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Application of Floyd Algorithm to find the Shortest Path for Marketing the Gum Arabic in Sudan

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Abstract:

This paper explores the application of Floyd's algorithm to optimize the marketing strategy for Gum Arabic in Sudan. Recognizing the unique challenges associated with Gum Arabic marketing, we leverage the algorithm's ability to identify the shortest paths and streamline transportation logistics. Using MATLAB as the implementation tool, we analyze the initial matrices, apply the algorithm iteratively, and uncover optimized routes that minimize distances. The results demonstrate significant improvements in market accessibility and transportation efficiency. Through visual representations, the study provides tangible evidence of Floyd's algorithm's effectiveness in real-world marketing contexts. The findings not only contribute to the enhancement of Gum Arabic distribution but also underscore the broader potential of algorithmic approaches in optimizing commodity marketing networks. This study not only addresses immediate marketing concerns but also encourages further exploration of algorithmic strategies in similar supply chain optimization scenarios. After completing the algorithm, we found that the route with smallest distance is (Nyala, Alaobied, Omdurman, Atbara, Bort Sudan) with length 1650 Km.

Keywords: The Gum Arabic, shortest path problem, Floyd's algorithm.

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تطبيق خوارزمية فلويد لإيجاد أقصر طريق لتسويق الصمغ العربي في السودان

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الملخص
يستكشف هذا البحث تطبيق خوارزمية فلويد لتحسين استراتيجية التسويق للصمغ العربي في السودان. إدراكًا للتحديات الفريدة المرتبطة بتسويق الصمغ العربي، فإننا نستفيد من قدرة الخوارزمية على تحديد أقصر المسارات وتبسيط لوجستيات النقل. باستخدام الماتلاب كأداة تنفيذ، نقوم بتحليل المصفوفات الأولية، وتطبيق الخوارزمية بشكل متكرر، والكشف عن المسارات المحسنة التي تقلل المسافات. وتظهر النتائج تحسينات كبيرة في إمكانية الوصول إلى الأسواق وكفاءة النقل. ومن خلال التمثيلات المرئية، تقدم الدراسة دليلاً ملموسًا على فعالية خوارزمية فلويد في سياقات التسويق في العالم الحقيقي. لا تساهم النتائج في تعزيز توزيع الصمغ العربي فحسب، بل تؤكد أيضًا على الإمكانات الأوسع لنهج الخوارزمية في تحسين شبكات تسويق السلع الأساسية. لا تتناول هذه الدراسة المخاوف التسويقية المباشرة فحسب، بل تشجع أيضًا على مواصلة استكشاف الاستراتيجيات الخوارزمية في سيناريوهات مماثلة لتحسين سلسلة التوريد. وبعد استكمال الخوارزمية وجدنا أن الطريق الأقل مسافة هو (نيالا، العبيد، أم درمان، عطبرة، بورت السودان) بطول 1650 كم.
الكلمات المفتاحية: الصمغ العربي، مسألة أقصر مسار، خوارزمية فلويد.

Introduction

The introduction lays the groundwork for understanding the significance of applying Floyd's algorithm to optimize the marketing of Gum Arabic in Sudan. It highlights the unique challenges associated with Gum Arabic marketing and the need for an efficient route optimization strategy. By employing Floyd's algorithm, the study aims to streamline marketing efforts, reduce transportation costs, and enhance overall market accessibility. The objectives of the study are clear, focusing on the practical application of this algorithm to address real-world marketing challenges. In the eighteenth century, seven bridges connected four regions in the former city of Kongsberg (now Kaliningrad) when the people went on strolls through town they wondered if there was a way to travel across all seven bridges and return to the starting point without crossing each bride twice. This problem was solved by Leonard Euler in 1736, which as a solution consisted of representing the problem by a graph, with the four regions represented by four vertices and the seven bridges by seven edges as follows [1], [2].

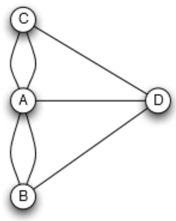

Figure 1: Simple model diagram.

Graph theory is a branch of mathematics that concerns a network of points connected by lines. The concept has expanded and is used in many applications such as chemical bonding, genetics, and computer science. The most visible application of basic graph theory is vacation planning. MapQuest, a popular site used for finding information like driving directions and quickest routes, relies heavily on graph theory concepts in order to create the routes. Graph theory is now a major tool in mathematical research, electrical engineering, computer programming and net [3]. The second issue relating to paths in digraphs is finding the best path between tow vertices. The simplest form of the problem is to compute the path between tow vertices that uses the fewest edges .We generalize somewhat and allow the edge of the digraph to be assigned weights that have nonnegative values. We call such a digraph a weighted digraph [4]. The weight of the path is then the sum of the weights of the edges in the path. The shortest path between two vertices u and v is called the distance from u to v. If there is no path from u to v, the distance is said to be infinity (∞). There are many algorithms for finding shortest paths in digraphs. The one we present here was developed by Floyd's [5], [6].

Floyds algorithm and application

In this paper we discuss the Floyds algorithm and application it for find the shortest path for marketing the gum Arabic in Sudan. The algorithm works by updating two matrices, namely D_k and Q_k, n times for a n - node network. The matrix D_k, in any iteration k, gives the value of the shortest distance (time) between all pairs of nodes (i, j) as obtained till the K^{th} iteration. The matrix Q_k has q_{ij}^k as its elements. The value of q_{ij}^k gives the immediate predecessor node from node i to node j on the shortest path as determined by the k^{th} iteration. D_0 and Q_0 give the starting matrices D_n and Q_n give the final matrices for an n-node system. The first task is to determine

D_0 and Q_0. D_0 is taken up first. The elements d_{ij} of matrix D_0 are defined as follows: If a link (branch) exists between nodes i and j the length of the shortest path between these nodes equals length $l\ (i,j)$ of branch (i,j) which connects them. Should there be several branches between nodes i and node j, the length of the shortest path d_{ij}^0 must equal the length of the shortest branch, i.e. [6]:

$$d_{ij}^0 = min[l_1(i,j), l_2(i,j), \dots \dots, l_m(i,j)\ ]$$

Where m is the number of branches between node i and node j.

It is clear that $d_{ij}^0 = 0$ when $i = j$. In the case when there is no direct link between node i and node j, we have no information at the beginning concerning the length of the shortest path between these two nodes so we treat them as though they were infinitely far from each other, that is,

$$d_{ij}^0 = \infty$$

Elements d_{0j}^0 of the predecessor matrix Q_0 are defined as follows:

If a link (branch) exists between nodes i and j the length of the shortest path between these nodes equals length $l\ (i,j)$ of branch (i,j) which connects them. Should there be several branches between nodes i and node j, the length of the shortest path d_{ij}^0 must equal the length of the shortest branch, i.e.:

$$d_{ij}^0 = min[l_1(i,j), l_2(i,j), \dots \dots, l_m(i,j)\ ]$$

First, we assume that $q_{0j}^0 = i$, for $i = j$, i.e. that for every pair of nodes (i,j) for $i = j$, the immediate predecessor of node *j* on the shortest path leading from node *i* node *j* is actually node *i*. After defining D_0 and Q_0 the following steps are used repeatedly to determine D_n and Q_n [4], [7].

Step (1): Let $k = 1$

Step (2): We calculate elements d_{ij}^k of the shortest path length matrix found after the k-th passage through algorithm D_k using the following equation:

$$d_{ij}^k = min\left[d_{ij}^{k-1} + d_{ik}^{k-1} + \cdots \dots + d_{kj}^{k-1}\ \right]$$

Step (3): Elements $d_{ij}^k$of predecessor matrix Q_k found after the *k-th* passage through the algorithm are calculated as follows:

$$d_{ij}^k = \begin{cases} q_{kj}^{k-1}, & for\ \ d_{ij}^k \neq d_{ij}^{k-1} \\ d_{ij}^k, & othre\ wise \end{cases}$$

Step (4): If $k = n$, the algorithm is finished. If $k < n$, increase k by 1, i.e. $K = k + 1$ and return to step 2.

