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Modeling and Comparison of a Beta-Type Stirling Engine Supplied By Air, Nitrogen and Helium Gases

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Abstract

In the present study, the results of modeling and the simulation of a beta-type Stirling engine have been performed. The gaseous media are air, nitrogen and helium, respectively. Since each of the gases have different molecular properties, the thermodynamic findings change a lot with the pressure and rotational speed. According to our theoretical model, a moderate heat stability of operating gases restricts peak temperatures up to 300 and 800 degrees Celsius. That gives an attainable efficiency for the cycles of the machine. The simulations have been performed for 1-5 bars for the above-mentioned gases. The model itself is a three-zone model having heater, regenerator and cooler. In each zone, time-dependent heat transfers are calculated under the boundary conditions. The effect of hot end temperature, speed of engine and pressure are examined for different pressures at different positions of the rotating shaft. The generated power has been determined for all sets of input parameters.

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1. Introduction

Solar power is the main renewable energy source for our planet. Two types of solar energies have grown intensely for last decades: Photovoltaic (PV) and solar concentrated systems (SCSs). Since both of them require large installation areas, other renewable energy techniques can be used for small areas [1]. The solar Stirling engines can generate output power according to its dimension. Besides many factors such as the filled gas, diameters of piston and regenerator can affect the output power, drastically. The main characteristics of a SE are mechanical power generation, weight and price. Mainly the output power and dimensions are important because these two parameters play an important role to identify the system.

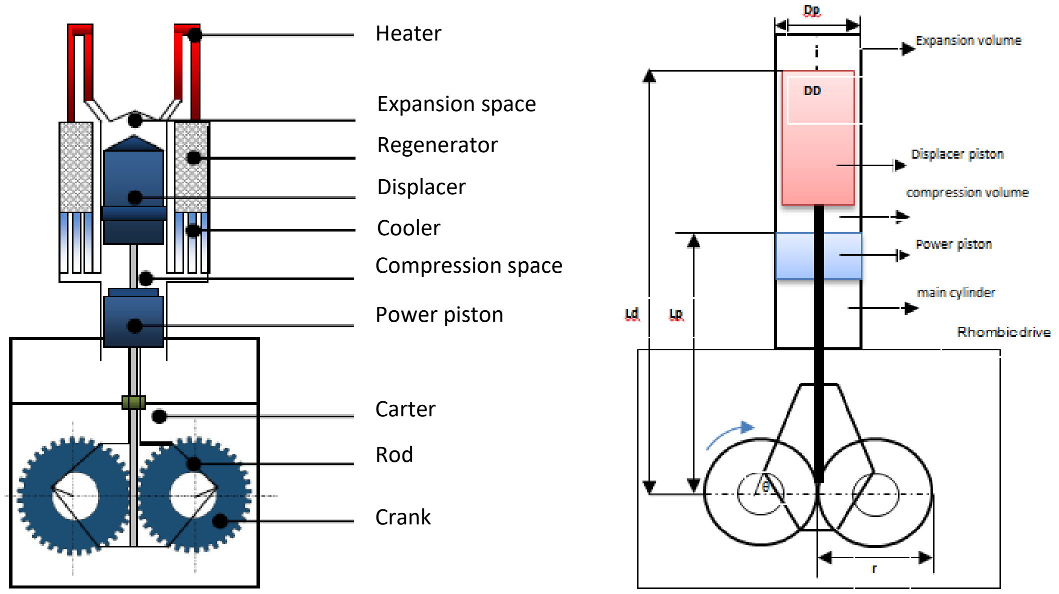
In the SE system designs, the main goal is to decide the optimization of output power and dimensions. Indeed, SEs are designed for an external heat source operation such as parabolic solar dishes. Differences in temperatures of two sources would affect two ends of the machine. For the high temperature part, the renewable energy sources such as solar or biomass are used frequently. According to literature, there exist a number of different working gas types such as air, helium, nitrogen, etc. [2-5]. The gas works on a closed thermodynamic cycle with expansion and compression cycles at high and low temperature levels [6-9].

Good energy performance is obtained for real gas Sterling cycles at the expense of the maximum pressure of the maximum cycle at the range of 100 to 300 bar. These extreme pressures have never had significant useful influences on the thermal energy conveyed in each unit of mechanical injecting power and reduced temperatures have a positive effect on the physical issues for the hottest parts of the engine.

