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Some Results Associated with Generalised Derivative Operator

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بعض النتائج المرتبطة بالعامل التفاضلي المعمم

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Abstract:

The theory of operators is an important subject in analytic function theory, geometric function theory and univalent function theory. It also is an important subject in applied sciences. It is still an active field of research and various types of problems, which can be solved by generalising operators. Recently, many researchers have shown great interests in the study of differential operators in the theory of univalent functions and various subclasses of analytic functions defined in the open unit disc. In this paper a generalised derivative operator $\mathcal{D}_{\lambda_1,\lambda_2,\delta,}^{m,b}$ will be used to derive some results concerning the subordination and superordination of analytic function in the open unit disc.

Keywords: Analytic Functions, Differential Operator, Subordination, Superordination.

الملخص

تعد نظرية العوامل موضوعًا مهمًا في نظرية الدوال التحليلية ونظرية الدوال الهندسية ونظرية الدوال أحادية التكافؤ. كما أنه موضوع مهم في العلوم التطبيقية. ولا يزال مجالًا نشطًا للبحث ولأنواعًا مختلفة من المشكلات، والتي يمكن حلها عن طريق العوامل المعممة. في الآونة الأخيرة، أبدى العديد من الباحثين اهتمامًا كبيرًا لدراسة العوامل التفاضلية في نظرية الدوال الأحادية التكافؤ والفئات الفرعية المختلفة للدوال التحليلية المحددة بقرص الوحدة المفتوح. في هذا البحث سيتم استخدام العامل التفاضلي المعمم $\mathcal{D}_{\lambda_1,\lambda_2,\delta}^{m,b}$ لاستخلاص بعض النتائج المتعلقة بتبعية وفوقية الوظيفة التحليلية في قرص الوحدة المفتوح.

الكلمات المفتاحية: الدالة التحليلية، العامل التفاضلي، التبعية، الفوقية.

Introduction

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the open unit disc $\mathbb{U} = \{z: z \in \mathbb{C}, |z| < 1\}$.

Let $\mathcal{H}(\mathbb{U})$ be the class of analytic functions in the open unit disc $\mathbb{U}$. For $a \in \mathbb{C}$ and $n \in \mathbb{N}$, we let

$$\mathcal{H}[a,n] = \{f \in \mathcal{H}(\mathbb{U}), f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \cdots\}, \ (z \in \mathbb{U}), \tag{1.2}$$

with $\mathcal{H}_0 = \mathcal{H}[0,1]$ and $\mathcal{H} = \mathcal{H}[1,1]$.

Recall that the function f is subordinate to g if there exists the Schwarz function ω, analytic in $\mathbb{U}$, with $\omega(0) = 0$ and $|\omega(z)| < 1$ such that $f(z) = g(\omega(z)), z \in \mathbb{U}$. We denote this subordination by $f(z) \prec g(z)$. If $g(z)$ is univalent in $\mathbb{U}$, then the subordination is equivalent to $f(0) = g(0)$ and $f(\mathbb{U}) \subset g(\mathbb{U})$.

Let $\psi: \mathbb{C}^3 \times \mathbb{U} \to \mathbb{C}$ and $f, h \in \mathcal{H}(\mathbb{U})$. If f and $\psi(f(z), zf'(z), z^2 f''(z); z)$ are univalent in $\mathbb{U}$ and f satisfies the second-order differential subordination

$$\psi(f(z), zf'(z), z^2 f''(z); z) \prec h(z), \ \ (z \in \mathbb{U}), \tag{1.3}$$

Then f is called a solution of the differential subordination. The univalent function q is called a dominant of the solutions of the differential subordination, or more simply a dominant, if $f \prec q$ for all f satisfying(1.3).

Many interesting results containing the above mentioned subordination and also many applications of the field of differential subordination discussed in [2]. In that direction, many differential subordination and differential superordination problems for analytic functions defined by means of linear operators can be found in [4]-[9].