Let us now look at the algorithm in a little more detail. In step (2), each time we go through the algorithms we are checking as to whether a shorter path exists between nodes *i* and *j* other than the path we already know about which was established during one of the earlier passages through the algorithm. If we establish that $d_{ij}^k \neq d_{ij}^{k-1}$, i.e. if we establish during the *k-th* passage through the algorithm that the length of the shortest path d_{ij}^k between nodes *i* and *j* is less than the length of the shortest path d_{ij}^k known previous to the *k-th* passage, we have to change the immediate predecessor node to node *j*. Since the length of the new shortest path is:

$$d_{ij}^k = d_{ij}^{k-1} + d_{kj}^{k-1}$$

it is clear that in this case node *k* is the new immediate predecessor node to *j*, and therefore:

$$q_{ij}^k = q_{kj}^{k-1}$$

This is actually done in the third algorithmic step. It is also clear that the immediate predecessor node to node *j* does not change if, at the end of step 2, we have established that no other new, shorter path exists. This means that:

$$q_{ij}^k = q_{kj}^{k-1} \text{ for } d_{ij}^k = q_{ij}^{k-1}$$

When we go through the algorithm *n* times (*n* is the number of nodes in the transportation network), elements d_{ij}^n of final matrix $D_n$will constitute the shortest path going from node i to node *j*.

The shortest path for marketing the gum Arabic in Sudan

Shortest path problem is a project using the application of graph theory for shortest path problem. The question is how efficiently graph theory can be used in route planning for a marketing of gum Arabic. The question is essentially two fold; what is route with the shortest distance between Nyala and Bortsudan and length of this route.

In Figure 2 **a, b, c ,d ,e, f, g, h, i, j, k, L, m,** and **n** represents the marketing cities .i.e. **a** → Nyala, **b**→ Elobeid, **c** → Ummruwaba, →**d** Tendelti, ***e*** → Rabk ,**f**→ Sinner, ***g*** →Wadennial, →**h** Eldamazeen, **i** → wad madni**, j** → Elgadarif**, k**→ Omdurman, **L**→Atbara, **m**→ cassala **n**→Bortsudana, and the distance between cities represents by 1cm≡100Km.

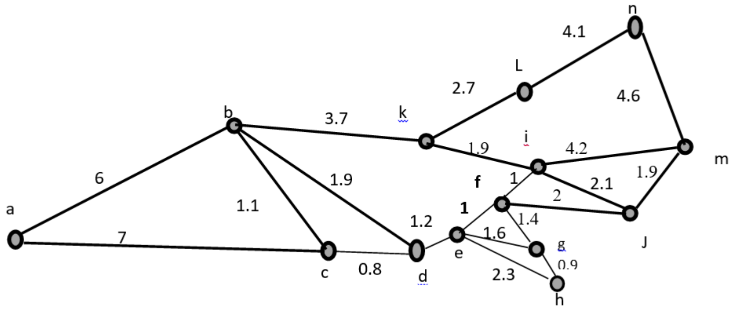

Figure 2: The distance between cities in Sudan.

We using Floyd's algorithm to finding the shortest path between Nyala and Bortsudan starting by the matrices D_0 and Q_0. The matrices D_0 and Q_0 give the initial representations of the graph. D_0 is symmetrical. Starting matrix D_0 is as follows:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	6	7	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
2	6	0	1.1	1.9	∞	∞	∞	∞	∞	∞	3.7	∞	∞	∞
3	7	1.1	0	0.8	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
4	∞	1.9	0.8	0	1.2	∞	∞	∞	∞	∞	∞	∞	∞	∞
5	∞	∞	∞	1.2	0	1	1.6	2.3	∞	∞	∞	∞	∞	∞
6	∞	∞	∞	∞	1	0	1.4	∞	1	2	∞	∞	∞	∞
7	∞	∞	∞	∞	1.6	1.4	0	0.9	∞	∞	∞	∞	∞	∞
8	∞	∞	∞	∞	2.3	∞	0.9	0	∞	∞	∞	∞	∞	∞
9	∞	∞	∞	∞	∞	1	∞	∞	0	2.1	1.9	∞	4.2	∞
10	∞	∞	∞	∞	∞	2	∞	∞	2.1	0	∞	∞	1.9	∞
11	∞	3.7	∞	∞	∞	∞	∞	∞	1.9	∞	0	2.7	∞	∞
12	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	2.7	0	∞	4.1
13	∞	∞	∞	∞	∞	∞	∞	∞	4.2	1.9	∞	∞	0	4.6
14	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	4.1	4.6	0

Starting matrix $Q_0$is as follows:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	–	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	–	2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	–	3	3	3	3	3	3	3	3	3	3	3
4	4	4	4	–	4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	–	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	–	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	–	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	–	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	–	9	9	9	9	9
10	10	10	10	10	10	10	∞	10	10	–	10	10	10	10
11	11	11	11	11	11	11	11	11	11	11	–	11	11	11
12	12	12	12	12	12	12	12	12	12	12	12	–	12	12
13	13	13	13	13	13	13	13	13	13	13	13	13	–	13
14	14	14	14	14	14	14	4	14	14	14	14	14	14	–

Starting matrix D_1 is as follows:

We now go to the first algorithm step. Let $k = 1$. As an illustration of step 2 we calculate the elements of the first row of matrix D_1.

$$d_{12}^1 = min\{d_{12}^0, d_{11}^0 + d_{12}^0\} = min\{6.0 + 6\} = 6$$
$$d_{13}^1 = min\{d_{13}^0, d_{11}^0 + d_{13}^0\} = min\{7.0 + 7\} = 7$$
$$d_{14}^1 = min\{d_{14}^0, d_{11}^0 + d_{14}^0\} = min\{\infty.0 + \infty\} = \infty$$

$$d_{15}^{1} = min\{d_{15}^{0}, d_{11}^{0} + d_{15}^{0}\} = min\{\infty. 0 + \infty\} = \infty$$
$$d_{16}^{1} = min\{d_{16}^{0}, d_{11}^{0} + d_{16}^{0}\} = min\{\infty. 0 + \infty\} = \infty$$
$$d_{17}^{1} = min\{d_{17}^{0}, d_{11}^{0} + d_{17}^{0}\} = min\{\infty. 0 + \infty\} = \infty$$
$$d_{18}^{1} = min\{d_{18}^{0}, d_{11}^{0} + d_{18}^{0}\} = min\{\infty. 0 + \infty\} = \infty$$
$$d_{19}^{1} = min\{d_{19}^{0}, d_{11}^{0} + d_{19}^{0}\} = min\{\infty. 0 + \infty\} = \infty$$
$$d_{110}^{1} = min\{d_{110}^{0}, d_{110}^{0} + d_{14}^{0}\} = min\{\infty. 0 + \infty\} = \infty$$
$$d_{111}^{1} = min\{d_{111}^{0}, d_{11}^{0} + d_{111}^{0}\} = min\{\infty. 0 + \infty\} = \infty$$
$$d_{112}^{1} = min\{d_{112}^{0}, d_{11}^{0} + d_{112}^{0}\} = min\{\infty. 0 + \infty\} = \infty$$
$$d_{114}^{1} = min\{d_{114}^{0}, d_{11}^{0} + d_{114}^{0}\} = min\{\infty. 0 + \infty\} = \infty$$

D_1 is as matrix follows:

$$\begin{array}{c|cccccccccccccc}
 & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{8} & \mathbf{9} & \mathbf{10} & \mathbf{11} & \mathbf{12} & \mathbf{13} & \mathbf{14} \\ \hline
\mathbf{1} & 0 & 6 & 7 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\mathbf{2} & 6 & 0 & 1.1 & 1.9 & \infty & \infty & \infty & \infty & \infty & \infty & 3.7 & \infty & \infty & \infty \\
\mathbf{3} & 7 & 1.1 & 0 & 0.8 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\mathbf{4} & \infty & 1.9 & 0.8 & 0 & 1.2 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
\mathbf{5} & \infty & \infty & \infty & 1.2 & 0 & 1 & 1.6 & 2.3 & \infty & \infty & \infty & \infty & \infty & \infty \\
\mathbf{6} & \infty & \infty & \infty & \infty & 1 & 0 & 1.4 & \infty & 1 & 2 & \infty & \infty & \infty & \infty \\
\mathbf{7} & \infty & \infty & \infty & \infty & 1.6 & 1.4 & 0 & 0.9 & \infty & \infty & \infty & \infty & \infty & \infty \\
\mathbf{8} & \infty & \infty & \infty & \infty & 2.3 & \infty & 0.9 & 0 & \infty & \infty & \infty & \infty & \infty & \infty \\
\mathbf{9} & \infty & \infty & \infty & \infty & \infty & 1 & \infty & \infty & 0 & 2.1 & 1.9 & \infty & 4.2 & \infty \\
\mathbf{10} & \infty & \infty & \infty & \infty & \infty & 2 & \infty & \infty & 2.1 & 0 & \infty & \infty & 1.9 & \infty \\
\mathbf{11} & \infty & 3.7 & \infty & \infty & \infty & \infty & \infty & \infty & 1.9 & \infty & 0 & 2.7 & \infty & \infty \\
\mathbf{12} & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 2.7 & 0 & \infty & 4.1 \\
\mathbf{13} & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 4.2 & 1.9 & \infty & \infty & 0 & 4.6 \\
\mathbf{14} & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 4.1 & 4.6 & 0
\end{array}$$

