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# Periodic, Rogue and Lump Soliton Solutions in Fiber Bragg Gratings for the Coupled Form of the Nonlinear $(2+1)$-Dimensional Kundu-Mukherjee-Naskar Equation

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Abstract:
In this article, the coupled version of the nonlinear (2+1)-dimensional Kundu-Mukherjee-Naskar equation in fiber bragg gratings was examined. To support the new optical solutions of the coupled form of the nonlinear (2+1)-dimensional Kundu-Mukherjee-Naskar equation in Fiber Bragg Gratings (FBGs), three well-known techniques were used: Periodic Cross-Kink wave, Rogue wave, and Lump interaction with kink and rogue waves. A novel type of traveling wave solution with interaction phenomena was produced by applying the proper functions of the solutions. Two and three-dimensional visualizations were used to depict the dynamics of the created solutions, demonstrating the dependability and effectiveness of the offered methods.

**Keywords:** Optical Solitons, Periodic Cross-Kink Wave, Rogue Wave, Lump Interaction With Kink And Rogue Waves, Fiber Bragg Gratings, Kundu-Mukherjee-Naskar Equation.

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# حلول سولوتونية دورية ، الموجة العملاقة و الدوال الكسرية المحلية المنتشرة في كل الاتجاهات في شبكات الالياف Bragg للشكل المزدوج لمعادلة كوندو- موخرجي - ناسكار (2+1) غير الخطية

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**الملخص**

في هذه المقالة، تم فحص النسخة المزدوجة لمعادلة كوندو- موخرجي – ناسكار (2+1) غير الخطية في شبكات الألياف Bragg. لدعم الحلول البصرية الجديدة للشكل المزدوج لمعادلة كوندو- موخرجي -ناسكار (2+1) غير الخطية في شبكات الألياف الضوئية (FBGs)، تم استخدام ثلاث تقنيات معروفة: موجة التشابك الدورية، والموجة العملاقة. والتفاعل المقطوع مع الأمواج الملتوية والعملاقة. تم إنتاج نوع جديد من حلول الموجات المتنقلة مع ظواهر التفاعل من خلال تطبيق الوظائف المناسبة للحلول. تم استخدام تصورات ثنائية وثلاثية الأبعاد لتصوير ديناميكيات الحلول التي تم إنشاؤها، مما يدل على موثوقية وفعالية الأساليب المقدمة.



**الكلمات المفتاحية:** المنعزلات الضوئية، الموجة المتقاطعة الدورية، الموجة العملاقة، التفاعل المقطوع مع الموجات المتعرجة والعملاقة، شبكات الألياف Bragg، معادلة كوندو- موخيرجي- ناسكا.


## 1. Introduction

Soliton waves occur due to a balance between dispersive and nonlinear effects. Recently, these waves have emerged in various fields including molecular material science, optics, liquids, plasma, atomic material science, and astronomy [1–27]. The speed and shape of the solitons remain constant over time. Owing to their applications, Fiber Bragg Gratings (FBGs) have garnered significant interest from physicists and engineers in recent years. FBGs are utilized in all-optical communication frameworks, thus they have been undergone extensively using a variety of assortment of strategies including, the three-wave method, double exponential, homoclinic breather, M-Shaped rational, M-Shaped with one kink, and M-Shaped with two kink [28], extended trial function method [29] etc. The study model in this article is a specific sort of NLSE with a particular shape of non-linearity and scattering terms [32–34].

In this paper, we investigate the Kundu-Mukherjee-Naskar model in FBGs for periodic cross-kink waves, rogue waves, and lump interaction with kink and rogue waves. The solutions addressed in this paper are novel because the Kundu-Mukherjee-Naskar model in FBGs has never been studied using the three methods used here. The extra findings in this paper improve the outcomes reported in reference [28]. The nonlinear (2+1)-dimensional Kundu-Mukherjee-Naskar model is written out as [32–34]:

$$i\psi_t + a\psi_{xy} + ib(\psi\psi_x^* - \psi^*\psi_x)\psi = 0, \tag{1.1}$$

Here $a, b$ are constants and $\psi(\mathrm{x},\mathrm{y},t)$ is a complex-valued function that depicts the wave profile. The symbol $i\psi(t)$ represents the linear temporal evolution, the dispersion term consedered by $a\psi_{xy}$, the sign for non-linearity is $ib(\psi\psi_x^* - \psi^*\psi_x)\psi$, and $i = \sqrt{-1}$. Following is the initial introduction of coupled vector version of (1.1)[1]:

$$\begin{aligned} &i\psi_t + a_1\phi_{xy} + i[(b_1\psi^2 + c_1\phi^2)\psi_x^* - (d_1|\psi|^2 + e_1|\phi|^2)\psi_x] + i\alpha_1\psi_x + \beta_1\phi + \sigma_1\psi^*\phi^2 = 0,\\ &i\phi_t + a_2\psi_{xy} + i[(b_2\phi^2 + c_2\psi^2)\phi_x^* - (d_2|\phi|^2 + e_2|\psi|^2)\phi_x] + i\alpha_2\phi_x + \beta_2\psi + \sigma_2\phi^*\psi^2 = 0, \end{aligned} \tag{1.2}$$

Here $\psi(x,y,t)$, $\phi(x,y,t)$ are complex functions that depicts the wave profiles and $a_j, b_j, c_j, d_j, e_j, \alpha_j, \beta_j, \sigma_j, (j = 1,2)$ are constants. Here $a_j$ is a representation of the dispersion terms coefficients. The variables $b_j, c_j, d_j, e_j (j = 1,2)$ serve as a representation of the non-linearity. The variables $a_j, \beta_j$, and $\sigma_j$ represent the inter-modal dispersions, the detuning, and the four-wave mixing respectively. The major goal of this article is to obtain Multi-waves, Double exponential and interaction phenomena, homoclinic breather approach, M- Shaped rational solutions, M-Shaped interaction with one kink, and M-Shaped interaction with two kink for Eq.(1.2). This article is organized as follows: Section 2 introduces the mathematical preliminaries. We provide the solutions of (1.2) in Sections 3,4,5,6,7, and 8. A few solutions are depicted numerically in Section 9. The conclusion is illustrated in Section 10.