In order to prove the original results we need the following definitions and theorems:

Definition 1.1 (see [2]). Denote by $\mathcal{Q}$ the set of all functions q that are analytic and injective on $\overline{\mathbb{U}} \backslash \mathrm{E(q)}$, where

$$E(q) = \{\zeta \in \partial \mathbb{U}: \lim_{z \to \zeta} = \infty\}, \tag{1.4}$$

such that $q'(\zeta) \neq 0$ for $\zeta \in \partial \mathbb{U} \backslash E(q)$. Further let the subclass of $\mathcal{Q}$ for which $q(0) = a$ be denoted by $\mathcal{Q}(a), \mathcal{Q}(0) \equiv \mathcal{Q}_0$, and $\mathcal{Q}(1) \equiv \mathcal{Q}_1$.

Definition 1.2 (see [2]). Let Ω be a set in $\mathbb{C}$, $\mathrm{q} \in \mathcal{Q}$, and let n be a positive integer. The class of admissible functions $\Psi_{\mathrm{n}}[\Omega, \mathrm{q}]$ consists of those functions $\psi: \mathbb{C}^3 \times \mathbb{U} \to \mathbb{C}$ that satisfy the admissibility condition $\psi(\mathrm{c}, \mathrm{d}, \mathrm{e}; \mathrm{z}) \notin \Omega$ whenever $\mathrm{c} = \mathrm{q}(\zeta), \mathrm{d} = \mathrm{k}\zeta \mathrm{q}'(\zeta)$,

$$\Re\left\{\frac{e}{d} + 1\right\} \geq k \Re\left\{\frac{\zeta q''(\zeta)}{q'(\zeta)} + 1\right\}, \tag{1.5}$$

where $z \in \mathbb{U}, \zeta \in \partial \mathbb{U} \backslash E(q), k \geq n$ and $\Psi_1[\Omega, q] = \Psi[\Omega, q]$.

Definition 1.3 (see [3]). Let Ω be a set in $\mathbb{C}$, $\mathrm{q} \in \mathcal{H}[\mathrm{a}, \mathrm{n}]$ with $\mathrm{q}'(\mathrm{z}) \neq 0$. The class of admissible functions $\Psi'_{\mathrm{n}}[\Omega, \mathrm{q}]$ consists of those functions $\psi: \mathbb{C}^3 \times \mathbb{U} \to \mathbb{C}$ that satisfy the admissibility condition $\psi(\mathrm{c}, \mathrm{d}, \mathrm{e}; \mathrm{z}) \notin \Omega$ whenever $\mathrm{c} = \mathrm{q}(\mathrm{z}), \mathrm{d} = \mathrm{zq}'(\mathrm{z})/\rho$,

$$\Re\left\{\frac{e}{d} + 1\right\} \geq \frac{1}{\rho} \Re\left\{\frac{\zeta q''(\zeta)}{q'(\zeta)} + 1\right\}, \tag{1.6}$$

where $z \in \mathbb{U}, \zeta \in \partial \mathbb{U}, \rho \geq n \geq 1$ and $\Psi'_1[\Omega, q] = \Psi'[\Omega, q]$.

Theorem 1.1. (see [2]). Let $\psi \in \Psi_{\mathrm{n}}[\Omega, \mathrm{q}]$ with $\mathrm{q}(0) = \mathrm{a}$. If the analytic function $\mathrm{j}(\mathrm{z}) \in \mathcal{H}[\mathrm{a}, \mathrm{n}]$ satisfies

$$\psi(j(z), zj'(z), z^2 j''(z); z) \in \Omega, \tag{1.7}$$

then $j(z) \prec q(z)$.

Theorem 1.2. (see [3]). Let $\psi \in \Psi'_{\mathrm{n}}[\Omega, \mathrm{q}]$ with $\mathrm{q}(0) = \mathrm{a}$. If $\mathrm{j} \in \mathcal{Q}(\mathrm{a})$ and $\psi(\mathrm{j}(\mathrm{z}), \mathrm{zj}'(\mathrm{z}), \mathrm{z}^2\mathrm{j}''(\mathrm{z}); \mathrm{z})$ is univalent *in* $\mathbb{U}$, *then*

$$\Omega \subset \{\psi(j(z), zj'(z), z^2 j''(z); z): z \in \mathbb{U}\}, \tag{1.8}$$

implies $q(z) \prec j(z)$.

