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Calculation of Positive and Negative Parity Energy Levels Using IVBM for Some Molybdenum Isotopes and Determination of Their Properties

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Abstract:
The calculations of the excited positive and negative parity bands of the $^{100-108}Mo$ isotopes were achieved by using the Interacting Vector Boson Model (IVBM) extension. The bands investigated in the model are extended to very high angular momenta as a result of their consideration as Octopole bands concerning simplistic classification of the basis states. The characteristic of collective behavior by using R ratio, and E-GOS for the $^{100-108}Mo$ isotopes have been determined. Furthermore the ΔI = 1 staggering as a function of the angular momentum (I) has been examined.

Keywords: IVBM, Molybdenum isotopes, E-GOS curve, ΔI = 1 staggering.

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حساب مستويات الطاقة ذات التماثل الموجب و السالب بإستخدام نموذج IVBM لبعض نظائر الموليبديوم و تحديد خواصها

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الملخص
تم حساب مستويات الطاقة المثارة ذات التماثل الموجب و السالب لنظائرالموليبديوم $^{100-108}Mo$ بإستخدام نموذج متجه البوزونات المتفاعلة IVBM . نطاقات الطاقة التي دُرست في هذا النموذج تمتد إلى عزوم زاوية عالية جداً نتيجة لإعتبارها نطاقات حزم مختلفة octopole bands. تم تحديد السلوك الجماعي بإستخدام النسبة R، و منحنى E-GOS . وتم دراسة التأرجح (ΔI = 1 staggering) كدالة في البرم I.

الكلمات المفتاحية: نموذج IVBM ، نظائر الموليبديوم، منحنى E-GOS ، التأرجح (ΔI = 1 staggering) .

Introduction

In recent years, the phase structure of quantum many-body systems has been the subject of extensive attention in experimental and theoretical circles. Different models have been used to describe quantum phase transitions in different many-body systems. In the early 1980s, a new phenomenological algebraic model was introduced called the interacting vector boson model (IVBM) **[1]**, which is based on two types of vector bosons: the proton p boson and the neutron n boson, representing collective excitation in the nucleus **[2].** IVBM was developed to characterize the basal and octopole zones of the nucleus. In even nuclei exhibiting octopole deformation the ground-state band (GSB), with energy levels $I^{\pi} = 0^{+}, 2^{+}, 4^{+}, \ldots$, is accompanied by a negative parity band (NPB) with energy levels $I^{\pi} = 1^{-}, 3^{-}, 5^{-} \ldots$. The two bands become interwoven, forming a single octupole band and characterized with energy levels as $I^{\pi} = 0^{+}, 1^{-}, 2^{+}, 3^{-}, 4^{+}, \ldots$ **[3] [4]**.

Nuclear shape phases are the manifestations of the collective motion modes of nuclei. Every characteristic quantity takes the same value no matter whether the calculation is carried out in fermion space or boson space. Quantities of interest are the normalized low-lying levels' energies and the electric quadrupole transition rates. The low-lying levels' energies are normalized to $E_{2_1^+}$, and the $B(E2)$s are normalized to $B(E2;\ 2_1^+ \rightarrow 0_1^+)$ **[5].** By using the ratio $R = E(4)/E(2)$ the characteristic of collective behavior can be known, If ($3.33 \geq R \geq 2.9$) the nuclei are rotational or near–rotational, while if ($2.4 \geq R \geq 2$) the nuclei are vibrational or near – vibrational**. [3]**.

Reference [6] introduced the relation between the gamma energy over spin $E\gamma\ (I \rightarrow I - 2)/I$ as a function of the spin I (E-GOS), which indicated good information about the evolution that occurs in the yrast line of the nuclei.

several staggering effects are known in nuclear spectroscopy, in octupole bands the levels with odd angular momentum and negative parity were displaced relatively to the levels with even angular momentum and positive parity, the odd levels do not lie at the energies predicted by an $E(I) = AI(I + 1)$, but are all systematically above or all systematically below the predicted energies. **[3]**. In this case each level with angular momentum I is displaced relatively to its neighbors with angular momentum $I = \pm 1$. This is called ΔI=1 staggering. There is also another kind of staggering (ΔI=2 staggering) has been observed in superdeformed nuclear bands that are level with angular momentum I is displaced relatively to its neighbors with angular momentum $I = \pm 2$ **[3] [7].**

In the present work, the excitation states for $^{100-108}Mo$ isotopes are calculated using IVBM and compared with their experimental counterparts, category of transitional for all isotopes has been determined, which has been used to investigate the $\Delta I = 1$ staggering for $^{100-108}Mo$ isotopes.