## 2. Mathematical preliminaries

The guiding principle for resolving the coupled equation under consideration is provided by

$$\psi(x,t) = w_1(\zeta), \tag{2.1}$$

$$\phi(x,t) = w_2(\zeta), \tag{2.2}$$

where $w_j, (j = 1,2)$ depicts the pulse forms, and

$$\zeta(x,t) = k_1x + k_2y - \nu t, \tag{2.3}$$

where $k_1, k_2$, and $\nu$ are non-zero constants.The direct cosines $k_1$ and $k_2$ denote the inverse soliton widths in the $x$ and $y$ directions, while $\nu$ is the soliton speed. By incorporating (2.2) and (2.3) into (1.2) we obtain

$$a_1 k_1 k_2 w_2'' + k_1 w_2 + \sigma_1 w_1 w_2^2 + i((\alpha_1 k_1 - \nu) w_1' + k_1(b_1 - d_1) w_1^2 w_1' + k_1(c_1 - e_1) w_2^2 w_1') = 0, \quad (2.4)$$

$$a_2 k_1 k_2 w_2'' + k_1 w_2 + \sigma_2 w_1 w_2^2 + i((\alpha_2 k_1 - \nu) w_1' + k_1(b_2 - d_2) w_1^2 w_1' + k_1(c_2 - e_2) w_2^2 w_1') = 0. \quad (2.5)$$

The result of combining equations (2.4) and (2.5) is

$$a_j k_1 k_2 w_l'' + k_1 w_l + \sigma_j w_j w_l^2 + i\left((\alpha_j k_1 - \nu) w_j' + k_1\left(b_j - d_j\right) w_j^2 w_j' + k_1\left(c_j - e_j\right) w_l^2 w_j'\right) = 0, \quad (2.6)$$

By applying the balancing principle to the situation where $j = 1,2$ and $l = 3 - j$, we obtain $w_l = A w_j$, $A \in \mathbb{R} - \{0,1\}$.

$$a_j k_1 k_2 A w_j'' + A k_1 w_j + A^2 \sigma_j w_j^3 + i((\alpha_j k_1 - \nu) w_j' + k_1(b_j - d_j + A^2(c_j - e_j)) w_j^2 w_j') = 0, \quad (2.7)$$

dividing up into real and imaginary components we obtain:

$$a_j k_1 k_2 w_j'' + k_1 w_j + A \sigma_j w_j^3 = 0, \quad (2.8)$$

$$(\alpha_j k_1 - \nu) w_j' + k_1(b_j - d_j + A^2(c_j - e_j)) w_j^2 w_j' = 0, \quad (2.9)$$

equation (2.9) provides us with $A = \pm\sqrt{\frac{d_j - b_j}{c_j - e_j}}$ and

$$\nu = \alpha_j k_1, \quad (2.10)$$

and the condition $(d_j - b_j)(c_j - e_j) > 0$ is obtained.

$$w_j = 2(\ln u)_\zeta, \quad (2.11)$$

equation (2.8) is transformed using the log transformation [38], and the result is

$$a_j k_1 k_2 (u^2 u_{\zeta\zeta\zeta} - 3 u u_\zeta u_{\zeta\zeta}) + (2 a_j k_1 k_2 + 4 A \sigma_j) u_\zeta^3 + k_1 u^2 u_\zeta = 0. \quad (2.12)$$

## 3. Periodic Cross-Kink wave

we study periodic cross-kink wave solutions which contain exponential function, trigonometric function, and hyprabolic function. setting

$$u = e^{-(a_3 + a_4\zeta)} + b_3\, e^{(a_3 + a_4\zeta)} + b_4\, cos(a_5 + a_6\zeta) + b_5\, cosh(a_7 + a_8\zeta) + a_9, \quad (3.1)$$

where $a_i (3 \leq i \leq 9)$ and $b_j (j = 3,4,5)$ are real parameters. Put $u$ into equation (2.12) and collecting all the paramameters of $e^{(a_3+a_4\zeta)} \cosh^2(a_7 + a_8\zeta)$, $e^{(a_3+a_4\zeta)}(sinh(a_7 + a_8\zeta) cosh(a_7 + a_8\zeta))$ , $e^{i(a_3+a_4\zeta)}$,$\sinh^i(a_7 + a_8\zeta)$,$\cosh^j(a_7 + a_8\zeta)$,$\cosh^j(a_7 + a_8\zeta)\; sinh(a_7 + a_8\zeta)$, $e^{j(a_3+a_4\zeta)}(cosh(a_7 + a_8\zeta))$, $e^{j(a_3+a_4\zeta)}(sinh(a_7 + a_8\zeta))$ for $(i = 1,2,3)$ , $(j = 1,2)$, and the powers of $\zeta$'s. We get system of equations which gives following values of parameters:

**Case 1**: $a_3 = a_3,\ b_3 = b_3,\ b_4 = b_4,\ b_5 = b_5,\ a_5 = a_5,\ a_6 = a_6,\ a_7 = a_7,\ a_8 = a_8,\ a_9 = a_9,\ a_4 = -\frac{1}{\sqrt{2 a_j k_2}}$ and $\sigma_j = -\frac{a_j k_1 k_2}{2A}$.