Here, a three zones model is developed with heater, regenerator and cooler. In every single zone, heat transfers, which are time-dependent, are discussed in this mode. One of the goals of the mode is to permit boundary trials of system and kinetic parameters of a variety of operating gases on a wide array of packing pressures and temperatures of the hot side. For that reason, it should be quite straightforward to examine a variety of sets of parameters and simultaneously grant correct results.
From the point of effectiveness, the SE systems have a good record of solar-to grid power conversion effectiveness [10]. Besides, the modeling of those systems is still interesting because of the improvement in optimization [11]. In this paper, a Stirling engine of a beta type is modeled and studied by using of air, nitrogen and helium working gases. The effect of hot source temperature, speed of engine and pressure is examined in immediate and instantaneous gas pressure analysis temperature at different positions to derive the adequate power. This confirms the acceptance of the interest of a variety of operating gases in addition to the multiple geometric composition of many utilizations. In addition, the approved mode grants for motor-level trials of geometry and working gas. It is used to improve Stirling motor for various temperature points and working gases.

2. Modeling study

The model of the Stirling engine (SE) has been considered by a thermodynamic cycle of a rhombic drive beta-configuration. According to his theory, a simplified engine model can be produced by considering five volumes [12]. In the present study, a similar method has been followed in order to obtain the working space volumes of the proposed design (Fig. 1(a)). Single-cylinder rhombic drive beta-configuration SE as shown in the figure is used with a different acting temperature and pressure using air, nitrogen and helium as working gases, and compare the output power in various cases. The main working spaces for that configuration are cold - end space (i.e. compression space), regenerator space, and a hot - end space (i.e. expansion space). In that model, the displacer piston movement changes the volume of the expansion space. The volume of regenerating space is also based on the connection between the displacer cylinder and displacer piston as seen in Fig. 1(b). The movement of both displacer and power pistons change the compression volume. The prediction of alternate displacement, volumetric displacement, cyclic energy flow engine, working fluid mass conservation, cycle pressure, and cyclic temperature are executed and discussed [13].



(a) (b)

Figure 1 Single cylinder rhombic drive beta-configuration Stirling engine
(a) Schematic diagram. (b) Geometric parameters.

Numerical model:

The numerical model consists of five main units. They are displacement equations, volumetric displacement equations, cyclic temperature equations, mass conservation equations, pressure equations and energy flow relations.

Displacements equations:

The displacer and power piston displacement equations are given below. Here, x_p and y_d are given as,

$$x(t) = \frac{x_0}{2}\left(1 + cos\left(\theta - \frac{\pi}{2}\right)\right); \quad (1)$$

$$y(t) = \frac{y_0}{2(1+cos\theta)}; \quad (2)$$

Note that these expressions are standard as in Ref. [13]. The derivatives of the displacements are derived as,

$$\dot{x}(t) = -\frac{x_0}{2}\left(1 + sin\left(\theta - \frac{\pi}{2}\right)\right); \quad (3)$$

$$\dot{y}(t) = -\frac{y_0}{2(1+sin\theta)}; \quad (4)$$

In above expressions, x(t), y(t) are the power piston and displacer displacements as functions of time. Besides, x0, y0, θ are the power piston and displacer piston strokes and piston crank angle.

Volumetric displacements:
Expansion and compression space volumes are denoted by V_e and V_c and can be calculated in terms of displacements and cross-sectional areas of the cylinders. The volumes should also be considered in that regard. The terms of expansion and compression space volumes can also be written as [14],

$$V_c(t) = V_{d\_c} + x(t)A_p(y_0 - y(t))A_d; \quad (5)$$

$$V_e(t) = V_{d\_e} + y(t)A_d; \quad (6)$$

$$V_c(t) = V_{d\_c} + 0.5V_{swp}\left(1 + cos\left(\theta - \frac{\pi}{2}\right)\right) + 0.5V_{swd}\left(1 - cos(\theta)\right); \quad (7)$$

$$V_e(t) = V_{d\_e} + 0.5V_{swd}\left(1 + cos(\theta)\right); \quad (8)$$

Here, while V_c, V_{dc}, V_e, V_{de}, V_{swp}, V_{swd} are compression space, compression space dead, expansion space, expansion space dead, power piston swept, displacer swept volumes, A_P and A_d are power piston contact and displacer contact areas. The derivative of volumes can be obtained as,

$$\dot{V}_c(t) = -0.5V_{swp}\omega \sin\left(\theta - \frac{\pi}{2}\right) + 0.5V_{swd}\omega \sin(\theta); \quad (9)$$

$$\dot{V}_e(t) = -0.5V_{swd}\omega \sin\theta; \quad (10)$$

Here, ω is angular velocity in rad/s.