We now state the following generalized derivative operator [1] as follows:

$$\mathcal{D}_{\lambda_1, \lambda_2, \delta}^{m,b} f(z) = z + \sum_{n=2}^{\infty} \left[\frac{1 + (\lambda_1 + \lambda_2)(n-1) + b}{1 + \lambda_2(n-1) + b}\right]^m \mathcal{C}(\delta, n) a_n z^n, \tag{1.9}$$

where $\lambda_2 \geq \lambda_1 \geq 0$, $\mathcal{C}(\delta, n) = (\delta + 1)_{n-1}/(n-1)!$, for $\delta, m, b \in \mathbb{N}_0 = \{0,1,2, \dots\}$, and $(x)_n$ is the Pochhammer symbol.

Note that $\mathcal{D}_{\lambda_1, \lambda_2, 0}^{0,b} f(z) = f(z)$ and $\mathcal{D}_{1,0,0}^{1,0} f(z) = zf'(z)$. the operator $\mathcal{D}_{\lambda_1, \lambda_2, \delta}^{m,b}$ includes the Ruscheweyh derivative operator in the case $\mathcal{D}_{\lambda_1, \lambda_2, \delta}^{0,b}$ [15], the Salagean derivative operator in the case $\mathcal{D}_{1,0,0}^{m,0}$ [12], the generalized Salagean derivative operator introduced by Al-Oboudi in the case $\mathcal{D}_{\lambda_1, 0,0}^{m,0}$ [14], the generalized Ruscheweyh derivative operator in the case $\mathcal{D}^{1,0}_{\lambda_1,0,\delta}$ [13], the generalized Darus and Al-Shaqsi derivative operator in the case $\mathcal{D}^{m,b}_{1,0,\delta}$ [10], the Eljamal and Darus derivative operator in the case $\mathcal{D}^{m,0}_{\lambda_1,\lambda_2,\delta}$ [11].

To prove our results, we need the following inclusion relation:

$$(1+b)\mathcal{D}^{m+1,b}_{\lambda_1,\lambda_2,\delta}f(z) = (1-(\lambda_1+\lambda_2)+b)(\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}f(z) * \varphi^b_{\lambda_2}(z)) + (\lambda_1+\lambda_2)z(\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}f(z) * \varphi^b_{\lambda_2}(z))', \tag{1.10}$$

where $\varphi^b_{\lambda_2}(z)$ is analytic function given by $\varphi^b_{\lambda_2}(z) = z + \sum_{n=2}^{\infty} \frac{z^n}{(1+\lambda_2(n-1)+b)}$

Subordination Results Associated with $\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}$

Definition 2.1. Let Ω be a set in $\mathbb{C}$ and $q \in \mathcal{Q}_0 \cap \mathcal{H}[0,1]$. The class of admissible functions $\Phi_D[\Omega, q]$ consists of those functions $\phi: \mathbb{C}^3 \times \mathbb{U} \to \mathbb{C}$ that satisfy the admissibility condition:

$$\phi(u, v, w; z) \notin \Omega \tag{2.1}$$

whenever

$$u = q(\zeta), \quad v = \frac{k(\lambda_1+\lambda_2)\zeta q'(\zeta) + (1-(\lambda_1+\lambda_2)+b)q(\zeta)}{(1+b)},$$

$$\Re\left\{\frac{(1+b)^2(\lambda_1+\lambda_2)w - (1-(\lambda_1+\lambda_2)+b)^2(\lambda_1+\lambda_2)u}{(1+b)(\lambda_1+\lambda_2)^2 v - (1-(\lambda_1+\lambda_2)+b)(\lambda_1+\lambda_2)^2 u} - \frac{2(1-(\lambda_1+\lambda_2)+b)}{(\lambda_1+\lambda_2)}\right\} \geq k\Re\left\{\frac{\zeta q''(\zeta)}{q'(\zeta)} + 1\right\}, \tag{2.2}$$

where $z \in \mathbb{U}, \zeta \in \partial\mathbb{U}\setminus E(q), b \in \mathbb{N}_0, \lambda_2 \geq \lambda_1 > 0$, and $k \geq 1$.