Putting these constants into (3.1) and then using (2.11), we deduce the solution for equation (1.2) as follows:

For $\sigma_1 = -\frac{a_1 k_1 k_2}{2A}$

$$\psi(x,y,t) = 2\frac{\frac{1}{\sqrt{2a_j k_2}} e^{-\left(a_3 - \frac{1}{\sqrt{2a_1k_2}}\zeta\right)} - \frac{b_1}{\sqrt{2a_jk_2}} e^{\left(a_3 - \frac{1}{\sqrt{2a_1k_2}}\zeta\right)} - a_6\, b_2\, sin(a_5 + a_6\zeta) + a_8\, b_3\, sinh(a_7 + a_8\zeta)}{e^{-\left(a_3 - \frac{1}{\sqrt{2a_1k_2}}\zeta\right)} + b_1\, e^{\left(a_3 - \frac{1}{\sqrt{2a_1k_2}}\zeta\right)} + b_2\, cos(a_5 + a_6\zeta) + b_3\, cosh(a_7 + a_8\zeta) + a_9}, \quad (3.2)$$

For $\sigma_2 = -\frac{a_2 k_1 k_2}{2A}$

$$\phi(x,y,t) = 2\frac{\frac{1}{\sqrt{2a_jk_2}}e^{-\left(a_3-\frac{1}{\sqrt{2a_2k_2}}\zeta\right)}-\frac{b_1}{\sqrt{2a_jk_2}}e^{\left(a_3-\frac{1}{\sqrt{2a_2k_2}}\zeta\right)}-a_6\,b_2\,\sin(a_5+a_6\zeta)+a_8\,b_3\,\sinh(a_7+a_8\zeta)}{e^{-\left(a_3-\frac{1}{\sqrt{2a_2k_2}}\zeta\right)}+b_1\,e^{\left(a_3-\frac{1}{\sqrt{2a_2k_2}}\zeta\right)}+b_2\,\cos(a_5+a_6\zeta)+b_3\,\cosh(a_7+a_8\zeta)+a_9}. \tag{3.3}$$

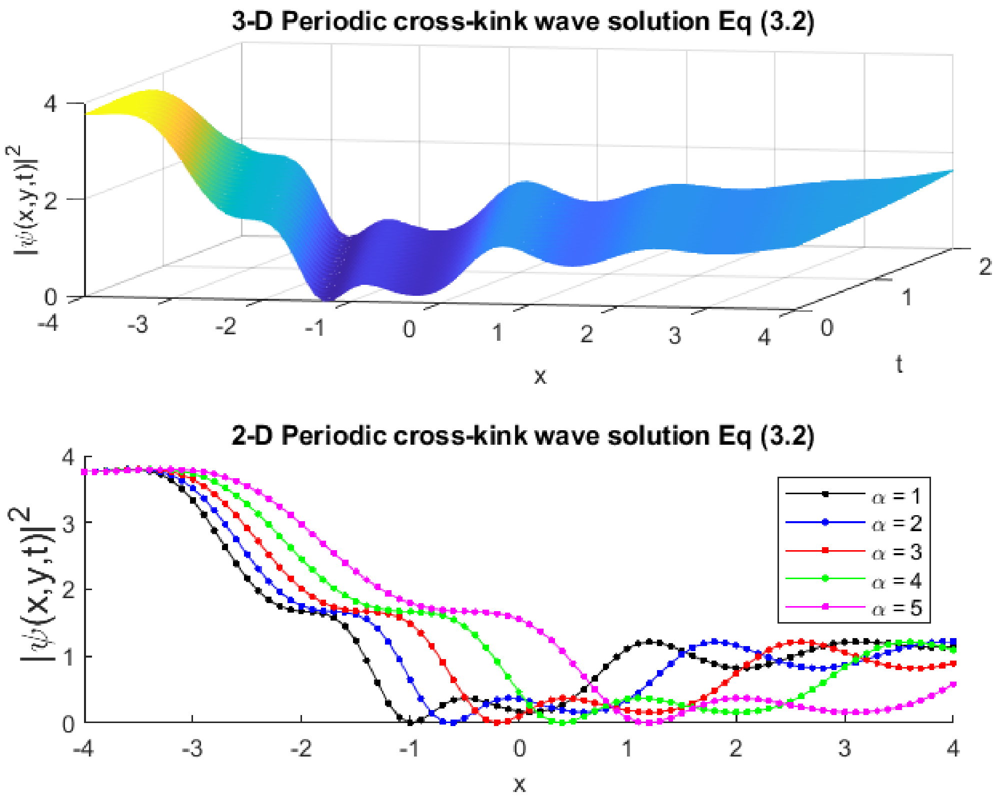


**Figure 1:** For $b_1 = 0.1,\ b_3 = 2,\ b_4 = -1,\ b_5 = 0.5,\ c_1 = 0.2,\ d_1 = 0.2, e_1 = 0.1,\ \alpha_1 = -2, k_1 = -0.55, k_2 = -0.9, a_3 = 2, a_5 = 1.2, a_6 = 2.5, a_7 = 0.9, a_8 = 1,\ a_9 = 3$, with $x \in [-4\,,4], y \in [-4\,,4]$, and $t \in [0\,,2]$.

**Case 2**: $a_3 = a_3,\ b_1 = b_1,\ b_2 = b_2,\ b_3 = b_3,\ a_5 = a_5,\ a_6 = a_6,\ a_7 = a_7,\ a_8 = a_8,\ a_9 = a_9,\ a_4 = \frac{1}{\sqrt{2a_jk_2}}$ and $\sigma_j = -\frac{a_jk_1k_2}{2A}$.

