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Optimizing Turbo Code Performance: Analyzing the Impact of Free Distance

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Abstract:
The performance of Turbo codes, a robust error correction method, is influenced by various factors. This study specifically explores the pivotal role of free distance in Turbo code performance, a crucial parameter defining error correction capabilities. Employing an extensive simulation framework encompassing realistic channel models and decoding algorithms, we scrutinize the impact of varied free distance values on Turbo codes' overall efficacy.

Our investigation spans a spectrum of free distance values, evaluating their influence on both bit error rate (BER) and frame error rate (FER) metrics across additive white Gaussian noise (AWGN) and fading channels, mirroring practical communication scenarios. The outcomes underscore a clear correlation: augmenting free distance enhances error correction, lowering BER and FER. Yet, this improvement incurs escalated decoding complexity, necessitating more iterations for higher free distance values.

Additionally, we delve into diverse interleaver designs' effects on Turbo code performance across varying free distances. Our findings illuminate certain interleaver configurations that enhance error correction capacities, even with lower free distance values. This emphasizes the significance of optimizing interleaver design to bolster performance under specific channel conditions.

This study offers crucial insights into how free distance influences Turbo code performance, elucidating the trade-off between error correction capabilities and decoding complexity. These insights empower researchers and engineers to make informed decisions while designing and implementing Turbo codes in real-world communication systems.

Keywords: Turbo codes, free distance, error correction, performance analysis, communication systems, bit error rate, frame error rate, AWGN channel.

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تحسين أداء توربو كود: تحليل تأثير المسافة الحرة

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الملــــــــخص

يخضع أداء رموزTurbo ، وهي طريقة أساسية لتصحيح الأخطاء تشتهر بفاعليتها، وتأثرها بعوامل مختلفة. تتعمق هده الدراسة في الدور المحوري للمسافة الحرة في أداء رموز Turbo, وهي عامل مهم في تحديد قدرته على تصحيح الأخطاء من خلال استخدام إطار محاكاة متطور يشمل نماذج قنوات وخوارزميات فك التشفير نقوم بفحص دقيق لتأثير قيم المسافة الحرة المتغيرة على الفاعلية الشاملة لرموز Turbo.

يمتد بحثنا الى مجموعة من قيم المسافة الحرة، ويقيم بدقة تأثيرها على كل من مقاييس معدل خطأBER ، معدل خطأ FERعبر قناة الضوضاء الغوسية البيضاء المضافة (AWGN) وقنوات التلاشي fading channelsمما يعكس بشكل وثيق سيناريوهات الاتصال العملية. تؤكد النتائج على وجود علاقة واضحة: زيادة المسافة الحرة تزيد من فعالية تصحيح الأخطاء، مما يؤدي إلى انخفاض معدل BER و FER ومع ذلك، فان هدا التحسن مصحوب بتعقيد متزايد لفك التشفير، مما يستلزم المزيد من التكرارات لقيم مسافة حرة أعلى.

علاوة على ذلك، فإننا هنا نتعمق في تأثيرات Interleaver designs المتنوعة على أداء رموزTurbo عبر مسافات حرة مختلفة. تسلط النتائج التي توصلنا اليها الضوء على Interleaver designs محددة تعزز قدرات تصحيح الأخطاء، حتى مع انخفض قيم المسافة الحرة. وهدا يؤكد أهمية تصميمات "Interleaver designs "لتحسين الأداء في ظل شروط قناة محددة.

تقدم هده الدراسة رؤى محورية حول كيفية تأثير المسافة الحرة على أداء رموز Turbo. وتوضيح المفاضلة بين قدرات تصحيح الأخطاء وتعقيد فك التشفير. توفر هده الرؤى للبحثين والمهندسين إرشادات قيمة لأتخاد قرارات مستنيرة أثناء تصميم وتنفيذ رموز Turbo في أنظمة الاتصالات في العالم الحقيقي.

الكلمـات المفتاحيـة: الكلمات المفتاحية: رموزTurbo، المسافة الحرة، تصحيح الأخطاء، تحليل أداء، انظمة الاتصال، معدل خطأ البت، معدل خطأ الإطار، قنوات الضوضاء الغوسية البيضاء المضافة.

