity, Pairwise Hausdorff Spaces, Pairwise Urysohn Spaces.

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الملخص

الغرض من هذا العمل هو وضع وتحديد فكرة الدوال ضعيفة الاستمرارية في الفضاء التوبولوجيا الثنائية. لقد تم عرض الشكل والخواص الاساسية للدوال ضعيفة الاستمرارية. في هذا البحث، الاستنتاجات الرئيسية تركزت على ضعف الاستمرارية في التوبولوجيا الثنائية. أيضًا، تمت دراسة علاقة ضعف الاستمرارية مع مسلمات الفصل في التوبولوجيا الثنائية. بالإضافة الي ذلك، تم فحص ضعف الاستمرارية مع فضاءات يورسون والفضاءات المرتبطة في التوبولوجيا الثنائية.

الكلمات المفتاحية: ضعيفة الاستمرارية في الفضاء التوبولوجيا الثنائي، شبه ضعيف الاستمرارية في الفضاء التوبولوجيا الثنائي، فضاءات هاوسدورف الثنائي، فضاءات يورسون الثنائي.

1. Introduction

Kelly defined and initiated the Bitopological space in 1968 [2]. Then, many authors have generalized several topological concepts to bitopology category such as covering properties [9], mappings [6,10] and multifunctions [11] and others. One of the most essential concepts in topology is that of continuity, so several kinds of generalizations of continuity have been studied and investigated as the notion of weak continuity which was introduced and investigated in topology by N. Levine [7], J. Chew [8], T. Noiri [3] and D. A. Rose [4]. These authors have studied its properties and its impacts on some topological concepts. In current work we will study the notion of weak continuity in bitopology by extending this idea to pairwise weak continuity. In addition, we state its characterizations and properties. Further, we obtain its relation with pairwise almost continuity and pairwise continuity. Additionally, we introduce its attitude with pairwise Urysohn, pairwise connected and pairwise Hausdorff spaces.

All through this work, (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or shortly X & Y respectively) index bitopologies where, unless certainly listed contrary, no separation axioms have been given.
Let U be a subset of X. The closure and the interior of U with respect to τ_i will be signified by τ_i-$cl(U)$ and τ_i-$int(U)$ where $i = 1, 2$.
It is well-known; the axioms of separation in bitopology play a fundamental part for the theory of bitopology. Spaciously, several authors have investigated the notion of separation axioms. Pairwise Urysohn space was described by M. K. Singal and R. A. Singal [5].

Next, we state Kelly's definition for pairwise Hausdorff and pairwise regular spaces as follows:

Definition 1. [2] If (X, τ_1, τ_2) is a Bitopological space, then X is pairwise Hausdorff if for every $x_1, x_2 \in X$ and $x_1 \neq x_2$, there is a set $U \in \tau_i$ and a set $V \in \tau_j$ so that $x_1 \in U, x_2 \in V$ together with $U \cap V = \emptyset$ for $i \neq j$ and $i, j = 1,2$.

Definition 2. [2] If X is a bitopological space, then it is a ij-regular if for any point $x \in X$ and a set $U \in \tau_i$ having x, there is a set $G \in \tau_i$ so that $x \in G \subset \tau_j cl$-$(G) \subset U$. The Bitopological space X is called p-regular whenever X is 12-regular and 21-regular.

Next, we present the meaning of continuous mapping in bitopology, which Kılıçman and Salleh have stated as the following definition.

Definition 3. [**6**] $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a i-continuous function if $f: (X, \tau_i) \to (Y, \sigma_i)$ is continuous where $i = 1,2$. f is said continuous if it is i-continuous for $i = 1,2$.

Definition 4. A space (X, τ_1, τ_2) is called (i, j)-Lindel$\ddot{o}$f if every τ_i-open cover of X can be reduced to a countable τ_j-open cover. (X, τ_1, τ_2) is pairwise Lindelöf if X is (1,2)- Lindelöf and (2,1)- Lindelöf.

The p-Urysohn space was described by A. R. Singal and M. K. Singal.