Putting these constants into (3.1) and then using (2.11), we deduce the solution for equation (1.2) as follows:

For $\sigma_1 = -\frac{a_1k_1k_2}{2A}$

$$\psi(x,y,t) = 2\frac{-\frac{1}{\sqrt{2a_jk_2}}e^{-\left(a_3+\frac{1}{\sqrt{2a_1k_2}}\zeta\right)}+\frac{b_1}{\sqrt{2a_jk_2}}e^{\left(a_3+\frac{1}{\sqrt{2a_1k_2}}\zeta\right)}-a_6\,b_2\,\sin(a_5+a_6\zeta)+a_8\,b_3\,\sinh(a_7+a_8\zeta)}{e^{-\left(a_3+\frac{1}{\sqrt{2a_1k_2}}\zeta\right)}+b_1\,e^{\left(a_3+\frac{1}{\sqrt{2a_1k_2}}\zeta\right)}+b_2\,\cos(a_5+a_6\zeta)+b_3\,\cosh(a_7+a_8\zeta)+a_9}, \tag{3.4}$$

For $\sigma_2 = -\frac{a_2k_1k_2}{2A}$

$$\phi(x,y,t) = 2\frac{-\frac{1}{\sqrt{2a_jk_2}}e^{-\left(a_3+\frac{1}{\sqrt{2a_2k_2}}\zeta\right)}+\frac{b_1}{\sqrt{2a_jk_2}}e^{\left(a_3+\frac{1}{\sqrt{2a_2k_2}}\zeta\right)}-a_6\,b_2\,sin(a_5+a_6\zeta)+a_8\,b_3\,sinh(a_7+a_8\zeta)}{e^{-\left(a_3+\frac{1}{\sqrt{2a_2k_2}}\zeta\right)}+b_1\,e^{\left(a_3+\frac{1}{\sqrt{2a_2k_2}}\zeta\right)}+b_2\,cos(a_5+a_6\zeta)+b_3\,cosh(a_7+a_8\zeta)+a_9}. \quad (3.5)$$

**Case 3**: $a_3 = a_3,\ b_1 = b_1,\ b_2 = b_2,\ b_3 = b_3,\ a_5 = a_5,\ a_6 = a_6,\ a_7 = a_7,\ a_8 = a_8,\ a_9 = a_9,\ a_4 = 0$ and $\sigma_j = -\frac{a_jk_1k_2}{2A}$.

Putting these constants into (3.1) and then using (2.11), we deduce the solution for equation (1.2) as follows:

For $\sigma_1 = -\frac{a_1k_1k_2}{2A}$ and $\sigma_2 = -\frac{a_2k_1k_2}{2A}$

$$\psi(x,y,t) = \phi(x,y,t) = 2\frac{-a_6\,b_2\,sin(a_5+a_6\zeta)+a_8\,b_3\,sinh(a_7+a_8\zeta)}{e^{-(a_3)}+b_1\,e^{(a_3)}+b_2\,cos(a_5+a_6\zeta)+b_3\,cosh(a_7+a_8\zeta)+a_9}. \quad (3.6)$$

## 4. Rogue wave

Substituting with

$$u = (a_3 + a_4\zeta)^2 + (a_5 + a_6\zeta)^2 + m_1\cosh(a_7 + a_8\zeta) + a_9, \quad (4.1)$$

into (2.12) with the help of symbolic accounts having all the coefficients of all powers of $\cosh(a_7 + a_8\zeta), \sinh(a_7 + a_8\zeta), \cosh(a_7 + a_8\zeta)\sinh(a_7 + a_8\zeta)$, $\cosh^2(a_7 + a_8\zeta)\sinh(a_7 + a_8\zeta)$, $\sinh^2(a_7 + a_8\zeta)$, $\sinh^3(a_7 + a_8\zeta)$ and the power of $\zeta$ we get a system of algebraic equations. By solving the system, we obtain:

**Case 1**: : $a_3 = a_3,\ a_4 = a_4,\ a_5 = a_5,\ a_6 = a_6,\ a_7 = a_7,\ a_9 = a_9, m_1 = m_1,\ a_8 = -\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_j}},\ \sigma_j = \sigma_j$ and $k_2 = -\frac{2A\sigma_j}{a_jk_1}$. provided $k_1(A\sigma_j) < 0$ and $a_jk_1 \neq 0$.

Putting these constants into (4.1) and then using (2.11), we deduce the solution for equation (1.2) as follows:

$$\psi(x,y,t) = 2\frac{2a_4(a_3+a_4\zeta)+2a_6(a_5+a_6\zeta)-\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_1}}\,m_1\sinh\left(a_7-\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_1}}\zeta\right)}{(a_3+a_4\zeta)^2+(a_5+a_6\zeta)^2+m_1\cosh\left(a_7-\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_1}}\zeta\right)+a_9}, \quad (4.2)$$

$$\phi(x,y,t) = 2\frac{2a_4(a_3+a_4\zeta)+2a_6(a_5+a_6\zeta)-\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_2}}\,m_1\sinh\left(a_7-\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_2}}\zeta\right)}{(a_3+a_4\zeta)^2+(a_5+a_6\zeta)^2+m_1\cosh\left(a_7-\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_2}}\zeta\right)+a_9}. \quad (4.3)$$

**Case 2**: $a_3 = a_3,\ a_4 = a_4,\ a_5 = a_5,\ a_6 = a_6,\ a_7 = a_7,\ a_9 = a_9, m_1 = m_1,\ a_8 = \frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_j}}$ and $k_2 = -\frac{2A\sigma_j}{a_jk_1}$. provided $k_1(A\sigma_j) < 0$ and $a_jk_1 \neq 0$.