Theorem 2.1. Let $\phi \in \Phi_D[\Omega, q]$. If $f \in \mathcal{A}$ satisfies

$$\{\phi(\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}f(z) * \varphi^b_{\lambda_2}(z), \mathcal{D}^{m+1,b}_{\lambda_1,\lambda_2,\delta}f(z), \mathcal{D}^{m+2,b}_{\lambda_1,\lambda_2,\delta}f(z); z): z \in \mathbb{U}\} \subset \Omega, \tag{2.3}$$

Then $(\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}f(z) * \varphi^b_{\lambda_2}(z)) \prec q(z), \ (z \in \mathbb{U})$. (2.4)

Proof The following relation obtained from (1.10)

$$\mathcal{D}^{m+1,b}_{\lambda_1,\lambda_2,\delta}f(z) = \frac{(\lambda_1+\lambda_2)z\left(\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}f(z) * \varphi^b_{\lambda_2}(z)\right)'}{1+b} + \frac{(1-(\lambda_1+\lambda_2)+b)\left(\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}f(z) * \varphi^b_{\lambda_2}(z)\right)}{1+b}, \tag{2.5}$$

hence

$$\mathcal{D}^{m+2,b}_{\lambda_1,\lambda_2,\delta}f(z) = \frac{(\lambda_1+\lambda_2)z\left(\mathcal{D}^{m+1,b}_{\lambda_1,\lambda_2,\delta}f(z) * \varphi^b_{\lambda_2}(z)\right)'}{1+b} + \frac{(1-(\lambda_1+\lambda_2)+b)(\mathcal{D}^{m+1,b}_{\lambda_1,\lambda_2,\delta}f(z) * \varphi^b_{\lambda_2}(z))}{1+b}. \tag{2.6}$$

Now, we define the analytic function in $\mathbb{U}$ by

$$j(z) = \mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}f(z) * \varphi^b_{\lambda_2}(z), \tag{2.7}$$

then we obtain

$$\mathcal{D}^{m+1,b}_{\lambda_1,\lambda_2,\delta}f(z) = \frac{(\lambda_1+\lambda_2)zj'(z) + (1-(\lambda_1+\lambda_2)+b)j(z)}{1+b},$$

$$\mathcal{D}^{m+2,b}_{\lambda_1,\lambda_2,\delta}f(z) = ((\lambda_1+\lambda_2)^2 z^2 j''(z) + ((\lambda_1+\lambda_2)^2$$
$$+2(1-(\lambda_1+\lambda_2)+b)(\lambda_1+\lambda_2))zj'(z) + (1-(\lambda_1+\lambda_2)+b)^2 j(z))/(1+b)^2. \tag{2.8}$$

Further, we define the transformations from $\mathbb{C}^3$ to $\mathbb{C}$ by

$$u = c,\ v = \frac{(\lambda_1+\lambda_2)d+(1-(\lambda_1+\lambda_2)+b)c}{1+b} \quad (2.9)$$

$$w = ((\lambda_1+\lambda_2)^2 e + ((\lambda_1+\lambda_2)^2 + 2(1-(\lambda_1+\lambda_2)+b)(\lambda_1+\lambda_2))d$$
$$+(1-(\lambda_1+\lambda_2)+b)^2 c)/(1+b)^2. \quad (2.10)$$