and Q_1 is as follows:

$$\begin{array}{c|cccccccccccccc}
 & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{8} & \mathbf{9} & \mathbf{10} & \mathbf{11} & \mathbf{12} & \mathbf{13} & \mathbf{14} \\ \hline
\mathbf{1} & - & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\mathbf{2} & 2 & - & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
3 & 3 & 3 & - & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\
\mathbf{4} & 4 & 4 & 4 & - & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 \\
\mathbf{5} & 5 & 5 & 5 & 5 & - & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\
\mathbf{6} & 6 & 6 & 6 & 6 & 6 & - & 6 & 6 & 6 & 6 & 6 & 6 & 6 & 6 \\
\mathbf{7} & 7 & 7 & 7 & 7 & 7 & 7 & - & 7 & 7 & 7 & 7 & 7 & 7 & 7 \\
\mathbf{8} & 8 & 8 & 8 & 8 & 8 & 8 & 8 & - & 8 & 8 & 8 & 8 & 8 & 8 \\
\mathbf{9} & 9 & 9 & 9 & 9 & 9 & 9 & 9 & 9 & - & 9 & 9 & 9 & 9 & 9 \\
\mathbf{10} & 10 & 10 & 10 & 10 & 10 & 10 & \infty & 10 & 10 & - & 10 & 10 & 10 & 10 \\
\mathbf{11} & 11 & 11 & 11 & 11 & 11 & 11 & 11 & 11 & 11 & 11 & - & 11 & 11 & 11 \\
\mathbf{12} & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & - & 12 & 12 \\
\mathbf{13} & 13 & 13 & 13 & 13 & 13 & 13 & 13 & 13 & 13 & 13 & 13 & 13 & - & 13 \\
\mathbf{14} & 14 & 14 & 14 & 14 & 14 & 14 & 4 & 14 & 14 & 14 & 14 & 14 & 14 & -
\end{array}$$

Step (2) set $k = 2$ then $D_2 =$

$$\begin{array}{c|cccccccccccccc}
 & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{8} & \mathbf{9} & \mathbf{10} & \mathbf{11} & \mathbf{12} & \mathbf{13} & \mathbf{14} \\ \hline
\mathbf{1} & 0 & 6 & 7 & (7.9) & \infty & \infty & \infty & \infty & \infty & \infty & (9.7) & \infty & \infty & \infty \\
\mathbf{2} & 6 & 0 & 1.1 & 1.9 & \infty & \infty & \infty & \infty & \infty & \infty & 3.7 & \infty & \infty & \infty \\
\mathbf{3} & 7 & 1.1 & 0 & 0.8 & \infty & \infty & \infty & \infty & \infty & \infty & (4.8) & \infty & \infty & \infty \\
\mathbf{4} & (7.9) & 1.9 & 0.8 & 0 & 1.2 & \infty & \infty & \infty & \infty & \infty & (5.6) & \infty & \infty & \infty \\
\mathbf{5} & \infty & \infty & \infty & 1.2 & 0 & 1 & 1.6 & 2.3 & \infty & \infty & \infty & \infty & \infty & \infty \\
\mathbf{6} & \infty & \infty & \infty & \infty & 1 & 0 & 1.4 & \infty & 1 & 2 & \infty & \infty & \infty & \infty \\
\mathbf{7} & \infty & \infty & \infty & \infty & 1.6 & 1.4 & 0 & 0.9 & \infty & \infty & \infty & \infty & \infty & \infty \\
\mathbf{8} & \infty & \infty & \infty & \infty & 2.3 & \infty & 0.9 & 0 & \infty & \infty & \infty & \infty & \infty & \infty \\
\mathbf{9} & \infty & \infty & \infty & \infty & \infty & 1 & \infty & \infty & 0 & 2.1 & 1.9 & \infty & 4.2 & \infty \\
\mathbf{10} & \infty & \infty & \infty & \infty & \infty & 2 & \infty & \infty & 2.1 & 0 & \infty & \infty & 1.9 & \infty \\
\mathbf{11} & (9.7) & 3.7 & (4.8) & (5.6) & \infty & \infty & \infty & \infty & 1.9 & \infty & 0 & 2.7 & \infty & \infty \\
\mathbf{12} & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 2.7 & 0 & \infty & 4.1 \\
\mathbf{13} & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 4.2 & 1.9 & \infty & \infty & 0 & 4.6 \\
\mathbf{14} & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 4.1 & 4.6 & 0
\end{array}$$

Q_2 is as follows:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	–	1	1	**2**	1	1	1	1	1	1	**2**	1	1	1
2	2	–	2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	–	3	3	3	3	3	3	3	**2**	3	3	3
4	**2**	4	4	–	4	4	4	4	4	4	**2**	4	4	4
5	5	5	5	5	–	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	–	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	–	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	–	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	–	9	9	9	9	9
10	10	10	10	10	10	10	∞	10	10	–	10	10	10	10
11	**2**	11	**2**	**2**	11	11	11	11	11	11	–	11	11	11
12	12	12	12	12	12	12	12	12	12	12	12	–	12	12
13	13	13	13	13	13	13	13	13	13	13	13	13	–	13
14	14	14	14	14	14	14	4	14	14	14	14	14	14	–

Step (3) set $k = 3$ then $D_3 =$

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	0	6	7	(**7.8**)	∞	∞	∞	∞	∞	∞	9.7	∞	∞	∞
2	6	0	1.1	1.9	∞	∞	∞	∞	∞	∞	3.7	∞	∞	∞
3	7	1.1	0	0.8	∞	∞	∞	∞	∞	∞	4.8	∞	∞	∞
4	(**7.8**)	1.9	0.8	0	1.2	∞	∞	∞	∞	∞	5.6	∞	∞	∞
5	∞	∞	∞	1.2	0	1	1.6	2.3	∞	∞	∞	∞	∞	∞
6	∞	∞	∞	∞	1	0	1.4	∞	1	2	∞	∞	∞	∞
7	∞	∞	∞	∞	1.6	1.4	0	0.9	∞	∞	∞	∞	∞	∞
8	∞	∞	∞	∞	2.3	∞	0.9	0	∞	∞	∞	∞	∞	∞
9	∞	∞	∞	∞	∞	1	∞	∞	0	2.1	1.9	∞	4.2	∞
10	∞	∞	∞	∞	∞	2	∞	∞	2.1	0	∞	∞	1.9	∞
11	9.7	3.7	4.8	5.6	∞	∞	∞	∞	1.9	∞	0	2.7	∞	∞
12	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	2.7	0	∞	4.1
13	∞	∞	∞	∞	∞	∞	∞	∞	4.2	1.9	∞	∞	0	4.6
14	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	4.1	4.6	0

$Q_3$is as follows:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	–	1	1	**3**	1	1	1	1	1	1	**2**	1	1	1
2	2	–	2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	–	3	3	3	3	3	3	3	**2**	3	3	3
4	**3**	4	4	–	4	4	4	4	4	4	**2**	4	4	4
5	5	5	5	5	–	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	–	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	–	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	–	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	–	9	9	9	9	9
10	10	10	10	10	10	10	∞	10	10	–	10	10	10	10
11	**2**	11	**2**	**2**	11	11	11	11	11	11	–	11	11	11
12	12	12	12	12	12	12	12	12	12	12	12	–	12	12
13	13	13	13	13	13	13	13	13	13	13	13	13	–	13
14	14	14	14	14	14	14	4	14	14	14	14	14	14	–

Step (4) set $k = 4$ then $D_4 =$

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	0	6	7	7.8	(9)	∞	∞	∞	∞	∞	9.7	∞	∞	∞
2	6	0	1.1	1.9	(3.1)	∞	∞	∞	∞	∞	3.7	∞	∞	∞
3	7	1.1	0	0.8	(2)	∞	∞	∞	∞	∞	4.8	∞	∞	∞
4	7.8	1.9	0.8	0	1.2	∞	∞	∞	∞	∞	5.6	∞	∞	∞
5	(9)	(3.1)	(2)	1.2	0	1	1.6	2.3	∞	∞	(6.8)	∞	∞	∞
6	∞	∞	∞	∞	1	0	1.4	∞	1	2	∞	∞	∞	∞
7	∞	∞	∞	∞	1.6	1.4	0	0.9	∞	∞	∞	∞	∞	∞
8	∞	∞	∞	∞	2.3	∞	0.9	0	∞	∞	∞	∞	∞	∞
9	∞	∞	∞	∞	∞	1	∞	∞	0	2.1	1.9	∞	4.2	∞
10	∞	∞	∞	∞	∞	2	∞	∞	2.1	0	∞	∞	1.9	∞
11	9.7	3.7	4.8	5.6	(6.8)	∞	∞	∞	1.9	∞	0	2.7	∞	∞
12	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	2.7	0	∞	4.1
13	∞	∞	∞	∞	∞	∞	∞	∞	4.2	1.9	∞	∞	0	4.6
14	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	4.1	4.6	0