Definition 4. [5] A Bitopological space (X, τ_1, τ_2) is (τ_i, τ_j)-Urysohn if for any two distinct points $x_1, x_2 \in X$, there exist a set $U \in \tau_i$ and a set $V \in \tau_j$ in X in order that $x_1 \in U$, $x_2 \in V$ and
τ_j- $cl(U) \cap \tau_i$- $cl(V) = \emptyset$.
A space X is called pairwise Urysohn if X is (1,2)- Urysohn and (2,1)- Urysohn.

Definition 5. [1] $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i, j)-weak ((i, j)-almost) continuous function iff as long as all $x \in X$ and every a neighborhood V as $f(x) \in V \in \tau_i$,there is a neighborhood U as $x \in U \in \tau_i$ where

$$f(U) \subset \sigma_j\text{-}cl(V) \ \ (f(U) \subset \sigma_i\text{-}\mathrm{int}(\sigma_j\text{-}\mathrm{cl}(V))).$$

f is termed p-weak (p-almost) continuity if it is (1,2)-weak ((1,2)-almost) and (2,1)-weak ((2,1)-almost).

Definition 6. [**6**] A space X is said to be τ_i-P-space if each countable intersection of τ_i- α-open sets is τ_i- α-open. X is said P- space if X τ_i-P-space for $i = 1,2$.

2. Characterizations of p-weak continuous functions.

Remark 1. Obviously, any i-continuity indicates (i, j)-weak continuity. However, the opposite is not always be reversed.

Example 1. Let $X = \{a_1, a_2, a_3\}$, $\tau_1 = \tau_2 = \{\emptyset, X, \{a_2\}\}$ and
$\sigma_1 = \sigma_2 = \{\emptyset, X, \{a_1\}\}$. Let $f: (X, \tau_1, \tau_2) \to (X, \sigma_1, \sigma_2)$ be a function such that $f(a_n) = a_n$, $n = 1,2,3$. So, f is (1,2)-weak continuity but not $1 -$continuity.

Theorem 1. $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i, j)-weak continuous iff $f^{-1}(V) \subseteq \tau_i\text{-}\mathrm{int}\,(f^{-1}(\sigma_j\text{-}\,cl(V)))$ for each $V \in \sigma_i$, $i, j = 1,2$.

Proof. ⇐ Suppose $x \in X$ and $V \in \sigma_i$ where $f(x) \in V$. By hypothesis, we get

$$x \in f^{-1}(V) \subseteq \tau_i\text{-}\ \mathrm{int}\,(f^{-1}(\sigma_j\text{-}\,cl(V))).$$

Put $U = \tau_i\text{-}\mathrm{int}\,(f^{-1}(\sigma_j\text{-}\,cl(V)))$, so $U$is τ_i-open set. Therefore, we get

$$U = \tau_i \text{ - int } (f^{-1}(\sigma_j\text{- } cl(V))) \subseteq f^{-1}(\sigma_j\text{- } cl(V))$$
$$\Rightarrow f(U) \subseteq \sigma_j\text{- } cl(V).$$

Thus, the function f is (i,j)-weak continuity.

$\Rightarrow$ Suppose that f is (i,j)-weak continuity. Let $V \in \sigma_i$ and $x \in f^{-1}(V) \Rightarrow f(x) \in V$. Due of f is (i,j)-weak continuity, we have $U \in \tau_i$ involving x so that

$$f(x) \in f(U) \subset \sigma_j \text{ - } cl(V)$$
$$\Rightarrow x \in U \subset f^{-1}(\sigma_j \text{ - } cl(V))$$
$$\Rightarrow x \in \tau_i\text{- } int(f^{-1}(\sigma_j \text{ - } cl(V))).$$

Consequently, $f^{-1}(V) \subseteq \tau_i\text{- int}(f^{-1}(\sigma_j \text{ - } cl(V)))$. ■

Theorem 2. If a function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (i,j)-weak continuity, so $\tau_j\text{- } cl\left(f^{-1}(V)\right) \subseteq f^{-1}(\sigma_j\text{- } cl(V))$ for any a subset $V \in \sigma_i$.