Material and methods

1. IVBM Theory:

This model illustrated a good description of the energies of different rotational bands for states with positive and negative parity **[8]**. Since the model is well known, in our calculations, we have used the interacting vector boson model (IVBM). The general formula for the Hamiltonian of the system is written in terms of the first and second order casmer operators of simplistic group Sp (12, R) **[2] [9]** as:

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 L^2 + \alpha_1 T_0^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where a, b, β_3 are the model parameters that describe the ground state band, while α_1, α_3 are parameters describing the octupole states, N the total number of bosons, while T and T_0 characterizing the pseudospin introduced to distinguish the two types of vector bosons which are the building blocks of the algebraic structure of the model.

The Hamiltonian is diagonal in the basis states with eigenvalues can be written **[9]:**

$$E(N, L, T, T_0) = aN + bN^2 + \alpha_3 T(T + 1) + \beta_3 L(L + 1) + \alpha_1 T_0^2 \quad \ldots\ldots\ldots\ldots (2)$$

The angular momentum operator T and L related to N as:

$$L = {}^{N}\!/_{2}$$

$$T = \frac{N}{2}, \frac{N}{2} - 1, \ldots\ldots\ldots, 0\ or\ 1$$

$$-T \leq T_0 \geq T$$

$$\lambda = 2T \quad , \quad \mu = \left({}^{N}\!/_{2}\right) - T$$

The values of L are given a standard way:

$$K = min(\lambda, \mu), min(\lambda, \mu) - 2, \ldots\ldots\ldots, 0\ or\ 1$$

$$L = 0, 2, \ldots\ldots\ldots\ldots, max(\lambda, \mu)\ for\ K = 0$$

$$L = max(\lambda, \mu), max(\lambda, \mu) - 1, \ldots\ldots..0\ or\ 1 \quad for\ K \neq 0$$

Since the energies (Eq. (2)) increase with increasing the number of vector N, the minimum values of the energies of the ground and octupole bands are obtained $N = 2L$ respectively.

I. For ground states we fix $T = 0$ and $T_0 = 0$ with $N = 0, 4, 8 \ldots$ $L = 0,2,4,6, \ldots \ldots$, the Hamiltonian written as:

$$H_{GSB} = aN + bN^2 + \beta_3 L(L+1) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3)$$

II. For octupole states we fix $T = 1$ and $T_0 = 0$ with $N = 2, 6, 10 \ldots$ $L = 1,3,5, \ldots \ldots$, the Hamiltonian written as:

$$H_{NPB} = aN + bN^2 + 2\alpha_3 + \beta_3 L(L+1) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (4)$$

The quantum number K is used to label the different bands in the energy spectra of nuclei.

To reduce the Hamiltonian terms (3,4), the following relationships can be substituted for **[10] [11]:**

$$\beta = 4b + \beta_3 \quad , \quad \gamma = 2a - 4b \quad , \quad \eta = 8b \quad , \quad \xi = 2a + b + 2\alpha_3$$

We get:

$$H_{GSB} = \beta L(I+1) + \gamma I \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (5)$$
$$H_{NPB} = \beta L(I+1) \quad + (\gamma + \eta)I + \xi \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (6)$$

Where β, ϒ, η, and ξ are parameters that can be found by program math lap.