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1. Introduction

In the field of digital communication, reliable and efficient transmission of data is of utmost importance. One critical aspect of achieving robust communication is through error correction coding, which compensates for channel-induced errors. Turbo codes were first introduced in 1993 by Berrou et al. currently, they are one of the most effective methods of generating codes. [1].

Turbo code is a capacity-approaching error correcting code and its bit error rate (BER) performance is close to the theoretical Shannon bound. The permutation sequence of the interleaver plays a crucial role in determining the turbo code's performance once the polynomials of the recursive convolutional code have been chosen. Reducing correlation between information bits and preventing burst mistakes brought on by the propagation of decoding faults are the two main goals of the interleaver. [2]

The performance of Turbo codes is influenced by various factors, and understanding their impact is crucial for optimizing system design and achieving reliable communication. Among these factors, the free distance of Turbo codes plays a significant role in determining their error correction capability. The free distance represents the minimum number of bit or symbol errors required for a Turbo decoder to fail in correcting the received data. [3]

This study delves into the impact of free distance on Turbo code performance, aiming to uncover insights into how Turbo codes behave across various scenarios by scrutinizing their relationship with key performance metrics like bit error rate (BER) and frame error rate (FER). Employing a comprehensive simulation framework inclusive of realistic channel models and decoding algorithms, our analysis encompasses evaluations of Turbo code performance in both additive white Gaussian noise (AWGN) channels and fading channels, mirroring real-world communication scenarios accurately.

The outcomes derived from our simulations yield valuable observations regarding the effect of free distance on Turbo code performance. Specifically, escalating the free distance significantly bolsters the error correction capabilities of Turbo codes, resulting in diminished BER and FER. However, it's essential to note that heightened free distance values correspondingly escalate decoding complexity, necessitating more iterations in the decoding process.

2. Theory and concepts

In this section, a brief definition of each term used in this research will be introduced, starting by turbo codes which is the main term of it.

Turbo codes are a class of forward error correction (FEC) codes that were introduced in 1993 by Berrou, Glavieux, and Thitimajshima. They exhibit remarkable error correction capabilities and have been widely adopted in various communication systems. Turbo codes are constructed using a parallel concatenation of two or more convolutional codes, with an interleaver placed between them.[5]

Turbo codes are naturally amenable to performance analysis from an ensemble perspective, mainly due to the existence of the interleaver; if we denote its length by K, then by varying its function $\pi: \{1, 2, \ldots, K\} \rightarrow \{1, 2,$

…, K} one may construct the ensemble of all K! equiprobable turbo codes corresponding to the constituent encoders given. The theoretical device that performs this task is the uniform interleaver. [6]

Continuing the definition, now it will be free distance, which is a fundamental parameter that characterizes the error correction capability of Turbo codes. It represents the minimum number of error events required for the Turbo decoder to fail in correcting the received data. In other words, a Turbo code with a higher free distance can correct a greater number of errors.

A turbo code is a long block code with the structure shown in Fig. 1. There are L input bits, and each of these bits is encoded q times. In the jth encoding, the L bits are sent through a permutation box Pj (often called an 'interleaver'), and then encoded via an (Nj L) block encoder Gj, which can be thought of as an L X Nj matrix. Since, in practice, Nj may be ≤ L for some or all values of j, we call the encoders Gj 'code fragment' encoders.[7]

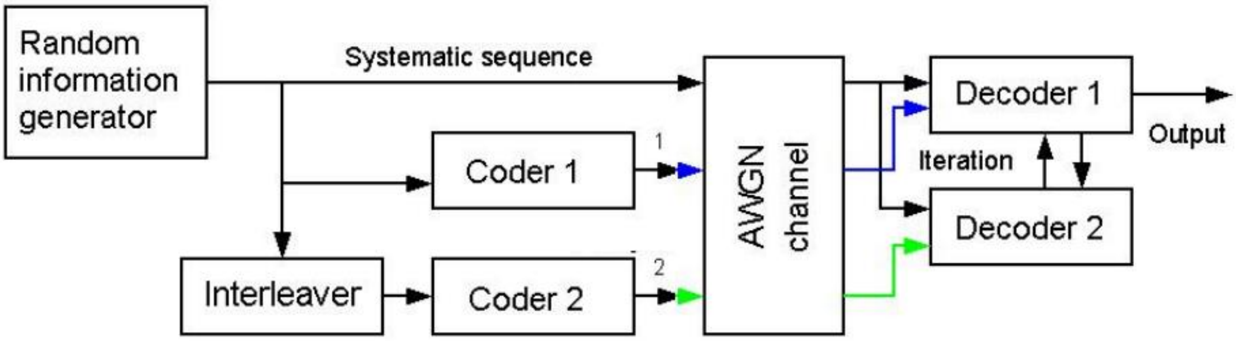

Figure 1. Block diagram of the implemented turbo code [8]