Proof. Consider that we have a point $x \in \tau_j\text{- } cl\left(f^{-1}(V)\right) \text{-} f^{-1}(\sigma_j\text{- } cl(V))$. So $f(x) \notin \sigma_j \text{ - } cl(V)$. Let $P = Y - \sigma_j\text{-}cl(V)$. Thus is a $P \in \sigma_j$ where $f(x) \in P$ and $V \cap P = \emptyset$.

Because f is (i,j)-weak continuous, there is a set $U \in \tau_i$ so that $x \in U$ and $f(U) \subset \sigma_j\text{- } cl(P)$. Hence, we get $f(U) \cap V = \emptyset$. In contrast, since $x \in \tau_j\text{- } cl\left(f^{-1}(V)\right)$, we have $U \cap f^{-1}(V) \neq \emptyset$ and then $f(U) \cap V \neq \emptyset$, so we get contradiction. Therefore

$$\tau_j\text{- } cl\left(f^{-1}(V)\right) \subset f^{-1}(\sigma_j\text{- } cl(V)). \blacksquare$$

Theorem 3. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a function. If $\tau_j\text{- } cl\left(f^{-1}(Q)\right) \subseteq f^{-1}(\sigma_j\text{- } cl(Q))$ for any σ_i-open subset Q of Y, hence f is (j,i)-weak continuity.

Proof. Assume that $V \in \sigma_j$. Put $Q = Y - \sigma_i\text{-}cl(V)$. Thus $\tau_j\text{- } cl(f^{-1}(Q) \subseteq f^{-1}(\sigma_j\text{- } cl(Q))$.
So, we get

$$\tau_j\text{- } cl(f^{-1}(Y \text{ - } \sigma_i\text{- } cl(V))) \subseteq f^{-1}(\sigma_j\text{-}cl(Y\text{- } \sigma_i\text{-}cl(V)))$$

$$\Rightarrow \tau_j\text{- } cl(X - f^{-1}(\sigma_i\text{- } cl(V))) \subseteq X - f^{-1}(\sigma_j\text{-}int(\sigma_i\text{-}cl(V)))$$

$$\Rightarrow X - (\tau_j\text{- } int(f^{-1}(\sigma_i\text{- } cl(V))) \subseteq X - f^{-1}(\sigma_j\text{-}int(\sigma_i\text{-}cl(V)))$$

$$\subseteq X\text{-}f^{-1}(V).$$

Therefore,

$$f^{-1}(V) \subseteq \tau_j\text{- } int(f^{-1}(\sigma_i\text{- } cl(V))).$$

From **Theorem 1**, f is (j,i)-weak continuous. ■

Theorem 4. If a function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (i,j)-weak continuity and i-open, hence f is (i,j)-almost continuity.

Proof. Suppose that $a \in X$ and $V \in \sigma_i$ having $f(a)$. Because f is (i,j)-weak continuity, there exists a subset $U \in \tau_i$ containing a so that $f(U) \subset \sigma_j\text{- cl}(V)$. Due of f must be i-open, we have
$f(U) = \sigma_i\text{- } int(f(U)) \subset \sigma_i\text{- } int(\sigma_j\text{- cl}(V))$.
So, f must be (i,j)-almost continuity. ■

Corollary 1. An i-open function is (i,j)-almost continuity iff is (i,j)-weakly continuity.

Definition 6. Suppose that f be a function from a bitopological space $(X, \tau_1 \tau_2)$ into a bitiopolgical setting (Y, σ_1, σ_2) . f is said to be (i,j)-subweakly continuity if $\tau_j\text{- } cl(f^{-1}(V)) \subseteq f^{-1}(\sigma_j\text{- } cl(V))$, for $V \in \mathcal{B}$ such that $\mathcal{B}$ is a σ_i-open basis for the bitopological space (Y, σ_1, σ_2). f is said p-subweakly continuity if it is (1,2)-subweakly continuity and (2,1)-subweakly continuity.