$Q_4$is as follows:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	–	1	1	**3**	**4**	1	1	1	1	1	**2**	1	1	1
2	2	–	2	2	**4**	2	2	2	2	2	2	2	2	2
3	3	3	–	3	**4**	3	3	3	3	3	**2**	3	3	3
4	**3**	4	4	–	4	4	4	4	4	4	**2**	4	4	4
5	**4**	**4**	**4**	5	–	5	5	5	5	5	**4**	5	5	5
6	6	6	6	6	6	–	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	–	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	–	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	–	9	9	9	9	9
10	10	10	10	10	10	10	∞	10	10	–	10	10	10	10
11	**2**	11	**2**	**2**	**4**	11	11	11	11	11	–	11	11	11
12	12	12	12	12	12	12	12	12	12	12	12	–	12	12
13	13	13	13	13	13	13	13	13	13	13	13	13	–	13
14	14	14	14	14	14	14	4	14	14	14	14	14	14	–

Step (5) set $k = 5$ then $D_5 =$

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	0	6	7	7.8	9	(10)	(10.6)	(11.3)	∞	∞	9.7	∞	∞	∞
2	6	0	1.1	1.9	3.1	(4.1)	(4.7)	(5.4)	∞	∞	3.7	∞	∞	∞
3	7	1.1	0	0.8	2	(3)	(3.6)	(4.3)	∞	∞	4.8	∞	∞	∞
4	7.8	1.9	0.8	0	1.2	(2.2)	(2.8)	(3.5)	∞	∞	5.6	∞	∞	∞
5	9	3.1	2	1.2	0	1	1.6	2.3	∞	∞	6.8	∞	∞	∞
6	(10)	(4.1)	(3)	(2.2)	1	0	1.4	∞	1	2	(7.8)	∞	∞	∞
7	(10.6)	(4.7)	(3.6)	(2.8)	1.6	1.4	0	0.9	∞	∞	(8.4)	∞	∞	∞
8	(11.3)	(5.4)	(4.3)	(3.5)	2.3	∞	0.9	0	∞	∞	(9.1)	∞	∞	∞
9	∞	∞	∞	∞	∞	1	∞	∞	0	2.1	1.9	∞	4.2	∞
10	∞	∞	∞	∞	∞	2	∞	∞	2.1	0	∞	∞	1.9	∞
11	9.7	3.7	4.8	5.6	6.8	(7.8)	(8.4)	(9.1)	1.9	∞	0	2.7	∞	∞
12	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	2.7	0	∞	4.1
13	∞	∞	∞	∞	∞	∞	∞	∞	4.2	1.9	∞	∞	0	4.6
14	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	4.1	4.6	0

$Q_5$is as follows:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	–	1	1	**3**	**4**	**5**	**5**	**5**	1	1	**2**	1	1	1
2	2	–	2	2	**4**	**5**	**5**	**5**	2	2	2	2	2	2
3	3	3	–	3	**4**	**5**	**5**	**5**	3	3	**2**	3	3	3
4	**3**	4	4	–	4	5	5	5	4	4	**2**	4	4	4
5	**4**	**4**	**4**	5	–	5	5	5	5	5	**4**	5	5	5
6	**5**	**5**	**5**	6	6	–	6	6	6	6	**5**	6	6	6
7	**5**	**5**	**5**	7	7	7	–	7	7	7	**5**	7	7	7
8	**5**	**5**	**5**	8	8	8	8	–	8	8	**5**	8	8	8
9	9	9	9	9	9	9	9	9	–	9	9	9	9	9
10	10	10	10	10	10	10	∞	10	10	–	10	10	10	10
11	**2**	11	**2**	**2**	**4**	**5**	**5**	**5**	11	11	–	11	11	11
12	12	12	12	12	12	12	12	12	12	12	12	–	12	12
13	13	13	13	13	13	13	13	13	13	13	13	13	–	13
14	14	14	14	14	14	14	4	14	14	14	14	14	14	–

Step (6) set $k = 6$ then $D_6 =$

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	0	6	7	7.8	9	10	10.6	11.3	(11)	(12)	9.7	∞	∞	∞
2	6	0	1.1	1.9	3.1	4.1	4.7	5.4	(5.1)	(6.1)	3.7	∞	∞	∞
3	7	1.1	0	0.8	2	(3	3.6	4.3	(4)	(5)	4.8	∞	∞	∞
4	7.8	1.9	0.8	0	1.2	2.2	2.8	3.5	(3.2)	(4.2)	5.6	∞	∞	∞
5	9	3.1	2	1.2	0	1	1.6	2.3	(2)	(3)	6.8	∞	∞	∞
6	10	4.1	3	2.2	1	0	1.4	∞	1	(2)	7.8	∞	∞	∞
7	10.6	4.7	3.6	2.8	1.6	1.4	0	0.9	(2.4)	(3.4)	8.4	∞	∞	∞
8	11.3	5.4	4.3	3.5	2.3	∞	0.9	0	(4.3)	(5.3)	9.1	∞	∞	∞
9	(11)	(5.1)	(4)	3.3	(2)	1	(2.4)	(4.3)	0	2.1	1.9	∞	4.2	∞
10	(12)	(6.1)	(5)	(4.2)	(3)	2	(3.4)	(5.3)	2.1	0	(9.8)	∞	1.9	∞
11	9.7	3.7	4.8	5.6	6.8	7.8	8.4	9.1	1.9	(9.8)	0	2.7	∞	∞
12	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	2.7	0	∞	4.1
13	∞	∞	∞	∞	∞	∞	∞	∞	4.2	1.9	∞	∞	0	4.6
14	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	4.1	4.6	0

Q_6 is as follows:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	–	1	1	3	4	5	5	5	**6**	**6**	**2**	1	1	1
2	2	–	2	2	4	5	5	5	**6**	**6**	**2**	2	2	2
3	3	3	–	3	**4**	**5**	**5**	**5**	**6**	**6**	**2**	3	3	3
4	**3**	4	4	–	4	4	4	4	**6**	**6**	**2**	4	4	4
5	**4**	**4**	**4**	5	–	5	5	5	**6**	**6**	**4**	5	5	5
6	**5**	**5**	**5**	6	6	–	6	6	6	6	5	6	6	6
7	**5**	**5**	**5**	7	7	7	–	7	**6**	**6**	**5**	7	7	7
8	**5**	**5**	**5**	8	8	8	8	–	**6**	**6**	**5**	8	8	8
9	**6**	**6**	**6**	**6**	**6**	9	**6**	**6**	–	**6**	9	9	9	9
10	**6**	**6**	**6**	**6**	**6**	10	**6**	**6**	**6**	–	**6**	10	10	10
11	**2**	11	**2**	**2**	**4**	**5**	**5**	**5**	11	**6**	–	11	11	11
12	12	12	12	12	12	12	12	12	12	12	12	–	12	12
13	13	13	13	13	13	13	13	13	13	13	13	13	–	13
14	14	14	14	14	14	14	4	14	14	14	14	14	14	–

Step (7) set $k = 7$ then $D_7 =$

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	0	6	7	7.8	9	(10)	(10.6)	11.3	(11)	(12)	9.7	∞	∞	∞
2	6	0	1.1	1.9	3.1	(4.1)	(4.7)	5.4	(5.1)	(6.1)	3.7	∞	∞	∞
3	7	1.1	0	0.8	2	(3)	3.6	4.3	(4)	(5)	4.8	∞	∞	∞
4	7.8	1.9	0.8	0	1.2	(2.2)	2.8	(3.5)	(3.3)	(4.2)	5.6	∞	∞	∞
5	9	3.1	2	1.2	0	1	1.6	2.3	(2)	(3)	6.8	∞	∞	∞
6	(10)	(4.1)	(3)	(2.2)	1	0	1.4	∞	1	(2)	(7.8)	∞	∞	∞
7	(10.6)	(4.7)	(3.6)	(2.8)	1.6	1.4	0	0.9	∞	(3.4)	(8.4)	∞	∞	∞
8	(11.3)	(5.4)	(4.3)	(3.5)	2.3	∞	0.9	0	∞	(5.3)	(9.1)	∞	∞	∞
9	(11)	(5.1)	(4)	(3.3)	(2)	1	(2.4)	(4.3)	0	2.1	1.9	∞	4.2	∞
10	(12)	(6.1)	(5)	(4.2)	(3)	2	(3.4)	(5.3)	2.1	0	∞	∞	1.9	∞
11	9.7	3.7	4.8	5.6	6.8	(7.8)	(8.4)	(9.1)	1.9	∞	0	2.7	∞	∞
12	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	2.7	0	∞	4.1
13	∞	∞	∞	∞	∞	∞	∞	∞	4.2	1.9	∞	∞	0	4.6
14	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	4.1	4.6	0