The performance of Turbo codes is typically evaluated using metrics such as the bit error rate (BER) which measures the probability of incorrectly decoding a single bit, and used to evaluate the quality of digital communication systems. It represents the ratio of the number of incorrectly received bits to the total number of transmitted bits. Lower BER values indicate better system performance. BER can be affected by factors such as modulation schemes, channel conditions, noise, and interference. Analyzing and comparing BER performance helps assess the robustness and reliability of different modulation schemes and system configurations. [9]

Talking about evaluating turbo codes and their performance, it is necessary to represent main types of noises, which could be applied on them, first type is Channel Additive White Gaussian Noise, it is a fundamental noise model used in information theory to simulate the effect of many random processes found in nature. Modifiers are words that describe specific qualities. The AWGN channel is a common channel model in which the only impediment to communication is a linear addition of wideband or white noise with constant spectral density and Gaussian amplitude distribution. It is an appropriate model for many satellite and deep space communications links. It's the most fundamental communication system model [10, 11].

Another type of noise is "fading", which refers to the rapid fluctuations in amplitude, phase, and multipath delays of a radio signal over a short period or travel distance, allowing large-scale path loss effects to be ignored. There are various types of fading channels depending on the circumstances; however, we will focus on Rayleigh fading channels, in which the impulse response may follow distributions of Rayleigh distributions (in which there is no Line of Sight (LOS) ray between transmitter and receiver [10,11].

Now it is time to introduce the design of turbo codes.

2.1. Turbo Encoder and Decoder

The turbo-code encoder consists of two identical RSC coders, C1 and C2, as shown in the figure that are connected using a concatenation scheme which is called as parallel concatenation. In the figure, it is shown that M is a memory register. The interleaver forces input bits dk to appear in different sequences [12].

The decoder is designed in a same way as the encoder. In this the two elementary decoders are interconnected to each other serially not in parallel as in case of decoder. The decoder DEC1 operates on lower speed R1 and for encoder C1 and follows a soft decision that causes delay L1. The same is the case for decoder DEC2 which cause a delay L2. To scatter error bursts that are coming from DEC1 output an interleaver is installed between the two decoders [4].

2.2. Interleaver Design:

Turbo codes with short block lengths perform adequately only if they have an appropriate interleaver design. Their performance is critically dependent on interleavers we use. The criteria used in the design of interleaver are:

a. Distance spectrum of code.
b. The correlation between the information input data and the soft output of each decoder corresponding to its parity bits.

Interleaving is a process in which data bits are arranged in a random or one of the deterministic formats. There are many types of Interleavers which we can use in our turbo codes can be classified into three types: [12]

a. Random Interleaving
b. Block Interleaving
c. Convolutional Interleaving

3. System and Mathematical Models

The following equations represent the basic mathematical model for a turbo code system with a 1/3 rate and 16-QAM modulation.

3.1. Encoder Equations

The encoder consists of two identical recursive systematic convolutional encoders. Let's denote the input data as u(k), and the encoded bits as x(k). For the 1/3 rate turbo code, each encoder produces three output bits (two parity bits and one systematic bit) for every input bit.

The state transition equations for the recursive systematic convolutional encoder can be represented as follows:

Encoder 1:

$$x_1(k) = u(k) \oplus s_1(k-1)$$
$$s_1(k) = u(k) \oplus s_1(k-1) \oplus s_2(k-1)$$

Encoder 2:

$$x_2(k) = u(k) \oplus s_2(k-1)$$
$$s_2(k) = x_1(k) \oplus s_1(k-1) \oplus s_2(k-1)$$

In these equations, $\oplus$ represents the bitwise XOR operation, $u(k)$ represents the input data at time k, $x_1(k)$ and $x_2(k)$ represent the encoded bits at time k from Encoder 1 and Encoder 2, respectively, and $s_1(k)$ and $s_2(k)$ represent the state variables at time k for Encoder 1 and Encoder 2, respectively.