Theorem 5. A function $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is (i,j)-subweakly continuity and (i,j)-almost continuity, thus f is (i,j)-weakly continuity.

Proof. For a bitopological space (Y,σ_1,σ_2), let a basis $\mathcal{B}\in\sigma_i$ relative to which f is (i,j)-subweakly continuous function. Let $V\in\mathcal{B}$, since f is (i,j)-almost continuous, we get

$$f^{-1}(V)\subseteq\tau_i\text{-}int(\sigma_j\text{-}cl(f^{-1}(V)))$$
$$\subseteq\tau_i\text{-}int(f^{-1}(\sigma_j\text{-}cl(V))).$$

Thus, f is (i,j)-weakly continuity function via **Theorem 1**.

3. Implication of pairwise weakly continuity with pairwise separation axioms.

Theorem 6. Each (i,j)-weakly continuous function from (τ_i,τ_j)-Lindelöf into any P-space (σ_i,σ_j)- Hausdorff space is (i,j)-closed.

Proof. consider f be (i,j)-weakly continuous function from (τ_i,τ_j)-Lindelöf space (X,τ_1,τ_2) into any (σ_i,σ_j)-Hausdorff space (Y,σ_1,σ_2). Let C be τ_i-closed set in (X,τ_1,τ_2) and let $b\notin f(C)$. For every $a\in X$, which $f(a)\in f(C)$, due to Hausdorffness of (Y,σ_1,σ_2), there is a neighborhood $V\in\sigma_i$ of $f(a)$ where $f(a)\notin\sigma_j\text{-}cl(V)$. because f *is* (i,j)-weakly continuity, there exists a neighborhood $U\in\tau_i$ having a so that $f(U)\subset\sigma_j\text{-}cl(V)$.

Because C is τ_i-closed set of a (τ_i,τ_j)-Lindelöf space (X,τ_1,τ_2),C is (τ_i,τ_j)-Lindelöf . Then, we have $C\subset\bigcup_{n\in k}U_{a_n}$such that K is countable set.
So, it implies that

$$f(C)\ \subset\ f\left(\bigcup_{n\in k}U_{a_n}\right)=\ \bigcup_{n\in k}f\left(\ U_{a_n}\right)\ \subset\ \bigcup_{n\in K}\sigma_j\ \text{-}cl\left(V_{f(a_n)}\right).$$

Since Y is a P- space, so $Y\text{-}\bigcup_{n\in K}\sigma_j\text{-}cl\left(V_{f(a_n)}\right)\in\sigma_j$ contains b where $[Y-\bigcup_{n\in K}\sigma_j\text{-}\ cl\left(V_{f(a_n)}\right)]\cap f(C)=\emptyset$. Then, $f(C)$ is σ_j-closed. Thus, f is (i,j)-closed. ■

Theorem 7. If a function $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is (i,j)-weak continuity and Y is (i,j)-regular space, so f is i- continuity.

Proof. For any $a\in X$ with $f(a)\in V$ so that V is σ_i-open set. Since Y is (i,j)-regular space, we get $f(a)\subseteq Q\subseteq\sigma_j\text{-}\ cl(Q)\subseteq V$ for $G\in\tau_i$. By weakness of f, we have a set $U\in\tau_i$ where

$$f(a)\in f(U)\ \subseteq\sigma_i\text{-}\ cl(Q)\subseteq V.$$

Therefore, f must be i-continuity at a. Because a is an arbitrarily point in X, so f is i- continuous.■

Example 2. Consider $X=Y=\mathcal{R}$ real numbers. Assume that $\tau_1=\tau_2$ the indiscrete topology. Let σ_1 be usual topology and σ_2 be the discrete topology. If $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is the identity function. Y is (1,2)-regular space, but f is 1 -discontinuity. By the previous Theorem, f is not (i,j)-weak continuous.

Theorem 8. Let $f:(X,\sigma_1,\sigma_2)\rightarrow(Y,\sigma_1,\sigma_2)$ be p-weak continuity. If Y is (i,j) $-$Hausdorff space, hence $E=\{(a_1,a_2)\in X\times X\colon\ f(x_1)=f(a_2)\}$ is (i,j)-closed in $X\times X$.