$Q_7$is as follows:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	−	1	1	3	4	5	5	5	1	**6**	**6**	1	1	1
2	2	−	2	2	4	5	5	5	2	**6**	**6**	2	2	2
3	3	3	−	3	**4**	**5**	**5**	**5**	3	**6**	**6**	3	3	3
4	**3**	4	4	−	4	4	4	4	4	**6**	**6**	4	4	4
5	**4**	**4**	**4**	5	−	5	5	5	5	**6**	**6**	5	5	5
6	**5**	**5**	**5**	6	6	−	6	6	6	6	5	6	6	6
7	**5**	**5**	**5**	7	7	7	−	7	7	**6**	**6**	7	7	7
8	**5**	**5**	**5**	8	8	8	8	−	8	**6**	**6**	8	8	8
9	**6**	**6**	**6**	**6**	**6**	9	**6**	**6**	−	9	9	9	9	9
10	**6**	**6**	**6**	**6**	**6**	10	**6**	**6**	10	−	10	10	10	10
11	**2**	11	**2**	**2**	**4**	**5**	**5**	**5**	11	11	−	11	11	11
12	12	12	12	12	12	12	12	12	12	12	12	−	12	12
13	13	13	13	13	13	13	13	13	13	13	13	13	−	13
14	14	14	14	14	14	14	4	14	14	14	14	14	14	−

Step (8) set $k = 8$ then $D_8 =$

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	0	6	7	7.8	9	10	10.6	11.3	11	12	9.7	∞	∞	∞
2	6	0	1.1	1.9	3.1	4.1	4.7	5.4	5.1	6.1	3.7	∞	∞	∞
3	7	1.1	0	0.8	2	3	3.6	4.3	4	5	4.8	∞	∞	∞
4	7.8	1.9	0.8	0	1.2	2.2	2.8	3.5	3.3	4.2	5.6	∞	∞	∞
5	9	3.1	2	1.2	0	1	1.6	2.3	2	3	6.8	∞	∞	∞
6	10	4.1	3	2.2	1	0	1.4	∞	1	2	7.8	∞	∞	∞
7	10.6	4.7	3.6	2.8	1.6	1.4	0	0.9	∞	3.4	8.4	∞	∞	∞
8	11.3	5.4	4.3	3.5	2.3	∞	0.9	0	∞	5.3	9.1	∞	∞	∞
9	11	5.1	4	3.3	2	1	2.4	4.3	0	2.1	1.9	∞	4.2	∞
10	12	6.1	5	4.2	3	2	3.4	5.3	2.1	0	∞	∞	1.9	∞
11	9.7	3.7	4.8	5.6	6.8	7.8	8.4	9.1	1.9	∞	0	2.7	∞	∞
12	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	2.7	0	∞	4.1
13	∞	∞	∞	∞	∞	∞	∞	∞	4.2	1.9	∞	∞	0	4.6
14	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	4.1	4.6	0

$Q_8$is as follows:

$$\begin{array}{cc} & \begin{matrix} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{8} & \mathbf{9} & \mathbf{10} & \mathbf{11} & \mathbf{12} & \mathbf{13} & \mathbf{14} \end{matrix} \\ \begin{matrix} \mathbf{1} \\ \mathbf{2} \\ 3 \\ \mathbf{4} \\ \mathbf{5} \\ \mathbf{6} \\ \mathbf{7} \\ \mathbf{8} \\ \mathbf{9} \\ \mathbf{10} \\ \mathbf{11} \\ \mathbf{12} \\ \mathbf{13} \\ \mathbf{14} \end{matrix} & \begin{bmatrix} - & 1 & 1 & 3 & 4 & 5 & 5 & 5 & 1 & 6 & 6 & 1 & 1 & 1 \\ 2 & - & 2 & 2 & 4 & 5 & 5 & 5 & 2 & 6 & 6 & 2 & 2 & 2 \\ 3 & 3 & - & 3 & 4 & 5 & 5 & 5 & 3 & 6 & 6 & 3 & 3 & 3 \\ \mathbf{3} & 4 & 4 & - & 4 & 4 & 4 & 4 & 4 & 6 & 6 & 4 & 4 & 4 \\ 4 & 4 & 4 & 5 & - & 5 & 5 & 5 & 5 & 6 & 6 & 5 & 5 & 5 \\ 5 & 5 & 5 & 6 & 6 & - & 6 & 6 & 6 & 6 & 5 & 6 & 6 & 6 \\ 5 & 5 & 5 & 7 & 7 & 7 & - & 7 & 7 & 6 & 6 & 7 & 7 & 7 \\ 5 & 5 & 5 & 8 & 8 & 8 & 8 & - & 8 & 6 & 6 & 8 & 8 & 8 \\ 6 & 6 & 6 & 6 & 6 & 9 & 6 & 6 & - & 9 & 9 & 9 & 9 & 9 \\ 6 & 6 & 6 & 6 & 6 & 10 & 6 & 6 & 10 & - & 10 & 10 & 10 & 10 \\ 2 & 11 & 2 & 2 & 4 & 5 & 5 & 5 & 11 & 11 & - & 11 & 11 & 11 \\ 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & - & 12 & 12 \\ 13 & 13 & 13 & 13 & 13 & 13 & 13 & 13 & 13 & 13 & 13 & 13 & - & 13 \\ 14 & 14 & 14 & 14 & 14 & 14 & 4 & 14 & 14 & 14 & 14 & 14 & 14 & - \end{bmatrix} \end{array}$$

Step (9) set $k = 9$ then $D_9 =$

$$\begin{array}{cc} & \begin{matrix} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{8} & \mathbf{9} & \mathbf{10} & \mathbf{11} & \mathbf{12} & \mathbf{13} & \mathbf{14} \end{matrix} \\ \begin{matrix} \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \\ \mathbf{4} \\ \mathbf{5} \\ \mathbf{6} \\ \mathbf{7} \\ \mathbf{8} \\ \mathbf{9} \\ \mathbf{10} \\ \mathbf{11} \\ \mathbf{12} \\ \mathbf{13} \\ \mathbf{14} \end{matrix} & \begin{bmatrix} 0 & 6 & 7 & 7.8 & 9 & 10 & 10.6 & 11.3 & 11 & 12 & 9.7 & \infty & (15.2) & \infty \\ 6 & 0 & 1.1 & 1.9 & 3.1 & 4.1 & 4.7 & 5.4 & 5.1 & 6.1 & 3.7 & \infty & (9.3) & \infty \\ 7 & 1.1 & 0 & 0.8 & 2 & 3 & 3.6 & 4.3 & 4 & 5 & 4.8 & \infty & (8.2) & \infty \\ 7.8 & 1.9 & 0.8 & 0 & 1.2 & 2.2 & 2.8 & 3.5 & 3.3 & 4.2 & (5.1) & \infty & (7.5) & \infty \\ 9 & 3.1 & 2 & 1.2 & 0 & 1 & 1.6 & 2.3 & 2 & 3 & (3.9) & \infty & (6.2) & \infty \\ 10 & 4.1 & 3 & 2.2 & 1 & 0 & 1.4 & \infty & 1 & 2 & (2.9) & \infty & (5.2) & \infty \\ 10.6 & 4.7 & 3.6 & 2.8 & 1.6 & 1.4 & 0 & 0.9 & \infty & 3.4 & (4.3) & \infty & (6.6) & \infty \\ 11.3 & 5.4 & 4.3 & 3.5 & 2.3 & \infty & 0.9 & 0 & \infty & 5.3 & (5.2) & \infty & (8.5) & \infty \\ 11 & 5.1 & 4 & 3.3 & 2 & 1 & 2.4 & 4.3 & 0 & 2.1 & 1.9 & \infty & 4.2 & \infty \\ 12 & 6.1 & 5 & 4.2 & 3 & 2 & 3.4 & 5.3 & 2.1 & 0 & (4) & \infty & 1.9 & \infty \\ 9.7 & 3.7 & 4.8 & 5.6 & 6.8 & 7.8 & 8.4 & 9.1 & 1.9 & \infty & 0 & 2.7 & (6.1) & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 2.7 & 0 & \infty & 4.1 \\ (15.2) & (9.3) & (8.2) & (7.5) & (6.2) & (5.2) & (6.6) & (8.5) & 4.2 & 1.9 & (6.1) & \infty & 0 & 4.6 \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 4.1 & 4.6 & 0 \end{bmatrix} \end{array}$$