3.2. Interleaver Equations

The interleaver rearranges the bits to introduce randomness into the code sequence. Let's denote the interleaved bits as y(k), which are obtained from the encoded bits x(k).

The interleaver equation can be represented as follows:

$$y(k) = x(pi(k))$$

In this equation, $pi(k)$ represents the interleaving permutation function that determines the mapping of the input bits to the interleaved bits.

3.3. Modulation Equations

For 16-QAM modulation, each group of four bits is mapped to a specific symbol from the 16-QAM constellation. Let's denote the modulated symbols as s(k), which are obtained from the interleaved bits y(k).

The modulation equation can be represented as follows:

$$s(k) = A * e^{\frac{1}{16}2\pi(m(k)-1)}$$

In this equation, A represents the amplitude of the 16-QAM constellation points, j represents the imaginary unit, m(k) represents the decimal value of the four-bit group at time k, and exp() represents the exponential function.

3.4. Demodulation Equations

The demodulation process involves estimating the transmitted symbols based on the received signal. Let's denote the received symbols as r(k), and the estimated symbols as ĉ(k).

The demodulation equation can be represented as follows:

$$\hat{c}(k) = \arg\min |r(k) - s(i)|$$

In this equation, s(i) represents the possible symbol from the 16-QAM constellation, and arg min represents the operation to find the symbol that minimizes the distance between the received symbol and the possible symbol.

3.5.**Decoder Equations**:

The turbo decoder performs soft-input, soft-output (SISO) decoding using the MAP algorithm. Let's denote the received symbols as r(k), the estimated symbols from the other decoder as ĉ(k), and the extrinsic information as e(k).

The decoder equations can be represented as follows:

Decoder 1:

$$p_1(k) = pr(u(k) = 1\ |r(k), \hat{c}(k), e(k))$$
$$p_2(k) = pr(u(k) = 0\ |r(k), \hat{c}(k), e(k))$$
$$e_1(k) = \frac{p_1(k)}{(p_1(k) + p_2(k))}$$

Decoder 2:

$$p_3(k) = pr(u(k) = 1\ |r(k), \hat{c}(k), e(k))$$
$$p_4(k) = pr(u(k) = 0\ |r(k), \hat{c}(k), e(k))$$
$$e_2(k) = \frac{p_3(k)}{(p_3(k) + p_4(k))}$$

In these equations, $p_1(k)$ and $p_2(k)$ represent the probabilities of u(k) being 1 and 0, respectively, for Decoder 1. Similarly, $p_3(k)$ and $p_4(k)$ represent the probabilities of u(k) being 1 and 0, respectively, for Decoder 2. $e_1(k)$ and $e_2(k)$ represent the extrinsic information for Encoder 1 and Encoder 2, respectively, which are calculated based on the probabilities.

4. Simulation and result

In this section, we present the simulation setup and discuss the results obtained from the simulation of free distance effect on turbo coding.

The following steps explains the pseudo code used to for the simulation:

For turbo coding, the number of iterations affects the effective free distance of the code. As the number of iterations increases, the turbo decoder has more opportunities to refine its estimates and correct errors. This can potentially improve the effective free distance of the turbo code, meaning that it can correct a greater number of bit errors before the decoding process fails.

Figure 2 shows the signal-to-noise rate SNR in dB vs Bet Error Rate BER for all iterations. While Figure 3 shows the best SNR vs BER, which is the fourth (iteration 15).