2. Staggering effect

Traditionally the odd–even staggering (ΔI = 1 staggering) in octupole bands, as well as in gamma bands, has been estimated quantitatively through the use of the expression **[12]**. By analogy, ΔI = 1 staggering effect in nuclei can be measured by the quantity **[3]**:

$$\Delta E_{1,\gamma}(I) = \frac{1}{16}(6E_{1,\gamma}(I) - 4E_{1,\gamma}(I-1) - 4E_{1,\gamma}(I+1) + E_{1,\gamma}(I-2) + E_{1,\gamma}(I+2) \ldots \ldots \ldots (7)$$

where

$$E_{1,\gamma}(I) = E((I+1) - E(I)$$

Results and discussion

The parameters β and γ have been calculated from equation (5) to find the energy levels with positive parity by fitting the experimental **[13]** values for the $^{100-108}Mo$ isotopes, therefore by using β and γ the parameters η and ξ have been determined from equation (6) to find the energy levels with negative parity by fitting the experimental values for last isotopes. **Table 1** shows the values of the parameters β, γ, ξ *and* η for the isotopes $^{100-108}Mo$. **Table 2** shows the comparison between the energy levels (positive and negative) parity calculated by IVBM model are compared with experimental data for $^{100-108}Mo$ isotopes. All these calculations for energy levels are in (MeV), the calculated values are in very good agreement with the experimental results. **Figure 1** shows compatibility between experimental and calculating energy levels. We have applied the formalism $R = E_4/E_2$ to describe collective behavior for the $^{100-108}Mo$ isotopes, the values of the R ratio are given in **Table 3** we noted that the values of R range from 2.1 to 2.6, meaning that the movement of these nuclei is from vibrational to γ unstable. E-GOS curve ($E\gamma / I$) as a function of the angular momentum (I) has been drawn in **Figure 2** for $^{100-108}Mo$ isotopes and compared with ideal limits of vibrational, rotational, and γ -soft. So, we noted that ^{100}Mo is pure vibrational (U(5)) but other isotopes are near vibrational (O(6)).

Table 1: The best parameters of IVBM for the ground state and octupole bands for $^{100-108}Mo$

A	π	β	γ	η	ξ
100	+	0.0081	0.2501	----	----
	-	0.0081	0.2501	0.2339	-1.7858
102	+	0.0168	0.102	----	----
	-	0.0168	0.1020	0.3359	-3.4956
104	+	0.0173	0.0564	----	----
	-	0.0173	0.0564	0.351	-10.5733

106	+	0.016	0.045	----	----
	-	0.016	0.045	0.24	-1.9
108	+	0.0164	0.055	----	----
	-	0.0164	0.055	0.45	-2.85

Table 2: The best energy levels (positive and negative) parity calculated by IVBM and compared with experimental data for $^{100-108}Mo$

I^{π}	100		102		104		106		108	
	E_{exp}	IVBM	E_{exp}	IVBM	E_{exp}	IVBM	E_{exp}	IVBM	E_{exp}	IVBM
1^-	---	1.1713	---	0.6606	---	0.2583	---	1.2396	---	0.3698
2^+	0.5355	0.5488	0.296	0.3048	0.1922	0.2166	0.1715	0.1860	0.19279	0.2084
3^-	1.908	1.7905	1.881	1.3324	---	0.9603	1.8172	1.7196	---	1.2698
4^+	1.1360	1.1624	0.743	0.744	0.5607	0.5716	0.5223	0.5000	0.56369	0.5480
5^-	2.339	2.4096	2.147	2.0042	1.884	1.6623	2.0904	2.1996	2.16225	2.1698
6^+	1.847	1.8408	1.324	1.3176	1.0799	1.0650	1.033	0.9420	1.0907	1.0188
7^-	2.843	3.0287	2.547	2.676	2.305	2.3643	2.4988	2.6796	3.1112	3.0698
8^+	2.6275	2.5840	2.018	2.0256	1.7218	1.696	1.6882	1.5120	1.7528	1.6208
9^-	3.299	3.6478	3.005	3.3478	2.867	3.0663	3.0415	3.1596	---	3.9698
10^+	3.367	3.392	2.418	2.868	2.4554	2.467	2.4724	2.2100	2.5295	2.3540
11^-	4.032	4.2669	3.614	4.0196	3.5558	3.7683	3.7076	3.6396	---	4.8698
12^+	4.0626	4.2648	3.625	3.8448	3.2534	3.3700	3.3691	3.0360	3.4014	3.2184
13^-	4.9398	4.8861	4.856	4.6914	4.6254	4.4703	---	4.1196	---	5.7698
14^+	4.875	5.2024	4.504	4.956	4.1147	4.4226	4.362	3.9900	4.3470	4.2140
15^-	---	5.5052	5.764	5.3632	5.2689	5.1723	---	4.5996	---	6.6698
16^+	5.84	6.2048	5.470	6.2016	5.0597	5.6080	5.4128	5.0720	5.3475	5.3408
17^-	---	6.1243	---	6.035	6.2844	5.8763	---	5.0796	---	7.5698
18^+	6.949	7.2720	---	7.5816	6.1112	6.93318	6.5010	6.2820	---	6.5988
19^-	---	6.7434	---	6.7068	---	6.5763	---	5.5596	---	8.4698
20^+	8.114	8.4040	---	9.096	7.2832	8.394	7.6600	7.6200	---	7.9880
21^-		7.3625		7.3786		7.2783	---	6.0396	---	9.3698