Cyclic temperature:
The gas temperature inside the compression, expansion, and regenerator spaces are determined by the equation of ideal gas state as follows:

$$T_c(t) = {}^{PV_c}/_{Rm_c} \quad (11)$$

$$T_e(t) = {}^{PV_e}/_{Rm_e} \quad (12)$$

$$T_r = \frac{(T_h - T_k)}{ln(T_h/T_k)} \quad (13)$$

The temperatures of the gas mass flow between the compression space and the cooler are determined by,

$$T_{c_k} = T_c \quad if\ m_{c_k} \leq 0 \quad (14)$$

$$T_{c_k} = T_k \quad if\ m_{c_k} > 0 \quad (15)$$

Note that m_{ck} is the gas mass flow rate between the compression and the cooler spaces in unit kg/s and T_c, T_k, T_{cr} are temperatures of gas in the compression, cooler spaces and the temperature of gas mass flow between the compression and the regenerator spaces in Kelvin degrees. The temperature of the gas mass flow between the heater and compression spaces can be obtained by,

$$T_{eh} = T_h \quad if\ m_{eh} \leq 0 \quad (16)$$

$$T_{eh} = T_e \quad if\ m_{eh} > 0 \quad (17)$$

As in Eqs. 14, 15. Here meh gives the gas mass flow rate between the expansion and the heater spaces. Besides, T_e, T_h, T_{eh} are the temperatures of gas in the expansion and heater spaces and the temperature of gas mass flow between the expansion and the heater spaces.

The mass conservation equations:
The conservation of mass for the control volumes can be summarized as follows:

$$m_e = {}^{PV_e}\!/_{(RT_h)}\,, \tag{18}$$

Where m_e denotes the working fluid mass in expansion space. For the compression space,

$$m_c = {}^{PV_c}\!/_{(RT_c)}\ \ , \tag{19}$$

Where m_c denotes the mass of working fluid in compression space and for the regenerator space,

$$m_r = {}^{PV_{c\_r}}\!\Big/_{(RT_r)} \tag{20}$$

Where m_r denotes the working fluid mass in regenerator space. For the heater and cooler parts,

$$m_h = {}^{PV_h}\!/_{(RT_h)} \tag{21}$$

Where m_h denotes the working fluid mass in heater and

$$m_k = {}^{PV_k}\!/_{(RT_k)} \tag{22}$$

Where m_k denotes the working fluid mass in cooler. The total mass in that case can be written as,

$$M = m_e + m_c + m_r + m_h + m_k \tag{23}$$

In order to simplify the simulations, one re-considers the reduced mass rates as follows:

$$\dot{m}_e = {}^{m_e \acute{P}}\!/_{P} \tag{24}$$

$$\dot{m}_c = {}^{\left(P\acute{V}_c + V_c \acute{P}/\gamma\right)}\!\Big/_{(RQ_k)} \tag{25}$$

$$\dot{m}_r = {}^{m_r \acute{P}}\!/_{P} \tag{26}$$

$$\dot{m}_r = {}^{m_h \acute{P}}\!/_{P} \tag{27}$$

$$\dot{m}_k = {}^{m_k \acute{P}}\!/_{P} \tag{28}$$

Pressure:
By recalling the ideal-gas equation,

$$PV = MRT \qquad , \tag{29}$$

The fluid pressure inside the cylinder can be found by,

$$P(t) = \frac{MR}{\left(\frac{V_c}{T_c}+\frac{V_k}{T_k}+\frac{V_r}{T_r}+\frac{V_h}{T_h}+\frac{V_e}{T_e}\right)} \tag{30}$$

Here,

$$T_r = \frac{(T_h - T_k)}{ln(T_h/T_k)} \tag{31}$$

Exists and the derivative of Eq. 30 yields to

$$\dot{P}(t) = \frac{\gamma\left(\acute{V}_c(t)/T_{c\_k} + \acute{V}_e(t)/T_{h\_e}\right)}{\left[V_e/T_{c_k} + \gamma(V_k/T_k + V_r/T_r + V_h/T_h) + V_e/T_{h\_e}\right]} \tag{32}$$

Cyclic energy flow:
The instantaneous indicated work output in the engine can be calculated in the following form:

$$\frac{dW_i}{dt} = \frac{dW_c}{dt} + \frac{dW_e}{dt} = P_c \frac{dV_c}{dt} + P_e \frac{dV_e}{dt} \tag{33}$$

Thus, the cyclic indicated work Wi can be stated as follows:

$$W_i = \int_0^{\tau} \left(\frac{dW_c}{dt} + \frac{dW_e}{dt}\right) dt = \int_0^{\tau} \left(P_c \frac{dV_c}{dt} + P_e \frac{dV_e}{dt}\right) dt \tag{34}$$

Finally, the indicated power can be formulated as,

$$P_i = W_i f \tag{35}$$

The simulation code has been written in MATLAB and it initially solves the equations in time domain. If averaged volume, power or pressure are required for the formulation, the averaged values for certain time span are taken into account.