Putting these constants into (4.1) and then using (2.11), we deduce the solution for equation (1.2) as follows:

$$\psi(x,y,t) = 2\frac{2a_4(a_3+a_4\zeta)+2a_6(a_5+a_6\zeta)+\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_1}}\,m_1\sinh\left(a_7+\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_1}}\zeta\right)}{(a_3+a_4\zeta)^2+(a_5+a_6\zeta)^2+m_1\cosh\left(a_7+\sqrt{\frac{-k_1}{A\sigma_1}}\zeta\right)+a_9}, \quad (4.4)$$

$$\phi(x,y,t) = 2\frac{2a_4(a_3+a_4\zeta)+2a_6(a_5+a_6\zeta)+\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_2}}\,m_1\sinh\left(a_7+\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_2}}\zeta\right)}{(a_3+a_4\zeta)^2+(a_5+a_6\zeta)^2+m_1\cosh\left(a_7+\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_2}}\zeta\right)+a_9}. \quad (4.5)$$

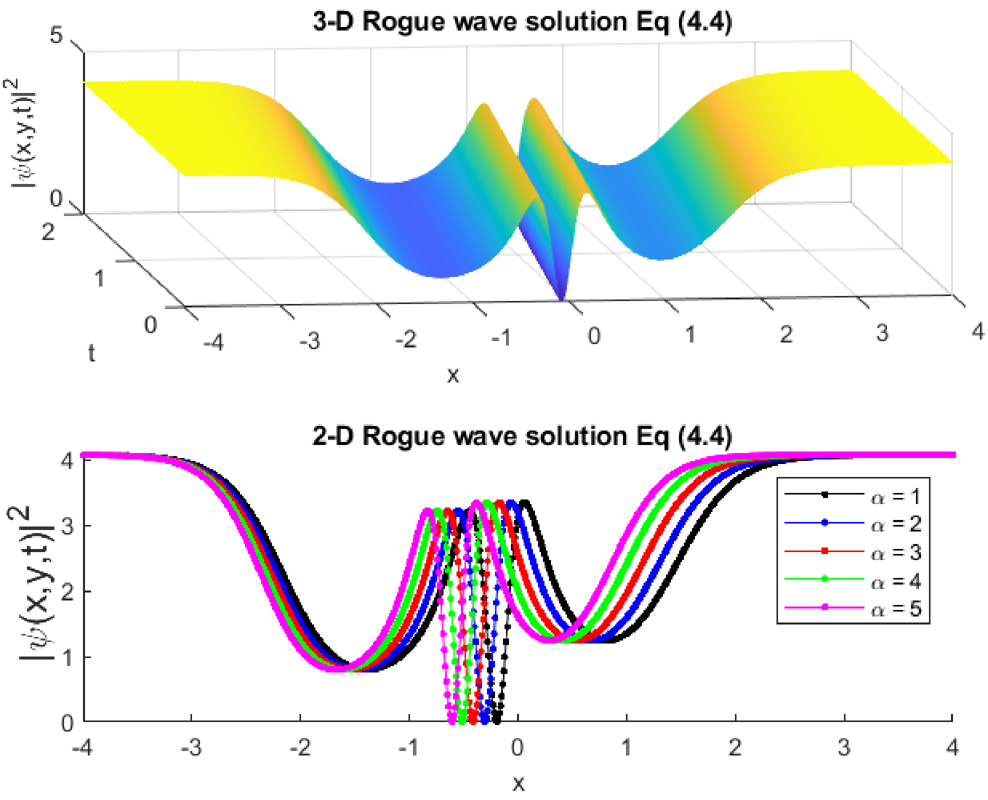


**Figure 2:** For $c_1 = 0.2,\ d_1 = 0.2, e_1 = 0.1,\ \alpha_1 = -2, k_1 = -0.55, k_2 = -0.9, a_1 = 0.1, a_3 = 2,\ a_4 = -0.52, a_5 = 1.2, a_6 = 2.5, a_7 = 0.9,\ \ a_9 = 3, \sigma_1 = 0.135, m_1 = 0.5$ with $x \in [-4\,,4], y \in [-4\,,4]$, and $t \in [0\,,2]$.

## 5. Lump interaction with kink and rogue wave

We obtain the solutions of Lump interaction with kink and rogue wave, which consist of the sum of exponential, quadratic and hyperbolic trigonometry function. We assume the following function

$$u = (a_3 + a_4\zeta)^2 + (a_5 + a_6\zeta)^2 + m_1 e^{(a_7 + a_8\zeta)} + m_2\, cosh(a_9 + a_{10}\zeta) + a_{11}, \tag{5.1}$$

where $a_i, (i = 3, \ldots, 11)$, $m_1$ and $m_2$ are constants to be determined later. Substituting into (2.12) with the help of symbolic accounts having all the coefficients of $\sinh^i(a_9 + a_{10}\zeta)$,$\cosh^j(a_9 + a_{10}\zeta)$, $\sinh(a_9 + a_{10}\zeta)\cosh^j(a_9 + a_{10}\zeta)$, $e^{i\,(a_7\zeta + a_8)}$, $e^{j\,(a_7\zeta + a_8)} cosh(a_9 + a_{10}\zeta)$ $e^{j\,(a_7\zeta + a_8)} sinh(a_9 + a_{10}\zeta)$, $e^{\,(a_7\zeta + a_8)} sinh(a_9 + a_{10}\zeta) cosh(a_9 + a_{10}\zeta), e^{\,(a_7\zeta + a_8)} \cosh^2(a_9 + a_{10}\zeta)$, for $(i = 1,2,3)$ , $(j = 1,2)$, and the powers of $\zeta$'s. We get system of equations which gives following values of parameters:

**Case 1**: $a_3 = a_3,\ a_4 = a_4,\ a_5 = a_5,\ a_6 = a_6,\ a_7 = a_7,\ \ a_8 = a_8,\ \ a_9 = a_9,\ m_1 = m_1,\ \ \ \ m_2 = m_2,\ a_{10} = -\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_j}},\ a_{11} = -(a_5^2 + a_3^2)$, and $k_2 = -\frac{2A\sigma_j}{a_j k_1}$. provided $k_1(A\sigma_j) < 0$ and $a_j k_1 \neq 0$.