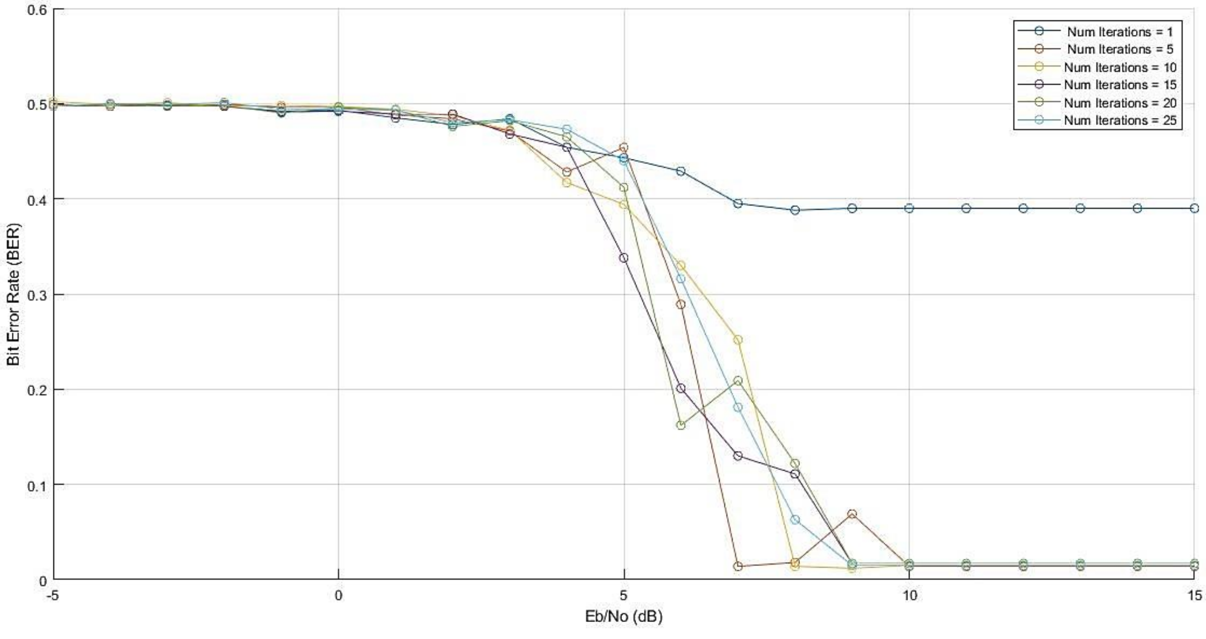

Figure 2 Eb/No in dB vs BER for all iterations

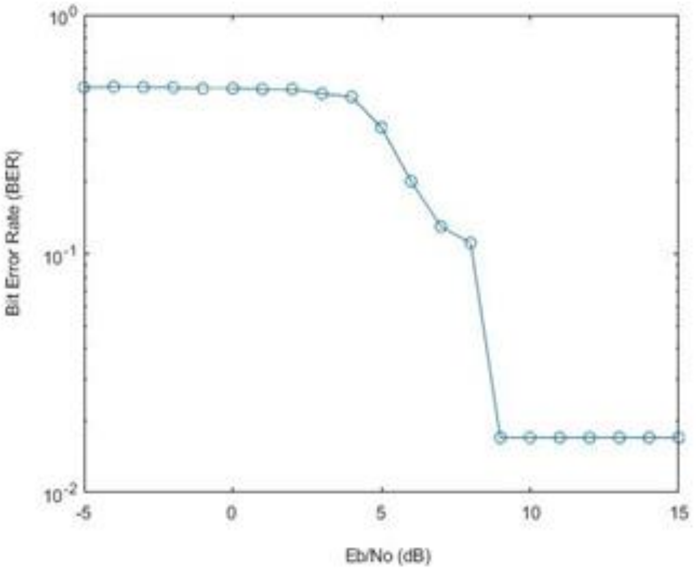

Figure 3 Eb/No in dB vs BER for the least mean of BER (The best Iteration).

However, it's important to note that the number of iterations is not the only factor that determines the free distance. The free distance also depends on other factors such as the code length, interleaver design, and the specific structure and parameters of the constituent codes used in the turbo code.

For more clarification, the effect of interleaver design will be focused-on in this step, Figure 4 shows the effect of changing the design of interleaver, random design, block design and convolutional design.