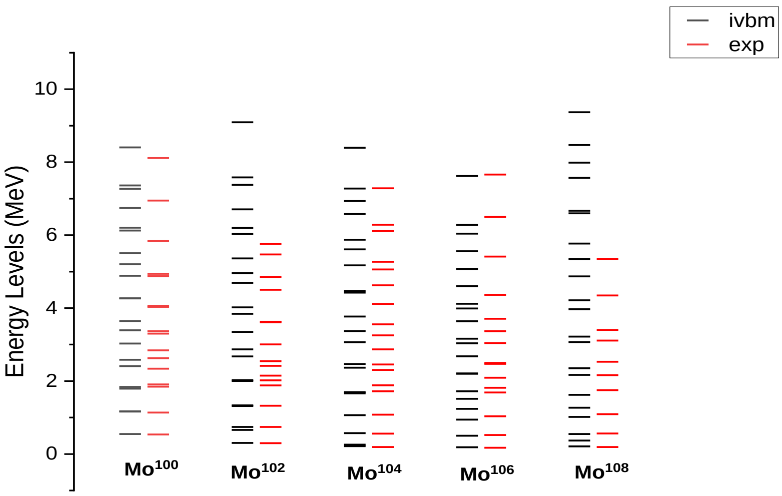

Figure 1 Fitting experimental and calculated data for $^{100-108}Mo$

Table 3: The ratio R for $^{100-108}Mo$ isotops.

A	E_2	E_4	R
100	0.548	1.162	2.118
102	0.304	0.744	2.440
104	0.216	0.571	2.638
106	0.186	0.500	2.688
108	0.208	0.548	2.629