Putting these constants into (5.1) and then using (2.11), we deduce the solution for equation (1.2) as follows:

$$\psi(x,y,t) = 2\frac{2a_4(a_3 + a_4\zeta) + 2a_6(a_5 + a_6\zeta) + a_8 m_1\, e^{(a_7 + a_8\zeta)} - \frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_1}}\, m_2\, sinh(a_9 - \frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_1}}\zeta)}{(a_3 + a_4\zeta)^2 + (a_5 + a_6\zeta)^2 + m_1\, e^{(a_7\zeta + a_8)} + m_2\, cosh(a_9 - \frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_1}}\zeta) + a_{11}}, \tag{5.2}$$

$$\phi(x,y,t) = 2\frac{2a_4(a_3 + a_4\zeta) + 2a_6(a_5 + a_6\zeta) + a_8 m_1\, e^{(a_7 + a_8\zeta)} - \frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_2}}\, m_2\, sinh(a_9 - \frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_2}}\zeta)}{(a_3 + a_4\zeta)^2 + (a_5 + a_6\zeta)^2 + m_1\, e^{(a_7\zeta + a_8)} + m_2\, cosh(a_9 - \frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_2}}\zeta) + a_{11}}. \tag{5.3}$$

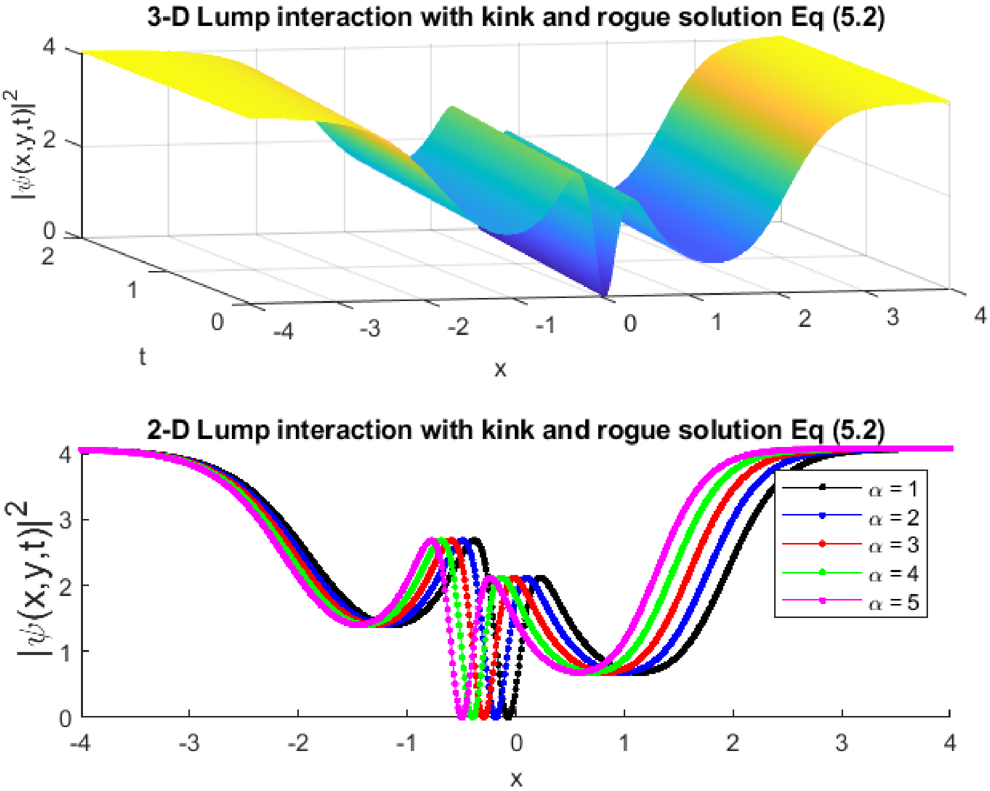


**Figure 3:** For $b_1 = 0.2,\ c_1 = 0.2,\ d_1 = 0.2, e_1 = 0.1,\ \alpha_1 = -2, k_1 = -0.55,\ a_1 = 0.1, a_3 = 2,\ a_4 = -0.52, a_5 = 1.2, a_6 = 2.5, a_7 = 2,\ a_8 = -0.52, a_9 = 0.9, \sigma_1 = 0.135, m_1 = 2, m_2 = 0.5$ with $x \in [-4\,,4], y \in [-4\,,4]$, and $t \in [0\,,2]$.

**Case 2**: : $a_3 = a_3,\ a_4 = a_4,\ a_5 = a_5,\ a_6 = a_6,\ a_7 = a_7,\ a_8 = a_8,\ a_9 = a_9,\ m_1 = m_1,\ \ m_2 = m_2,\ a_{10} = \frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_j}},\ a_{11} = -(a_5^2 + a_3^2)$, and $k_2 = -\frac{2A\sigma_j}{a_j k_1}$. provided $k_1(A\sigma_j) < 0$ and $a_j k_1 \neq 0$.