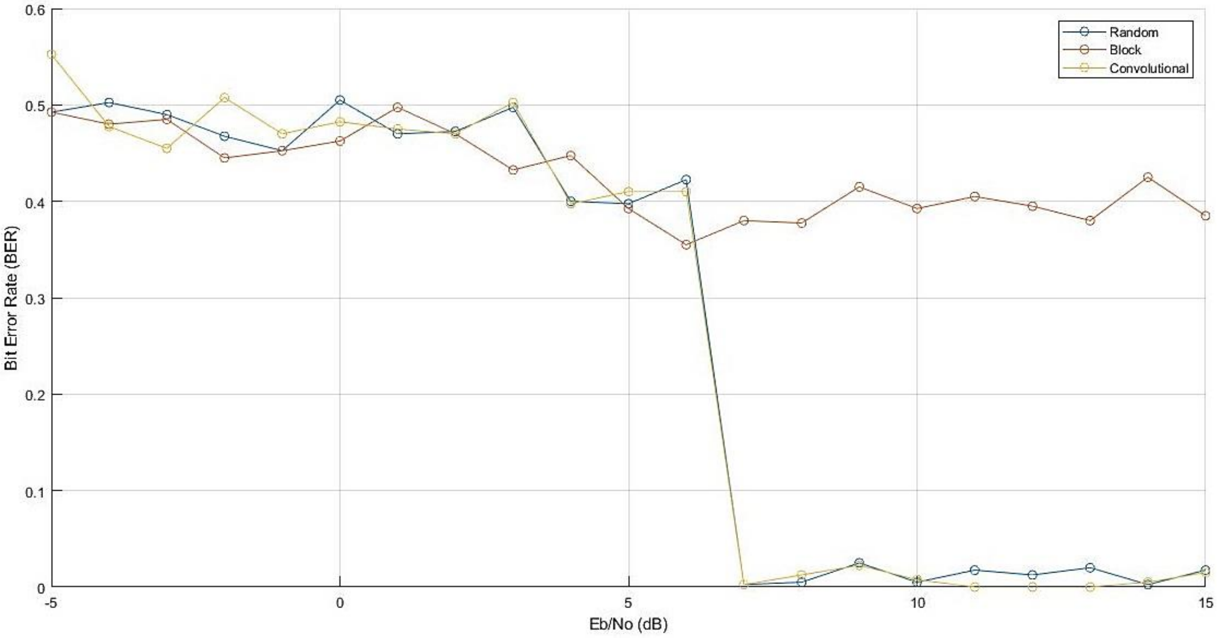

Figure 4 Eb/No in dB vs BER for three purposed interleaver designs.

Although the code length is not the most significant factor to investigate, but Figure 5 shows the comparison of multiple code lengths of the performance of turbo coding.