Proof. If $(a_1,a_2)\ \in X\times X\text{-}\ E$, so $f(a_1)\neq f(a_2)$. Since (Y,σ_1,σ_2) is (i,j)-Hausdorff space, we have a set $V\in\sigma_i$ having $f(a_1)$ and a set $W\in\sigma_j$ having $f(a_2)$ where $V\cap W=\emptyset$. Because of f is p-weak continuity, there exists two sets where $U\in\tau_i$ and $U_2\in\tau_j$ holding a_1 and a_2 respectively, where $f(U_1)\subseteq\sigma_j\text{-}\ cl(V)$ and $f(U_2)\subseteq\sigma_i\text{-}\ cl(W)$. So, $(U_1\times U_2)\cap\ E=\emptyset$. Therefore, $U_1\times U_2$ is (i,j)-open in $X\times X$ containing (a_1,a_2). It follows that E is (i,j)-closed in $X\times X$.■

Theorem 9. Consider that a function $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ is p-weak continuity as-well-as one-to-one. If (Y,σ_1,σ_2) is (σ_i,σ_j)-Urysohn space, then (X,τ_1,τ_2) is (τ_i,τ_j)-Hausdorff space.

Proof. Let x_1 , $x_2\ \in X$ such that $x_1\neq x_2$. Since the function f is one-to-one, we have $f(x_1)\neq f(x_2)$. Due to Urysohntness of Y, there exist two sets $V_1\in\sigma_i$ and $V_2\in\sigma_j$ in Y where $f(x_1)\in V_1,f(x_2)\in V_2$ and

$$\sigma_j\text{-}\ cl(V_1)\cap\sigma_i\text{-}\ cl(V_2)=\emptyset.$$

$$\Rightarrow \tau_i\text{-}\, int(f^{-1}(\sigma_j\text{-}\, cl(V_1))) \cap \tau_j\text{-}\, int(f^{-1}(\sigma_i\text{-}\, cl(V_2))) = \emptyset.$$

Since f is pairwise weak continuous, it follows

$$x_1 \in f^{-1}\big((V_1)\big) \subset \tau_i\text{-}\, int(f^{-1}(\sigma_j\text{-}\, cl(V_1)))$$
$$x_2 \in f^{-1}\big((V_2)\big) \subset \tau_j\text{-}\, int(f^{-1}(\sigma_i\text{-}\, cl(V_2))).$$

Therefore, X is (τ_i, τ_j)-Hausdorff space.■

Theorem 10. If (X, τ_1, τ_2) is (τ_i, τ_j)-connected space and $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is p-weak continuity and onto, so Y is (σ_i, σ_j)-connected space.

Proof. Consider that a space Y is not (σ_i, σ_j)-connected space, so we obtain two sets $V_1 \in \sigma_i$ and $V_2 \in \sigma_j$ in Y where $V_1 \cap V_2 = \emptyset$ also $V_1 \cup V_2 = Y$. Now, $f^{-1}(V_1) \cap f^{-1}(V_2) = \emptyset$ and $f^{-1}(V_1) \cup f^{-1}(V_2) = X$. Because f is onto, $f^{-1}(V_k) \neq \emptyset$ as $k = 1,2$. By weakness of f, so

$$f^{-1}\big((V_1)\big) \subset \tau_i\text{-}\, int(f^{-1}(\sigma_j\text{-}\, cl(V_1)))$$
$$f^{-1}\big((V_2)\big) \subset \tau_j\text{-}\, int(f^{-1}(\sigma_i\text{-}\, cl(V_2))) \quad .$$

Because V_1 is σ_i −open and σ_j −closed set and also V_2 is σ_j −open and σ_i −closed set in Y, we have

$$f^{-1}\big((V_1)\big) \subset \tau_i\text{-}\, int\big(f^{-1}(V_1)\big)$$
$$f^{-1}\big((V_2)\big) \subset \tau_j\text{-}\, int\big(f^{-1}(V_2)\big).$$