$Q_9$is as follows:

$$\begin{array}{cc} & \begin{matrix} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{8} & \mathbf{9} & \mathbf{10} & \mathbf{11} & \mathbf{12} & \mathbf{13} & \mathbf{14} \end{matrix} \\ \begin{matrix} \mathbf{1} \\ \mathbf{2} \\ 3 \\ \mathbf{4} \\ \mathbf{5} \\ \mathbf{6} \\ \mathbf{7} \\ \mathbf{8} \\ \mathbf{9} \\ \mathbf{10} \\ \mathbf{11} \\ \mathbf{12} \\ \mathbf{13} \\ \mathbf{14} \end{matrix} & \begin{bmatrix} - & 1 & 1 & 3 & 4 & 5 & 5 & 5 & 6 & 6 & 2 & 1 & \mathbf{9} & 1 \\ 2 & - & 2 & 2 & 4 & 5 & 5 & 5 & 6 & 6 & 2 & 2 & \mathbf{9} & 2 \\ 3 & 3 & - & 3 & 4 & 5 & 5 & 5 & 6 & 6 & 2 & 3 & \mathbf{9} & 3 \\ \mathbf{3} & 4 & 4 & - & 4 & 4 & 4 & 4 & 6 & 6 & \mathbf{9} & 4 & \mathbf{9} & 4 \\ 4 & 4 & 4 & 5 & - & 5 & 5 & 5 & 6 & 6 & \mathbf{9} & 5 & \mathbf{9} & 5 \\ 5 & 5 & 5 & 6 & 6 & - & 6 & 6 & 6 & 6 & \mathbf{9} & 6 & \mathbf{9} & 6 \\ 5 & 5 & 5 & 7 & 7 & 7 & - & 7 & 6 & 6 & \mathbf{9} & 7 & \mathbf{9} & 7 \\ 5 & 5 & 5 & 8 & 8 & 8 & 8 & - & 6 & 6 & \mathbf{9} & 8 & \mathbf{9} & 8 \\ 6 & 6 & 6 & 6 & 6 & 9 & 6 & 6 & - & 9 & 9 & 9 & 9 & 9 \\ 6 & 6 & 6 & 6 & 6 & 10 & 6 & 6 & 10 & - & 9 & 10 & 10 & 10 \\ 2 & 11 & 2 & \mathbf{9} & \mathbf{9} & \mathbf{9} & \mathbf{9} & \mathbf{9} & 11 & \mathbf{9} & - & 11 & \mathbf{9} & 11 \\ 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & - & 12 & 12 \\ \mathbf{9} & \mathbf{9} & \mathbf{9} & \mathbf{9} & \mathbf{9} & \mathbf{9} & \mathbf{9} & \mathbf{9} & 13 & 13 & \mathbf{9} & 13 & - & 13 \\ 14 & 14 & 14 & 14 & 14 & 14 & 4 & 14 & 14 & 14 & 14 & 14 & 14 & - \end{bmatrix} \end{array}$$

Step (10) set $k = 10$ then $Q_{10}$is as follows:

$$\begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \end{matrix} & \begin{bmatrix} - & 1 & 1 & 3 & 4 & 5 & 5 & 5 & 1 & 6 & 6 & 1 & 10 & 1 \\ 2 & - & 2 & 2 & 4 & 5 & 5 & 5 & 2 & 6 & 6 & 2 & 10 & 2 \\ 3 & 3 & - & 3 & 4 & 5 & 5 & 5 & 3 & 6 & 6 & 3 & 10 & 3 \\ 3 & 4 & 4 & - & 4 & 4 & 4 & 4 & 4 & 6 & 6 & 4 & 10 & 4 \\ 4 & 4 & 4 & 5 & - & 5 & 5 & 5 & 5 & 6 & 6 & 5 & 10 & 5 \\ 5 & 5 & 5 & 6 & 6 & - & 6 & 6 & 6 & 6 & 5 & 6 & 10 & 6 \\ 5 & 5 & 5 & 7 & 7 & 7 & - & 7 & 7 & 6 & 6 & 7 & 10 & 7 \\ 5 & 5 & 5 & 8 & 8 & 8 & 8 & - & 8 & 6 & 6 & 8 & 10 & 8 \\ 6 & 6 & 6 & 6 & 6 & 9 & 6 & 6 & - & 9 & 9 & 9 & 10 & 9 \\ 6 & 6 & 6 & 6 & 6 & 10 & 6 & 6 & 10 & - & 10 & 10 & 10 & 10 \\ 2 & 11 & 2 & 2 & 4 & 5 & 5 & 5 & 11 & 11 & - & 11 & 10 & 11 \\ 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & 12 & - & 12 & 12 \\ 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 13 & 10 & 13 & - & 13 \\ 14 & 14 & 14 & 14 & 14 & 14 & 4 & 14 & 14 & 14 & 14 & 14 & 14 & - \end{bmatrix} \end{array}$$

Step (11) set $k = 11$ then $D_{11} =$

$$\begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \end{matrix} & \begin{bmatrix} 0 & 6 & 7 & 7.8 & 9 & 10 & 10.6 & 11.3 & 11 & 12 & 9.7 & (12.4) & 13.9 & \infty \\ 6 & 0 & 1.1 & 1.9 & 3.1 & 4.1 & 4.7 & 5.4 & 5.1 & 6.1 & 3.7 & (6.4) & 8 & \infty \\ 7 & 1.1 & 0 & 0.8 & 2 & 3 & 3.6 & 4.3 & 4 & 5 & 4.8 & (7.5) & 6.9 & \infty \\ 7.8 & 1.9 & 0.8 & 0 & 1.2 & 2.2 & 2.8 & 3.5 & 3.3 & 4.2 & 5.6 & (8.3) & 6.1 & \infty \\ 9 & 3.1 & 2 & 1.2 & 0 & 1 & 1.6 & 2.3 & 2 & 3 & 6.8 & (6.6) & 4.9 & \infty \\ 10 & 4.1 & 3 & 2.2 & 1 & 0 & 1.4 & \infty & 1 & 2 & 7.8 & (10.5) & 3.9 & \infty \\ 10.6 & 4.7 & 3.6 & 2.8 & 1.6 & 1.4 & 0 & 0.9 & \infty & 3.4 & 8.4 & (11.1) & 5.3 & \infty \\ 11.3 & 5.4 & 4.3 & 3.5 & 2.3 & \infty & 0.9 & 0 & \infty & 5.3 & 9.1 & (11.8) & 7.2 & \infty \\ 11 & 5.1 & 4 & 3.3 & 2 & 1 & 2.4 & 4.3 & 0 & 2.1 & 1.9 & (4.6) & 4 & \infty \\ 12 & 6.1 & 5 & 4.2 & 3 & 2 & 3.4 & 5.3 & 2.1 & 0 & \infty & (12.5) & 1.9 & \infty \\ 9.7 & 3.7 & 4.8 & 5.6 & 6.8 & 7.8 & 8.4 & 9.1 & 1.9 & \infty & 0 & 2.7 & 6.1 & \infty \\ (12.4) & (6.4) & (7.5) & (8.3) & (6.6) & (10.5) & (11.1) & (11.8) & (4.6) & (12.5) & 2.7 & 0 & (8.6) & 4.1 \\ 13.9 & 8 & 8.2 & 6.1 & 4.9 & 3.9 & 5.3 & 7.2 & (4) & 1.9 & 6.1 & (8.6) & 0 & 4.6 \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 4.1 & 4.6 & 0 \end{bmatrix} \end{array}$$