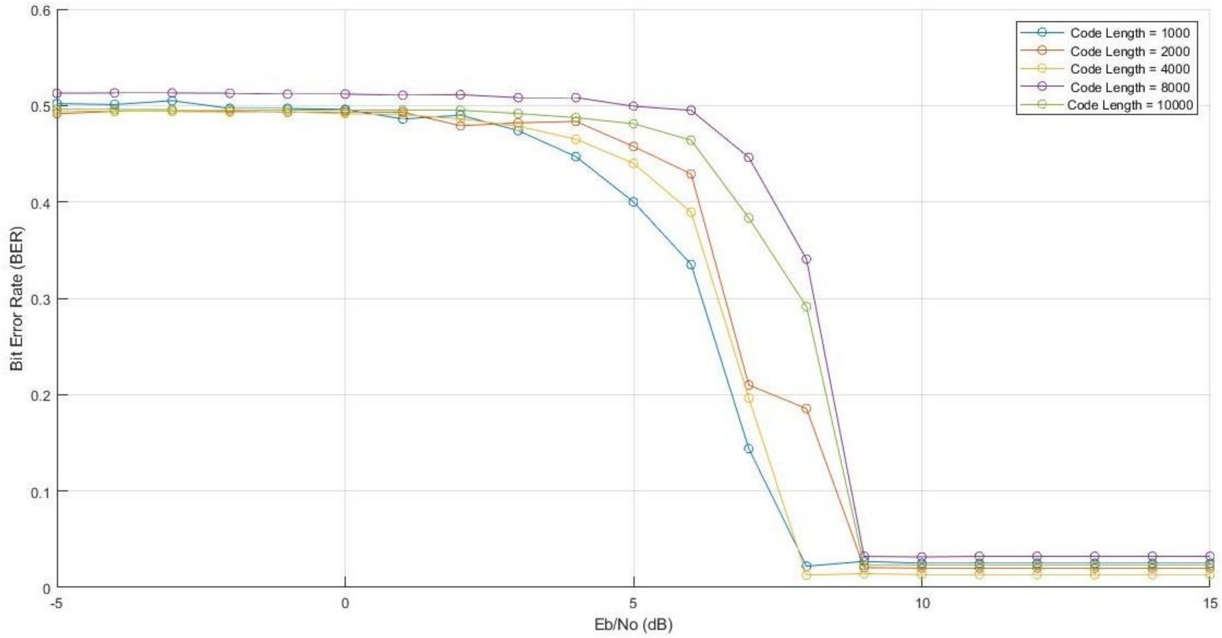

Figure 5 Eb/No in dB vs BER for all code lengths purposed.

Finally, merging the three factors gives more obvious results, Figure 6 for example shows the Eb/No in dB vs BER for code length of 400, 15 Iterations, Random Interleaver.
And Fig. 7 shows the Eb/No in dB vs BER for code length of 400, 5 Iterations, Block Interleaver.
While Figure 8 shows the Eb/No in dB vs BER for code length of 600, 10 Iterations, Convolutional Interleaver

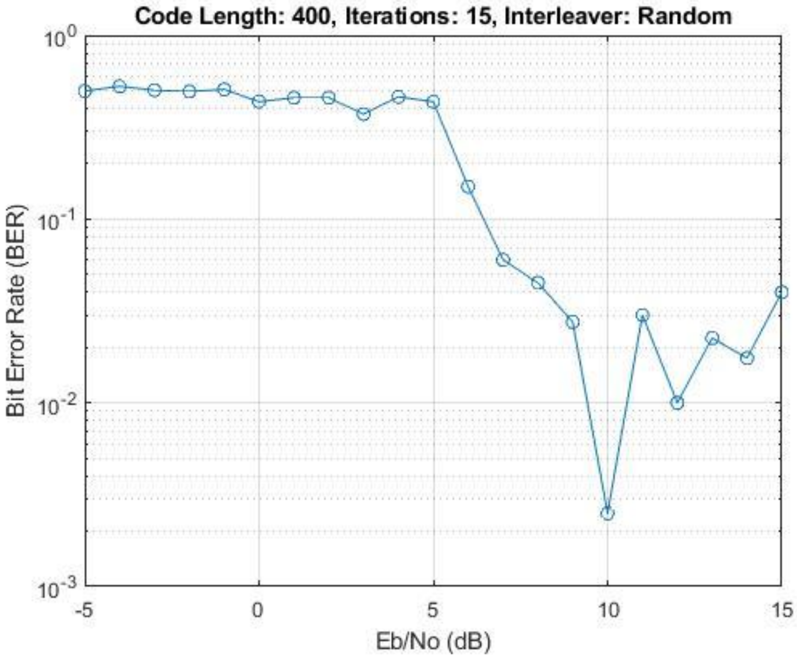

Figure 6 Eb/No in dB vs BER for code length of 400, 15 Iterations, Random Interleaver.

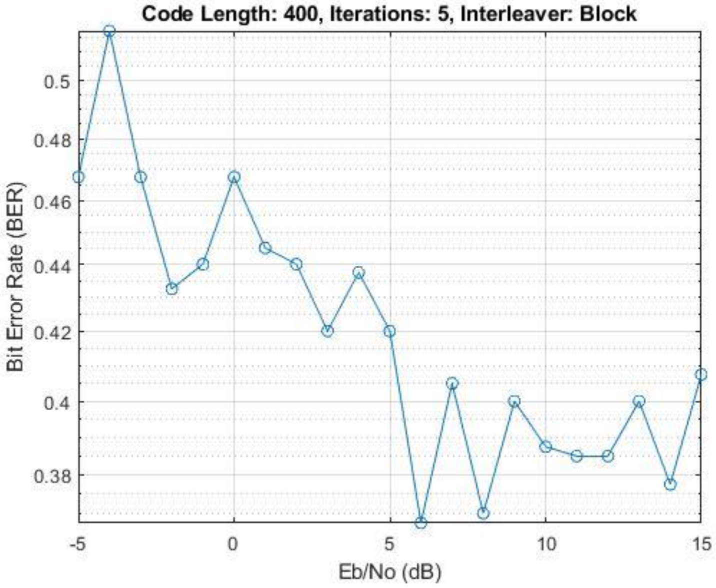

Figure 7 Eb/No in dB vs BER for code length of 400, 5 Iterations, Block Interleaver.