Putting these constants into (5.1) and then using (2.11), we deduce the solution for equation (1.2) as follows:

$$\psi(x,y,t) = 2\,\frac{2a_4(a_3+a_4\zeta)+2a_6(a_5+a_6\zeta)+a_8 m_1\, e^{(a_7+a_8\zeta)}+\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_1}}\, m_2\, \sinh\!\left(a_9+\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_1}}\zeta\right)}{(a_3+a_4\zeta)^2+(a_5+a_6\zeta)^2+m_1\, e^{(a_7\zeta+a_8)}+m_2\, \cosh\!\left(a_9+\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_1}}\zeta\right)+a_{11}}, \tag{5.4}$$

$$\phi(x,y,t) = 2\,\frac{2a_4(a_3+a_4\zeta)+2a_6(a_5+a_6\zeta)+a_8 m_1\, e^{(a_7+a_8\zeta)}+\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_2}}\, m_2\, \sinh\!\left(a_9-\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_2}}\zeta\right)}{(a_3+a_4\zeta)^2+(a_5+a_6\zeta)^2+m_1\, e^{(a_7\zeta+a_8)}+m_2\, \cosh\!\left(a_9+\frac{1}{2}\sqrt{\frac{-k_1}{A\sigma_2}}\zeta\right)+a_{11}}. \tag{5.5}$$

## 6. Numerical simulation

In this section, we investigate the coupled form of the nonlinear (2+1)-dimensional Kundu-Mukherjee-Naskar equation in fiber bragg gratings for Periodic Cross-Kink solutions, Rogue solutions, and Lump interaction with kink and rogue solutions using the log transformation (2.11). Utilizing the proper functions of the solutions yields Periodic Cross-Kink solutions Eqs.(3.2 – 3.6), Rogue solutions Eqs.(4.2 – 4.5), and Lump interaction with kink and rogue solutions Eqs.(5.2 – 5.5). Figs. 1 represents (2&3)-D Periodic Cross-Kink soliton solutions. Figs. 2 represents (2&3)-D Rogue (kink-bright-dark)soliton solutions. Figs. 3 illustrate (2&3)-D Lump interaction with kink and rogue (kink-bright-dark)soliton solutions.

## 7. Conclusion

In this study, we retrieved new optical solutions for the nonlinear (2+1)-dimensional Kundu-Mukherjee-Naskar equation in Fiber Bragg Gratings using three effective techniques: Periodic Cross-Kink wave, Rogue wave, and Lump interaction with kink and rogue waves. New Periodic Cross-Kink soliton solutions, Rogue (kink-bright-dark) soliton solutions, and lump interaction with kink and rogue (kink-bright-dark) soliton solutions were extracted. Visual representations of the solutions were also included. To the best of our knowledge, this study is the first to identify the outcomes of this model.

## References


[1] M.E.Z. Elsayed, M.A.S. Reham and E.M.A. Mohamed, "Optical solitons in Bragg gratings fibers for the nonlinear (2+1)-dimensional Kundu-Mukherjee-Naskar equation using two integration schemes", Optical and Quantum Electronics, 2022, 54(1), 16.

[2] M.H. Saleh and A.A. Altwaty, "Optical Solitons of The Extended Gerdjikov-Ivanov Equation In DWDM System By Extended Simplest Equation Method", Appl. Math. Inf. Sci., 2020, 14(5), 1 - 7.

[3] A.A. Altwaty, M.H. Saleh and B. Dumitry, "Soliton and wave solutions to the extended Gerdjikov-Ivanov equation in DWDM system with auxiliary equation method", Math. Sci.Lett., 2020, 9(3), 57 - 63.

[4] A.A. Altwaty, M.H. Saleh and R.K.M. Bader, "Optical solitons with ractional temporal evolution in fiber bragg gratings with generalized anti-cubic nonlinearity by the fractional Riccati method", Results in Physics, 2021, 22(20), 103872.

[5] M.H. Saleh and A.A. Altwaty, "Solitons and other solutions to the extended Gerdjikov-Ivanov equation in DWDM system by the exp(-\phi(\zeta))-expansion method", Ricerche di Matematica, 2022. DOI:10.1007/s11587-022-00701-9.

[6] A.A. Altwaty, "Numerous new Jacobi elliptic function solutions for fractional space-time perturbed Gerdjikov-Ivanov equation with conformable derivative", Int. J. Nonlinear Anal. Appl., 2022, 13(2), 1009 - 1021.

[7] T. Ananya, S. Subbadacshan, R. Hadi, A. Hijaz and I. Mustafa, "New multi-wave solutions of the conformable LPD model with nonlinear wave phenomena arise in mathematical physics", Journal of Ocean Engineering and Science, 2022. DOI: 10.1016/j.joes.2022.04.016.

[8] S.T.R. Rizvi, A.R. Seadawy, M. Younis, I. Ali, S. Althobaiti and S.F. Mahmoud, "Soliton solutions,painleve analysis and conservation laws for a nonlinear evolution equation",Results Phys, 2021, 23, 103999.

[9] S.T.R. Rizvi, I. Bibi and M. Bekir, "Interaction properties of solitons for a couple of nonlinear evolution equations", Chin Phys B, 2021, 30(1), 010502.

[10] S.T.R. Rizvi, A.R. Seadawy, I. Bibi and M. Younis, "Chirped and chirp-free optical solitons for heisenberg ferromagnetic spin chains model", Modern Phys Lett, 2021, 35(8), 2150139.

[11] Q. Zhou, H. Triki, J. Xu, Z.L. Zeng, W. Liu and A. Biswas, "Perturbation of chirped localized waves in a dual-power law nonlinear medium", Chaos, Solitons & Fractals, 2022, 160, 112198.

[12] Q. Zhou, Y. Zhong, H. Triki, Y. Sun, J. Xu, W. Liu and A. Biswas, "Chirped Bright and Kink Solitons in Nonlinear Optical Fibers with Weak Nonlocality and Cubic-Quantic-Septic Nonlinearity", Chin. Phys. Lett., 2022, 39, 044202.

[13] H. Triki, Q. Zhou, W. Liu, A. Biswas, L. Moraru, Y. Yldrm, H.M. Alshehri and M.R. Belic, "Chirped optical soliton propagation in birefringent fibers modeled by coupled Fokas-Lenells system", Chaos Solitons & Fractals, 2022, 155, 111751.

[14] A.K. Daoui, A. Messouber, H. Triki, Q. Zhou, A. Biswas, W. Liu, A.K. Alzahrani and M.R. Belic, " Propagation of chirped periodic and localized waves with higher-order effects through optical fibers", Chaos Solitons & Fractals, 2021, 146, 110873.