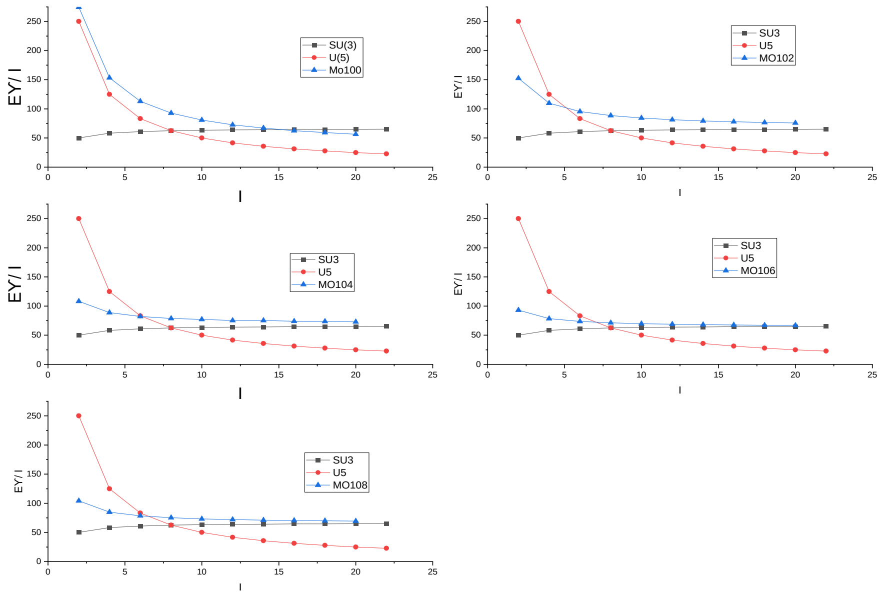

Figure 2 E-GOS curves of $^{100-108}Mo$ isotopes

To find $\Delta I = 1$ staggering effect for these nuclei we used (Eq. (7)). As shown in **Fig (3),** we found that for odd I the behavior of the staggering amplitude is exactly the opposite of the one described for even I. The amplitude starts from a negative value and then becomes consequently positive (because of the second term), negative (because of the third term), again positive (because of the fourth term), and so on. On the other hand, we observed in the case of ^{102}Mo and ^{106}Mo the decrease slowly of the staggering with increasing I, but ^{104}Mo and ^{108}Mo increase slowly of the staggering with increasing I, while ^{100}Mo decreases quickly. One can get a similar impression from parts of the patterns shown as, in the cases of ^{102}Mo (in the region $I = 3 - 9$), ^{104}Mo (for $I = 4 - 12$), ^{108}Mo (for $I = 5 - 15$). As well as staggering patterns resembling the "beat" structure have been seen in several bands as shown in **Figure 3**. In all cases bands not influenced by bandcrossing effects have been considered, in order to make sure that the observed effects are "pure" single-band effects. The only exception is ^{108}Mo. The "beat" pattern appears in vibrational nuclei.

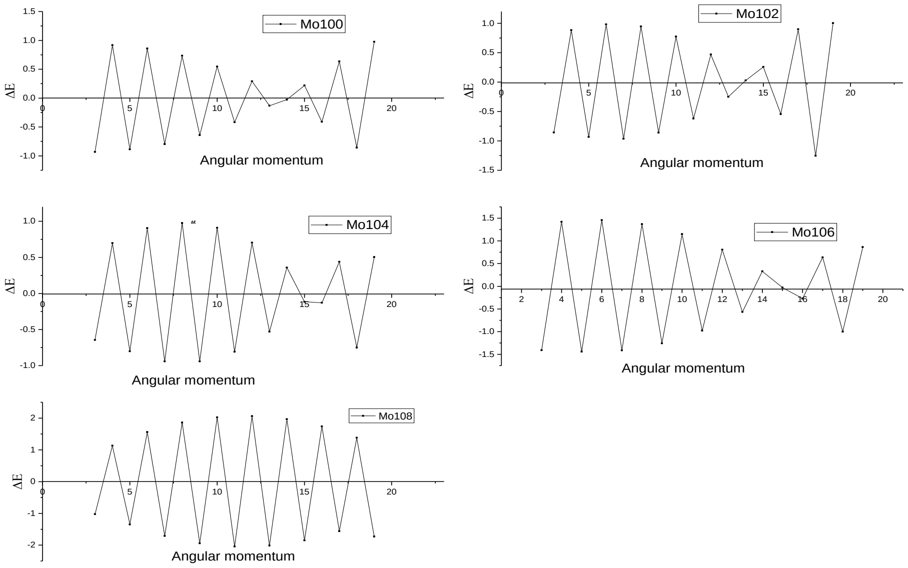

Figure 3 $\Delta E_{1,\gamma}(I)$ in MeV calculated from Eq. (6), for octupole bands of $^{100-108}Mo$

Conclusion

We have applied the Interacting Vector Boson Model for the description of the octupole bands in some even-even Mo nuclei that under-investigated up to very high spins, the calculated values are in very good agreement with the experimental results. To describe the property of $^{100-108}Mo$ isotopes, the formalism of $R = E_4/E_2$ and E-GOS curve are plotted and compared to the ideal limits of vibrational, rotational, and γ -soft. it is interesting to see that these isotopes have good vibrational and near-vibrational characteristics. Staggering patterns resembling the "beat" structure have been seen in several bands, the only exception is ^{108}Mo.

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