Q_{11} is as follows:

$$\begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \end{matrix} & \begin{bmatrix} - & 1 & 1 & 3 & 4 & 5 & 5 & 5 & 1 & 6 & 6 & 11 & 10 & 1 \\ 2 & - & 2 & 2 & 4 & 5 & 5 & 5 & 2 & 6 & 6 & 11 & 10 & 2 \\ 3 & 3 & - & 3 & 4 & 5 & 5 & 5 & 3 & 6 & 6 & 11 & 9 & 3 \\ 3 & 4 & 4 & - & 4 & 4 & 4 & 4 & 4 & 6 & 6 & 11 & 10 & 4 \\ 4 & 4 & 4 & 5 & - & 5 & 5 & 5 & 5 & 6 & 6 & 11 & 10 & 5 \\ 5 & 5 & 5 & 6 & 6 & - & 6 & 6 & 6 & 6 & 5 & 11 & 10 & 6 \\ 5 & 5 & 5 & 7 & 7 & 7 & - & 7 & 7 & 6 & 6 & 11 & 10 & 7 \\ 5 & 5 & 5 & 8 & 8 & 8 & 8 & - & 8 & 6 & 6 & 11 & 10 & 8 \\ 6 & 6 & 6 & 6 & 6 & 9 & 6 & 6 & - & 9 & 9 & 11 & 10 & 9 \\ 6 & 6 & 6 & 6 & 6 & 10 & 6 & 6 & 10 & - & 10 & 11 & 10 & 10 \\ 2 & 11 & 2 & 2 & 4 & 5 & 5 & 5 & 11 & 11 & - & 11 & 9 & 11 \\ 11 & 11 & 11 & 11 & 11 & 11 & 11 & 11 & 11 & 11 & 12 & - & 11 & 12 \\ 10 & 10 & 10 & 10 & 10 & 10 & 10 & 10 & 13 & 13 & 9 & 11 & - & 13 \\ 14 & 14 & 14 & 14 & 14 & 14 & 4 & 14 & 14 & 14 & 14 & 14 & 14 & - \end{bmatrix} \end{array}$$

Step (12) set $k = 12$ then $D_{12} =$

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	0	6	7	7.8	9	10	10.6	11.3	11	12	9.7	12.4	13.9	(16.5)
2	6	0	1.1	1.9	3.1	4.1	4.7	5.4	5.1	6.1	3.7	6.4	8	(10.5)
3	7	1.1	0	0.8	2	3	3.6	4.3	4	5	4.8	7.5	8.2	(11.5)
4	7.8	1.9	0.8	0	1.2	2.2	2.8	3.5	3.3	4.2	5.6	8.3	6.1	(12)
5	9	3.1	2	1.2	0	1	1.6	2.3	2	3	6.8	6.6	4.9	(10.7)
6	10	4.1	3	2.2	1	0	1.4	∞	1	2	7.8	10.5	3.9	(9.7)
7	10.6	4.7	3.6	2.8	1.6	1.4	0	0.9	∞	3.4	8.4	11.1	5.3	(11.1)
8	11.3	5.4	4.3	3.5	2.3	∞	0.9	0	∞	5.3	9.1	11.8	7.2	(12)
9	11	5.1	4	3.3	2	1	2.4	4.3	0	2.1	1.9	4.6	4	(8.7)
10	12	6.1	5	4.2	3	2	3.4	5.3	2.1	0	∞	12.5	1.9	(10.8)
11	9.7	3.7	4.8	5.6	6.8	7.8	8.4	9.1	1.9	∞	0	2.7	6.1	(6.8)
12	12.4	6.4	7.5	8.3	6.6	10.5	11.1	11.8	4.6	12.5	2.7	0	∞	4.1
13	13.9	8	8.2	6.1	4.9	3.9	5.3	7.2	4	1.9	6.1	∞	0	4.6
14	(16.5)	(10.5)	(11.5)	(12)	(10.7)	(9.7)	(11.1)	(12)	(8.7)	(10.8)	(6.8)	4.1	4.6	0

Q_{12} is as follows:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	–	1	1	3	4	5	5	5	1	6	6	11	10	**12**
2	2	–	2	2	4	5	5	5	2	6	6	11	10	**12**
3	3	3	–	3	4	5	5	5	3	6	6	11	9	**12**
4	**3**	4	4	–	4	4	4	4	4	6	6	11	10	**12**
5	4	4	4	5	–	5	5	5	5	6	6	11	10	**12**
6	5	5	5	6	6	–	6	6	6	6	5	11	10	**12**
7	5	5	5	7	7	7	–	7	7	6	6	11	10	**12**
8	5	5	5	8	8	8	8	–	8	6	6	11	10	**12**
9	6	6	6	6	6	9	6	6	–	9	9	11	10	**12**
10	6	6	6	6	6	10	6	6	10	–	10	11	10	**12**
11	2	11	2	2	4	5	5	5	11	11	–	11	9	**12**
12	11	11	11	11	11	11	11	11	11	11	12	–	12	12
13	10	10	10	10	10	10	10	10	13	13	9	13	–	13
14	**12**	**12**	**12**	**12**	**12**	**12**	**12**	**12**	**12**	**12**	**12**	14	14	–

Step (13) set $k = 13$ then $D_{13} =$

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1	0	6	7	7.8	9	10	10.6	11.3	11	12	9.7	12.4	13.9	16.5
2	6	0	1.1	1.9	3.1	4.1	4.7	5.4	5.1	6.1	3.7	6.4	8	10.5
3	7	1.1	0	0.8	2	3	3.6	4.3	4	5	4.8	7.5	8.2	(11.5)
4	7.8	1.9	0.8	0	1.2	2.2	2.8	3.5	3.3	4.2	5.6	8.3	6.1	(10.7)
5	9	3.1	2	1.2	0	1	1.6	2.3	2	3	6.8	6.6	4.9	(9.5)
6	10	4.1	3	2.2	1	0	1.4	∞	1	2	7.8	10.5	3.9	(8.5)
7	10.6	4.7	3.6	2.8	1.6	1.4	0	0.9	∞	3.4	8.4	11.1	5.3	(9.9)
8	11.3	5.4	4.3	3.5	2.3	∞	0.9	0	∞	5.3	9.1	11.8	7.2	(11.8)
9	11	5.1	4	3.3	2	1	2.4	4.3	0	2.1	1.9	4.6	4	(8.6)
10	12	6.1	5	4.2	3	2	3.4	5.3	2.1	0	∞	12.5	1.9	(6.5)
11	9.7	3.7	4.8	5.6	6.8	7.8	8.4	9.1	1.9	∞	0	2.7	6.1	6.8
12	12.4	6.4	7.5	8.3	6.6	10.5	11.1	11.8	4.6	12.5	2.7	0	∞	4.1
13	13.9	8	8.2	6.1	4.9	3.9	5.3	7.2	4	1.9	6.1	∞	0	4.6
14	16.5	10.5	(10.7)	(9.5)	(8.5)	(9.9)	(15.2)	(11.8)	(8.6)	(6.5)	6.8	4.1	4.6	0

Q_{13} is as follows:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	–	1	1	3	4	5	5	5	1	6	6	11	10	12
2	2	–	2	2	4	5	5	5	2	6	6	11	10	12
3	3	3	–	3	4	5	5	5	3	6	6	11	9	13
4	3	4	4	–	4	4	4	4	4	6	6	11	10	13
5	4	4	4	5	–	5	5	5	5	6	6	11	10	13
6	5	5	5	6	6	–	6	6	6	6	5	11	10	13
7	5	5	5	7	7	7	–	7	7	6	6	11	10	13
8	5	5	5	8	8	8	8	–	8	6	6	11	10	13
9	6	6	6	6	6	9	6	6	–	9	9	11	10	13
10	6	6	6	6	6	10	6	6	10	–	10	11	10	13
11	2	11	2	2	4	5	5	5	11	11	–	11	9	12
12	11	11	11	11	11	11	11	11	11	11	12	–	12	12
13	10	10	10	10	10	10	10	10	13	13	9	13	–	13
14	12	13	13	13	13	13	13	13	13	13	12	14	14	–

Step (14) set $k = 14$ then $D_{14} =$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	6	7	7.8	9	10	10.6	11.3	11	12	9.7	12.4	13.9	16.5
2	6	0	1.1	1.9	3.1	4.1	4.7	5.4	5.1	6.1	3.7	6.4	8	10.5
3	7	1.1	0	0.8	2	3	3.6	4.3	4	5	4.8	7.5	6.9	11.5
4	7.8	1.9	0.8	0	1.2	2.2	2.8	3.5	3.2	4.2	5.1	7.9	6.1	10.7
5	9	3.1	2	1.2	0	1	1.6	2.3	2	3	3.9	6.6	4.9	9.5
6	10	4.1	3	2.2	1	0	1.4	2.3	1	2	2.9	5.6	3.9	8.5
7	10.6	4.7	3.6	2.8	1.6	1.4	0	0.9	2.4	3.4	4.3	7	5.3	9.9
8	11.3	5.4	4.3	3.5	2.3	2.3	0.9	0	3.3	4.3	5.2	7.9	7.2	11.8
9	11	5.1	4	3.2	2	1	2.4	3.3	0	2.1	1.9	4.6	4	8.6
10	12	6.1	5	4.2	3	2	3.4	4.3	2.1	0	4	6.7	1.9	6.5
11	9.7	3.7	4.8	5.1	3.9	2.9	4.3	5.2	1.9	4	0	2.7	5.9	6.8
12	12.4	6.4	7.5	7.9	6.6	5.6	7	7.9	4.6	6.7	2.7	0	8.6	4.1
13	13.9	8	6.9	6.1	4.9	3.9	5.3	7.2	4	1.9	5.9	8.6	0	4.6
14	16.5	10.5	11.5	10.7	9.5	8.5	9.9	11.8	8.6	6.5	6.8	4.1	4.6	0