[15] J. Yang, Z. Huang, Y. Zhu, Q. Zhou, J. Li, Y. Sun, H.M. Alshehri and A. Biswas, "Soliton - soliton interaction and its influence on soliton amplitude and period", Results in Physics, 2021, 30, 104831.

[16] Q. Zhou, Y. Sun, H. Triki, Y. Zhong, Z. Zeng and M. Mirzazadeh, "Study on propagation properties of one-soliton in a multimode fiber with higher-order effects", Results in Physics, 2022, 41, 105898.

[17] Q. Zhou, M. Xu, Y. Sun, Y. Zhong and M. Mirzazadeh, "Generation and transformation of dark solitons, anti-dark solitons and dark double-hump solitons", Nonlinear Dyn., 2022, 110, 1747 - 1752.

[18] Q. Zhou, "Influence of Parameters of Optical Fibers on Optical Soliton Interactions", Chin. Phys. Lett., 2022, 39, 010501.

[19] W. Feng, L. Chen, G. Ma and Q. Zhou, "Study on weakening optical soliton interaction in nonlinear Optics", Nonlinear Dyn., 2022, 108, 2483 - 2488.

[20] C.C. Ding, Q. Zhou, H. Triki and Z.H Hu, "Interaction dynamics of optical dark bound solitons for a defocusing Lakshmanan-Porsezian-Daniel equation", Optics Express, 2022, 30, 40712 - 40727.

[21] A. Kundu, A Mukherjee and T. Naskar, "Modelling rogue waves through exact dynamical lump soliton controlled by ocean currents", Proc. R. Soc. A., 2014, 470, 20130576.
[22] X. Wang and J. Wei, "Three types of Darboux transformation and general soliton solutions for the space-shifted nonlocal PT symmetric nonlinear Schrdinger equation", Appl. Math. Lett., 2022, 130, 107998.
[23] S.S. Tabatabaei, " Generalized Lyapunov stability and designing pseudo-state/order estimator for incommensurate variable order systems", Commun. Nonlinear Sci. Numer. Simulat, 2022, 107, 106172.
[24] Z. Zho and L. He, "Resonance Y-type soliton and hybrid solutions of a (2+1)-dimensional asymmetrical Nizhnik-Novikov-Veselov equation", Appl. Math. Lett., 2021, 122, 107497.
[25] Z. Zhao and L. He, "Nonlinear superposition between lump waves and other waves of the (2 + 1)-dimensional asymmetrical Nizhnik-Novikov-Veselov equation", Nonlinear Dyn., 2022, 108, 555 - 568.
[26] Z. Zhao and L. He, "Lie symmetry, nonlocal symmetry analysis, and interaction of solutions of a (2+1)-dimensional KdVmKdV equation", Theoret. and Math. Phys., 2021, 206, 142 - 162.
[27] Z. Zhao, "Conservation laws and nonlocally related systems of the Hunter-Saxton equation for liquid crystal", Anal. Math. Phys, 2019, 9, 2311 - 2327.
[28] A. Altwaty, "Optical solitons in Fiber Bragg Gratings for the coupled form of the nonlinear (2 + 1)-dimensional Kundu-Mukherjee-Naskar equation via four powerful techniques", Results in Physics, 2023, 44, 106205.
[29] A. Biswas, " Optical soliton perturbation with Radhakrishnan-Kundu-Lakshmanan equation by traveling wave hypothesis", Optik, 2018, 171, 217 - 220.
[30] A. Biswas, M. Ekici, A. Sonmezoglu and M.R. Belic, " Solitons in Optical fiber Bragg gratings with dispersive reflectivity by extended trial function method", Optik, 2019, 182, 88 - 94.
[31] A. Biswas, j. Vega-Guzman, M.F. Mahmood, M. Ekici, Q. Zhou, S.P. Belic and M.R. Belic, " Optical solitons in fiber Bragg gratings with dispersive reflectivity for parabolic law nonlinearity using undetermined coefficients", Optik, 2019, 182, 39 - 44.
[32] M. Ekici, A. Sonmezoglu, A. Biswas and M.R. Belic, " Optical solitons in (2+1)-Dimensions with Kundu-Mukherjee-Naskar equation by extended trial function scheme", Chin. J. Phys., 2019, 57, 72 - 77.
[33] Y. Yldrm, " Optical solitons to Kundu-Mukherjee-Naskar model with trial equation approach", Optik, 2019, 183, 1061 - 1065.
[34] Y. Yldrm, "Optical solitons to Kundu-Mukherjee-Naskar model with modified simple equation approach", Optik, 2019, 184, 247 - 252.
[35] A. Altwaty and P.W. Eloe, "The role of concavity in applications of Avery type fixed point theorems to higher order differential equations", Journal of Mathematical Inequalities, 2012, 6(1), 79 - 90.
[36] H. Wang, "Lump and interaction solutions to the (2+1)-dimensional Burger's equation", Applied Mathematics Letters, 2018, 85, 2734.
[37] K. El-Rashidy, R. Seadawy, S. Althobaiti and M.M. Makhloufa, " Multiwave, Kinky breathers and multi-peak soliton solutions for the nonlinear Hirota dynamical system", Results in Physics, 2020, 19, 103678.
[38] I. Ahmed, R. Seadawy and D. Lu, " M-shaped rational solitons and their interaction with kink waves in the Fokas-lenells equation", Physica Scripta, 2019, 94, 055205.
[39] S.T.R. Rizvi, R. Seadawy, F. Ashraf, M. Younis, H. Iqbal and D. Baleanu, "Lump and Interaction solutions of a geophysical Korteweg–de Vries equation", Results in Physics, 2020, 19, 103661.