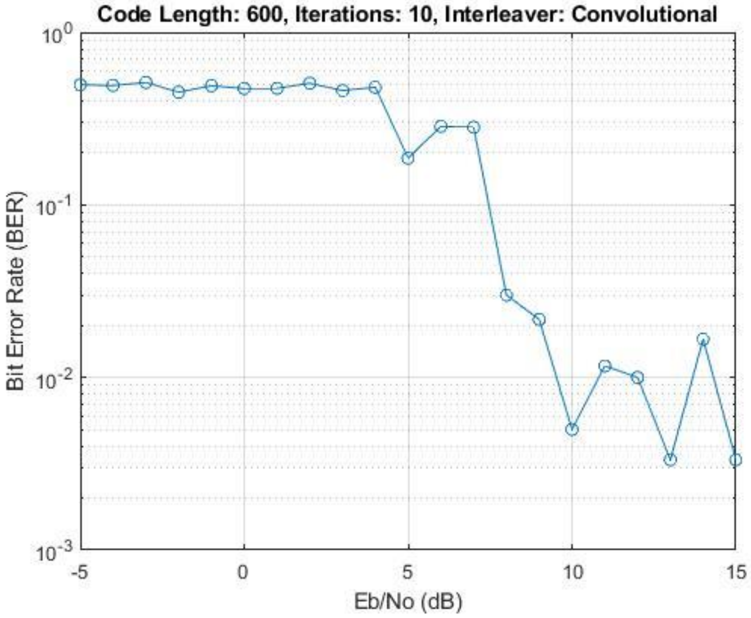

Figure 8 Eb/No in dB vs BER for code length of 600, 10 Iterations, Conolutional Interleaver.

5. Results and discussion

In this section a full analysis of the effect of the free distance on turbo coding with respect to number of iterations, code length, and interleaver design will be done.

Let’s discuss the effect of the free distance on turbo coding. The free distance has a direct impact on the performance of turbo codes. A higher free distance generally leads to better error correction performance. Turbo codes with a larger free distance can correct more errors and operate closer to their theoretical limits. The free distance is influenced by factors such as the number of iterations, code length, interleaver design, and the specific constituent codes used in the turbo code.

By increasing the code length, the free distance can be enhanced, allowing the turbo code to correct more errors. Similarly, the choice of interleaver design can affect the free distance. Some interleaver designs may provide better randomization of errors, leading to an improved free distance and error correction performance.

It is worth noting that the number of iterations also impacts the error correction capability of turbo codes. Increasing the number of iterations can help approach the performance limits set by the free distance. However, the number of iterations alone cannot compensate for a significantly small free distance. In such cases, even with a large number of iterations, the turbo code may not be able to correct a high number of errors effectively.

6. Conclusion

In conclusion, the effect of free distance on turbo coding is influenced by three key factors: the number of iterations, the code length, and the interleaver design. Firstly, the number of iterations plays a crucial role in turbo coding performance. Increasing the number of iterations allows the iterative decoding process to converge to a more reliable estimate of the transmitted data. This results in improved error correction capability and a higher likelihood of achieving the full potential of the turbo code's free distance. Secondly, the code length has a significant impact on turbo coding. Longer code lengths provide greater redundancy, enabling the turbo code to achieve higher coding gain and improved error correction capability. However, longer code lengths also lead to increased decoder complexity and decoding latency, which must be carefully considered in practical system design. Lastly, the interleaver design affects the free distance and overall performance of turbo codes. Different interleaver designs distribute the coded bits in distinct patterns, impacting the resilience against burst errors and the interleaver gain. The choice of interleaver design should be carefully evaluated to optimize the error correction performance and maximize the turbo code's free distance.

Overall, the free distance of turbo coding is influenced by the interplay of the number of iterations, the code length, and the interleaver design. Balancing these factors is essential to achieve the desired error correction capability while considering constraints such as complexity, latency, and bandwidth efficiency. By understanding and appropriately managing these factors, system designers can harness the full potential of turbo coding and optimize its performance for various communication applications.

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