3. Results and Discussion
The performance of the Stirling engine has been studied in different operating conditions according to various applied hot end temperatures such as T_h (300 ~ 800 K) and applied pressure (1,2,3,4,5 bar) using air, nitrogen and helium gases. The model can examine various applied pressures and engine speeds as in the sketched diagram of Fig. 2. By using various input values, the thermo-mechanical part has been solved according to input specific heat and gas constants as listed in table 1. As a result, comparison of output power using three input gases with respect to applied temperature, engine speed and pressure, besides changing of volumes can be obtained as outputs.

Table 1. *Specific Heat and Individual Gas Constant of Gases*

No	Gas	Specific Heat				Specific Heat Ratio		Individual Gas constant **- R -**
		cp (kJ/(kg K))	*cv (kJ/(kg K))*	*cp (Btu/(lbmoF))*	*cv (Btu/(lbmoF))*	*κ = p / cv*	*cp - cv (kJ/(kg K))*	*cp - cv (ft lbf/(lbmoR))*
1	Air	1.01	0.718	0.24	0.17	1.40	0.287	53.34
2	Nitrogen	1.04	0.743	0.25	0.18	1.40	0.297	54.99
3	Helium	5.19	3.12	1.25	0.75	1.667	2.08	386.3

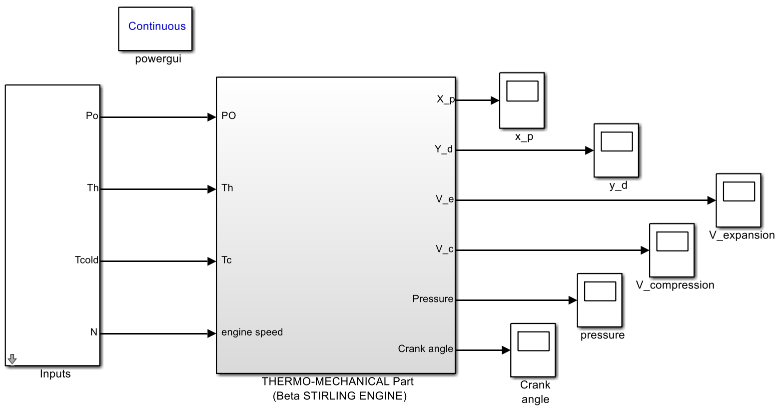


Figure 2 Simulation model of the study system

Fig. 3. Shows the expansion and compression cylinder volumes changes versus the crank angle of the engine. Note that pistons work systematically with a perfect sinusoidal form and the temperature has been kept at 700 K in this case of simulation. In the plot, θ = 0 corresponds to the top dead center of the displacer, which means that the large quantity of the gas has been in the compression volume. The working gas is bumped from compression volume to the hot volume on the top of the displacer while the displacer shifted down. A negative work on the piston is exist during this operation, and that process continues until piston reaches to its top dead center, which is also denoted as PTDC.