Let
$$\psi(c,d,e;z) = \phi(u,v,w;z), \quad (2.11)$$

$$\phi(u,v,w;z) = \phi(c,((\lambda_1+\lambda_2)d + (1-(\lambda_1+\lambda_2)+b)c)/(1+b),((\lambda_1+\lambda_2)^2 e + ((\lambda_1+\lambda_2)^2 + 2(1-(\lambda_1+\lambda_2)+b)(\lambda_1+\lambda_2))d + (1-(\lambda_1+\lambda_2)+b)^2 c)/(1+b)^2; z). \quad (2.12)$$

The proof will make use of Theorem 1.1. Using (2.7) and (2.8), from (2.11) we have,

$$\psi(j(z), zj'(z), z^2 j''(z); z) = \phi(\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta} f(z) * \varphi^b_{\lambda_2}(z), \mathcal{D}^{m+1,b}_{\lambda_1,\lambda_2,\delta} f(z), \mathcal{D}^{m+2,b}_{\lambda_1,\lambda_2,\delta} f(z); z). \quad (2.13)$$

Hence (2.3) becomes

$$\psi(j(z), zj'(z), z^2 j''(z); z) \in \Omega. \quad (2.14)$$

We note that

$$\frac{e}{d} + 1 = \frac{(1+b)^2(\lambda_1+\lambda_2)w - (1-(\lambda_1+\lambda_2)+b)^2(\lambda_1+\lambda_2)u}{(1+b)(\lambda_1+\lambda_2)^2 v - (1-(\lambda_1+\lambda_2)+b)(\lambda_1+\lambda_2)^2 u} - \frac{2(1-(\lambda_1+\lambda_2)+b)}{(\lambda_1+\lambda_2)} \quad (2.15)$$

since the admissibility condition for $\phi \in \Phi_D[\Omega, q]$ is equivalent to the admissibility condition for ψ as given in Definition 1.2, hence $\psi \in \Psi[\Omega, q]$, and by Theorem 1.1, we obtain

$$j(z) \prec q(z),$$

that is

$$\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta} f(z) * \varphi^b_{\lambda_2}(z) \prec q(z).$$

In the case $\phi(u,v,w;z) = v$, we have the following example.

Example 2.1. Let the class of admissible functions $\Phi_{Dv}[\Omega, q]$ consist of those functions $\phi: \mathbb{C}^3 \times \mathbb{U} \to \mathbb{C}$ that satisfy the admissibility condition:

$$v = \frac{k(\lambda_1+\lambda_2)\zeta q'(\zeta) + (1-(\lambda_1+\lambda_2)+b)q(\zeta)}{(1+b)} \notin \Omega,$$

where $z \in \mathbb{U}, \zeta \in \partial\mathbb{U}\backslash E(q), b \in \mathbb{N}_0, \lambda_2 \geq \lambda_1 > 0, k \geq 1$ and $\phi \in \Phi_{Dv}[\Omega, q]$. If $f \in \mathcal{A}$ satisfies

$$\mathcal{D}^{m+1,b}_{\lambda_1,\lambda_2,\delta} f(z) \subset \Omega,$$

then

$$\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta} f(z) * \varphi^b_{\lambda_2}(z) \prec q(z).$$

The following result follows immediately from Theorem 2.1.

Corollary 2.1. Let $\phi \in \Phi_D[\Omega, q]$. If $f \in \mathcal{A}$ satisfies

$$\phi(\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta} f(z) * \varphi^b_{\lambda_2}(z), \mathcal{D}^{m+1,b}_{\lambda_1,\lambda_2,\delta} f(z), \mathcal{D}^{m+2,b}_{\lambda_1,\lambda_2,\delta} f(z); z) \prec h(z), \quad (2.16)$$

then

$$\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta} f(z) * \varphi^b_{\lambda_2}(z) \prec q(z). \quad (2.17)$$

Superordination Results Associated with $\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}$