Then, $f^{-1}\big((V_1)\big) \in \tau_i$ and $f^{-1}\big((V_2)\big) \in \tau_j$. It means the space X must not be (τ_i, τ_j)-connectedness. It is contrary to that X is (τ_i, τ_j)-connected space. Thus, (Y, σ_1, σ_2) is (σ_i, σ_j)-connected space.■

Theorem 11. Suppose that (Y, σ_1, σ_2) is (σ_i, σ_j) −locally connected space and a function $f: X \to Y$ is a onto p-almost continuity. If f and f^{-1} both preserve connected sets, so f is (i, j)- weak continuity.

Proof. Consider $V \in \sigma_i$ as connected basic. If $x \in \tau_j\text{-}\, cl\big(f^{-1}(V)\big) - f^{-1}(\sigma_j\text{-}\, cl(V))$, so there exists a connected set $W \in \sigma_i$ where $f(x) \in W$ also $V \cap W = \emptyset$. Because $f^{-1}(V) \neq \emptyset$ and $f^{-1}(W) \neq \emptyset$, then $(V \cup W) \cap f(X)$ is disconnected. Since f and f^{-1} both preserve connected sets, $f^{-1}(V)$ and $f^{-1}(W)$ are connected and $\tau_j\text{-}\, cl\big(f^{-1}(V)\big) \cap f^{-1}(W) \neq \emptyset$. Therefore $f^{-1}(V) \cup f^{-1}(W) = f^{-1}(V \cup W)$ is connected. Then $f\big(f^{-1}(V \cup W)\big) = (V \cup W) \cap f(X)$ is connected. This is contradiction, so that

$$\tau_j\text{-}\, cl\big(f^{-1}(V)\big) \subseteq f^{-1}(\sigma_j\text{-}\, cl(V)).$$

Thus f should be (i, j)- subweak continuity. A function f is (i, j)- weak via **Theorem 5.■**

4. Conclusion

In this work, we consider the notion of weak continuity in bitopology. Further, we study its characterizations and basic properties. Also, we set some relations between p-weak continuity with some kind of continuity in bitopology. In addition, we study its attitude with pairwise Urysohness, pairwise connectedness and pairwise Hausdorffness.

5. References

[1] S. Bose, D. Sinha (1982). Pairwise almost continuous map and weakly continuous map in bitopological spaces, Bull. Cal. Math. Soc. 74, pp. 195–206.

[2] J.C. Kelly (1963). Bitopological spaces, Proc. London Math. Soc. 3 (13) , pp. 71–89.

[3] T. Noiri (1974). On weakly continuous mappings, Proc. Amer. Math. Soc., 46, pp. 120-124.

[4] D. A. Rose (1984). Weak continuity and almost continuity, Internat. J. & Math. Sci., Vol. 2, pp. 311-318.

[5] M. K. Singal and A. R Singal (1970). Some more separation axioms in bitopological spaces , Annales de la Societe Scientifique de Bruxelles. 84, pp. 207-230.

[6] Kılıçman, A. and Salleh, Z. (2010). Mappings and decompositions of pairwise continuity on pairwise nearly Lindelöf spaces, Albanian J. Math., 4(2), 31-47.

[7] N. Levine (1961). A decomposition of continuity in topological spaces, Amer. Math. Monthly, 63, pp.44-66.

[8] J. Chew, and J. Tong (1991). Some remarks on weak continuity, Amer. Math. Monthly 98, pp. 931-934.
[9] Hend M. Bouseliana and A. Kiliçman (2015). A comparison of paralindelöf-type properties in bitopological spaces. Applied Mathematics and Information Science, 9(5), 1787-1792.
[10] Hend M. Bouseliana and A. Kiliçman (2015). Mappings on pairwise para-lindelöf bitopological Spaces, Journal of the Egyptian Mathematical Society, 23, 377-381.
[11] Hend M. Bouseliana and A. Kiliçman (2013). On pairwise super continuous multifunctions, AIP Conf. Proc. 1557: 10.1063/1.4823939, 376-379.