Q_{14} is as follows:

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	–	1	1	3	4	5	5	5	1	6	6	11	10	12
2	2	–	2	2	4	5	5	5	2	6	6	11	10	12
3	3	3	–	3	4	5	5	5	3	6	6	11	9	12
4	3	4	4	–	4	4	4	4	4	6	6	11	10	12
5	4	4	4	5	–	5	5	5	5	6	6	11	10	12
6	5	5	5	6	6	–	6	6	6	6	5	11	10	12
7	5	5	5	7	7	7	–	7	7	6	6	11	10	12
8	5	5	5	8	8	8	8	–	8	6	6	11	10	12
9	6	6	6	6	6	9	6	6	–	9	9	11	10	12
10	6	6	6	6	6	10	6	6	10	–	10	11	10	12
11	2	11	2	2	4	5	5	5	11	11	–	11	9	12
12	11	11	11	11	11	11	11	11	11	11	12	–	12	12
13	10	10	10	10	10	10	10	10	13	13	9	13	–	13
14	12	12	12	12	12	12	12	12	12	12	12	14	14	–

The final matrices D_{14} and Q_{14} contain all the information needed to determine the shortest route between any two nodes in the graph. From node 1 to node 14 is $d_{1,14} = 16.5cm$, since 1 cm ≡100km→ $d_{1,14} = 1650\ km$ ie the shortest distance between Nyala and Botsudan is 1650 km. to determine the associated route, recall that a linked through at least one other intermediate node, because $q = 12 \neq 14$, the route is initially given as 1→12 → 14 , Now because $q_{1,12} = 11 \neq 12$ the segment (1,12) is not a direct link, and 1→12 is replaced with 1→11→ 12 and $q_{1,11} = 2 \neq 11$ the segment (1,11) is not a direct link, and 1→11 is replaced with 1→2→ 11 → 12 → 14 defines the shortest route.(i.e) the shortest route is (Nyala, Alaobied, Omdurman, Atbara, and Bort-Sudan.

MATLAB to implement Floyd's algorithm for finding the shortest path

To visualize the graphs and the shortest paths using MATLAB, you can use the graph and plot functions. Below is a complete MATLAB code that implements Floyd's algorithm and visualizes the graphs [5]–[7]:

Table 1 MATLAB code that implements Floyd's algorithm and visualizes the graphs

```
function visualizeShortestRoutes(D_0, Q_0)
  D = D_0; Q = Q_0; % Step (1): Initialize matrices
  % Step (2): Apply Floyd's Algorithm
  n = size(D_0, 1);
  for k = 1:n
    for i = 1:n
      for j = 1:n
        if D(i, j) > D(i, k) + D(k, j)
          D(i, j) = D(i, k) + D(k, j);
          Q(i, j) = Q(k, j);
        end
      end
    end
  end
  % Step (3): Display final matrices
  disp('Final Distance Matrix (D):');  disp(D);
  disp('Final Path Matrix (Q):');  disp(Q);
  % Visualize the routes
  G = graph(D);
  figure;
  % Plot the graph with edge weights
  h = plot(G, 'EdgeLabel', G.Edges.Weight, 'Layout', 'force', 'EdgeColor', [0, 0,
0]);
  title('Graph with Shortest Routes');
  highlightShortestRoutes(h, G, Q); % Highlight the shortest routes
  labelEdges(G);     % Display edge weights
  labelNodes(G);  % Add labels to nodes
  legend('Shortest Routes', 'Location', 'Best'); % Add a legend
end
function highlightShortestRoutes(h, G, Q)
  [rows, cols] = find(Q ~= 0);
  for i = 1:length(rows)
    route = shortestpath(G, rows(i), cols(i));
    highlight(h, 'Edges', findedge(G, route), 'EdgeColor', 'r', 'LineWidth', 2);
  end
end
function labelEdges(G)
  weights = G.Edges.Weight;
  for i = 1:G.numedges
    labelEdge = sprintf('%.1f', weights(i));
    text(G.Edges.EndNodes(i, 1), G.Edges.EndNodes(i, 2), labelEdge, ...
      'Color', [0, 0, 0], 'FontSize', 8, 'FontWeight', 'bold');
  end
end
function labelNodes(G)
  for i = 1:G.numnodes
    labelNode = sprintf('%d', i);
    text(G.Nodes.XData(i), G.Nodes.YData(i), labelNode, ...
      'Color', [0, 0, 0], 'FontSize', 10, 'FontWeight', 'bold');
  end
end
```

This code in table 1 defines a function visualizeShortestRoutes that determines the shortest paths between all pairs of vertices using Floyd's algorithm and then visualizes the routes. The highlightShortestRoutes function is responsible for finding and highlighting the shortest routes. Also, will display the final distance and path matrices and a graph the Figure 3 with highlighted shortest routes.

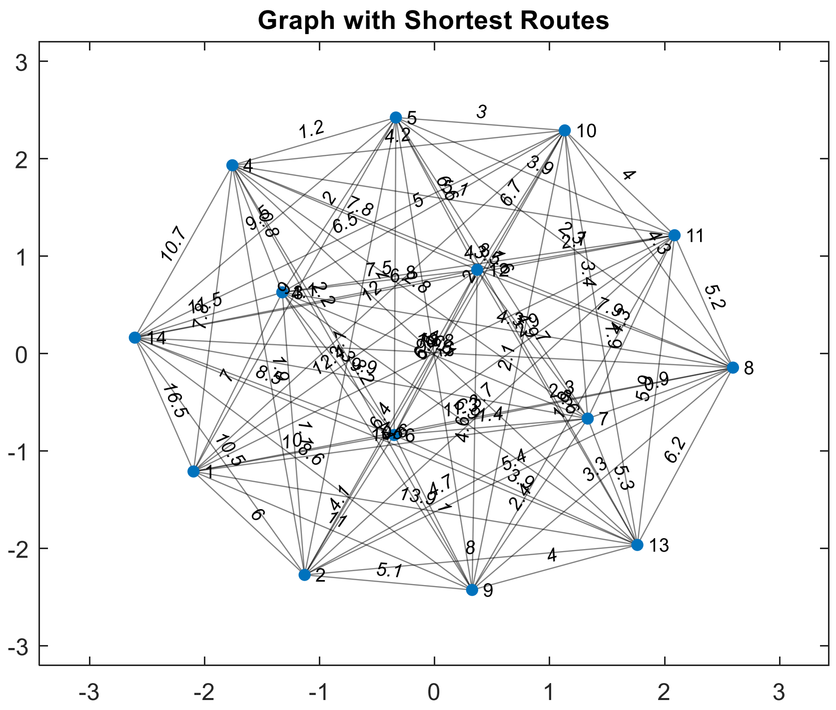

Figure 3: Graph with Shortest Routes.

Results and discussion

The results section presents compelling evidence of the effectiveness of Floyd's algorithm in optimizing the shortest path for Gum Arabic marketing in Sudan. Through the application of MATLAB, the study successfully identifies routes that minimize distances, providing a systematic and data-driven approach to marketing logistics. Visual representations, such as graphs and charts, illustrate the optimized routes and the corresponding reductions in transportation distances. The outcomes underscore the algorithm's potential to significantly impact the efficiency and cost-effectiveness of Gum Arabic distribution, contributing valuable insights to the field.

Conclusion

In conclusion, the study demonstrates the practical application of Floyd's algorithm as a powerful tool for enhancing the marketing of Gum Arabic in Sudan. The findings reveal that the algorithm facilitates the identification of optimal routes, leading to reduced transportation costs and improved market accessibility. The results align with the objectives set forth in the introduction, validating the utility of Floyd's algorithm in the context of Gum Arabic marketing. The study contributes valuable insights to the field, providing a basis for further research and encouraging the adoption of algorithmic approaches in the optimization of commodity distribution networks.

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