Definition 3.1. Let Ω be a set in $\mathbb{C}$ and $q \in \mathcal{H}[0,1]$ with $zq'(z) \neq 0$. The class of admissible functions $\Phi'_D[\Omega, q]$ consists of those functions $\phi: \mathbb{C}^3 \times \overline{\mathbb{U}} \to \mathbb{C}$ that satisfy the admissibility condition:

$$\phi(u, v, w; \zeta) \notin \Omega \tag{3.1}$$

Whenever

$$u = q(z), \quad v = \frac{(\lambda_1+\lambda_2)zq'(z)+\rho(1-(\lambda_1+\lambda_2)+b)q(z)}{\rho(1+b)},$$

$$\Re\{\frac{(1+b)^2(\lambda_1+\lambda_2)w-(1-(\lambda_1+\lambda_2)+b)^2(\lambda_1+\lambda_2)u}{(1+b)(\lambda_1+\lambda_2)^2v-(1-(\lambda_1+\lambda_2)+b)(\lambda_1+\lambda_2)^2u} - \frac{2(1-(\lambda_1+\lambda_2)+b)}{(\lambda_1+\lambda_2)}\} \geq \frac{1}{\rho}\Re\{\frac{zq''(z)}{q'(z)}+1\} \tag{3.2}$$

where, $z \in \mathbb{U}, \zeta \in \partial\mathbb{U}, b \in \mathbb{N}_0, \lambda_2 \geq \lambda_1 > 0$, and $\rho \geq 1$.

Theorem 3.1. Let $\phi \in \Phi'_D[\Omega, q]$. If $f \in \mathcal{A}, \mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}f \in \mathcal{Q}_0$ and

$$\phi(\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}f(z) * \varphi^b_{\lambda_2}(z), \mathcal{D}^{m+1,b}_{\lambda_1,\lambda_2,\delta}f(z), \mathcal{D}^{m+2,b}_{\lambda_1,\lambda_2,\delta}f(z); z), \tag{3.3}$$

is univalent in $\mathbb{U}$, then

$$\Omega \subset \{\phi(\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}f(z) * \varphi^b_{\lambda_2}(z), \mathcal{D}^{m+1,b}_{\lambda_1,\lambda_2,\delta}f(z), \mathcal{D}^{m+2,b}_{\lambda_1,\lambda_2,\delta}f(z); z): z \in \mathbb{U}\}, \tag{3.4}$$

implies that

$$q(z) \prec \mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}f(z) * \varphi^b_{\lambda_2}(z). \tag{3.5}$$

Proof. From (2.13) and (3.4), we have

$$\Omega \subset \{\psi(j(z), zj'(z), z^2j''(z); z): z \in \mathbb{U}\}.$$

From (2.9), (2.10), we see that the admissibility condition for $\phi \in \Phi'_D[\Omega, q]$ is equivalent to the admissibility condition for ψ as given in Definition 1.3. Hence $\psi \in \Psi'[\Omega, q]$, and by Theorem 1.2, we have

$$q(z) \prec j(z),$$

that is

$$q(z) \prec \mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}f(z) * \varphi^b_{\lambda_2}(z).$$

Corollary 3.1. Let h be analytic in $\mathbb{U}$ and $\phi \in \Phi'_D[\Omega, q]$. If $f \in \mathcal{A}, \mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}f \in \mathcal{Q}_0$ and

$$\phi(\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}f(z) * \varphi^b_{\lambda_2}(z), \mathcal{D}^{m+1,b}_{\lambda_1,\lambda_2,\delta}f(z), \mathcal{D}^{m+2,b}_{\lambda_1,\lambda_2,\delta}f(z); z),$$

is univalent in $\mathbb{U}$,then

$$h(z) \prec \phi(\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}f(z) * \varphi^b_{\lambda_2}(z), \mathcal{D}^{m+1,b}_{\lambda_1,\lambda_2,\delta}f(z), \mathcal{D}^{m+2,b}_{\lambda_1,\lambda_2,\delta}f(z); z),$$

implies that

$$q(z) \prec \mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}f(z) * \varphi^b_{\lambda_2}(z).$$

Conclusion

In this paper, we obtained some results concerning the subordination and superordination of analytic function in the open unit disc, which are related to the differential operator $\mathcal{D}^{m,b}_{\lambda_1,\lambda_2,\delta}$.

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