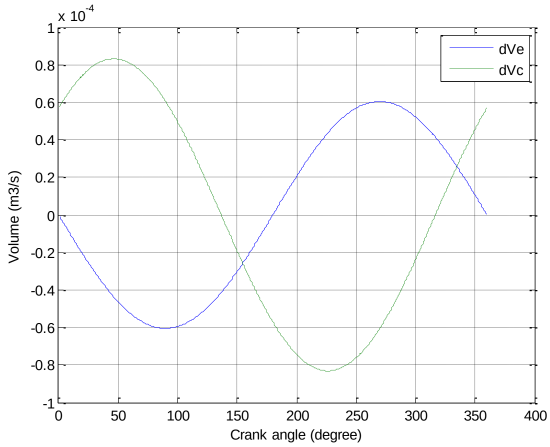
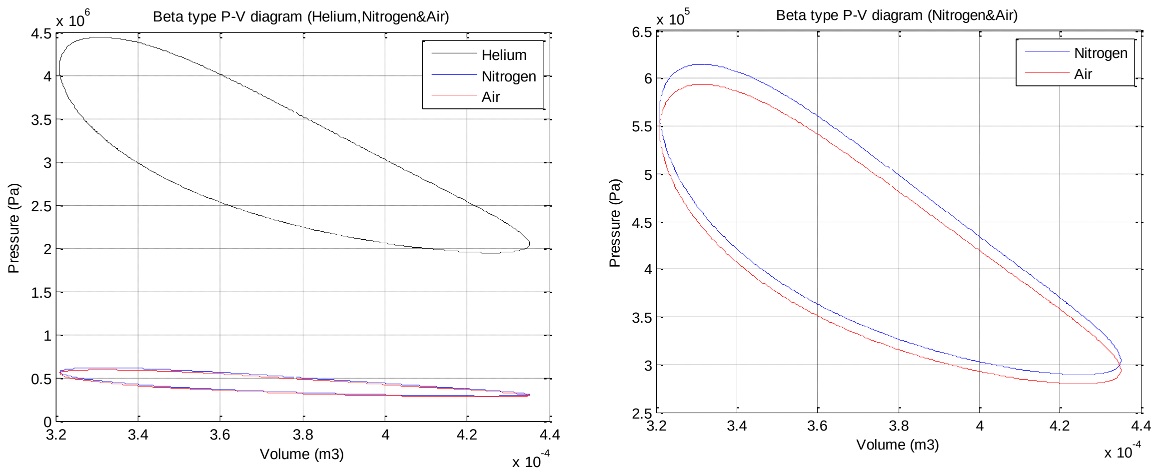


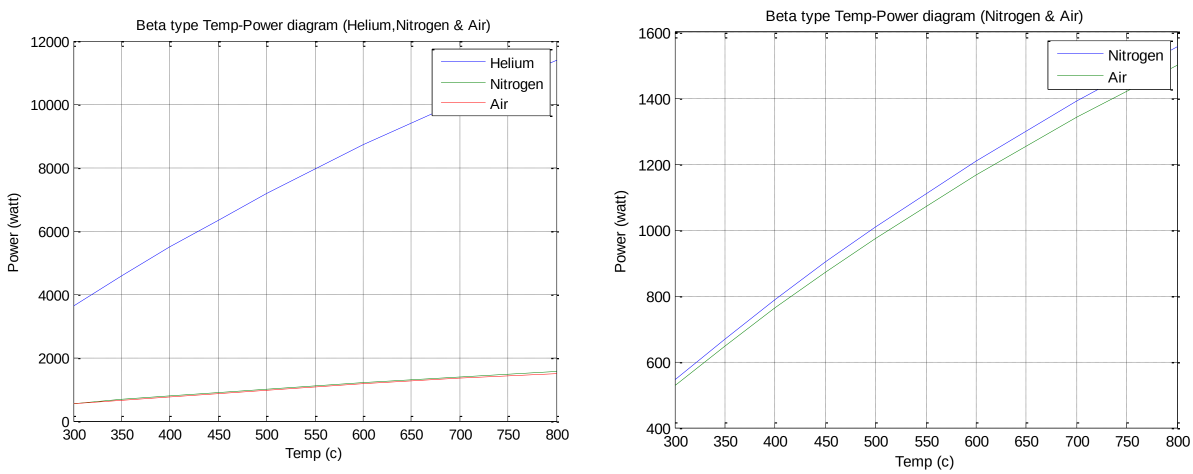
Figure 3 (a) Changing of expansion and compression volume versus crank angles.

Fig. 4(a, b) show the *P-V* diagrams for different gases. While the cycling area for air and nitrogen is narrow, it has been enlarged for helium. Although, both diagrams have been obtained under 4 bars charge pressure, 700 rpm engine speed and a 30 °C cold end temperature, the generated pressure in case of helium is about eight times greater than that generated by air or nitrogen. Air and nitrogen are more save but helium gas molecules are more active due to their light masses. Also, helium has higher thermal conductivity than air.



(a) (b)

Figure 4 Pressure vs Cylinder volume, (a) Air, nitrogen and helium. (b) Air and nitrogen



(a) (b)

Figure 5 Output power various applied hot end temperature 300 ~ 800 K for (a) Air, nitrogen and helium. (b) Air and nitrogen

Figs. 6 proves that the increasing temperature also affects the efficiency of the Stirling engine. For instance, efficiency can be increased upto 23% from 14% by enhancing the temperature. When the hot end temperature is increased to 900 K, the efficiency increasing around two times. Thus, the temperature-dependence is an important finding for such engines.

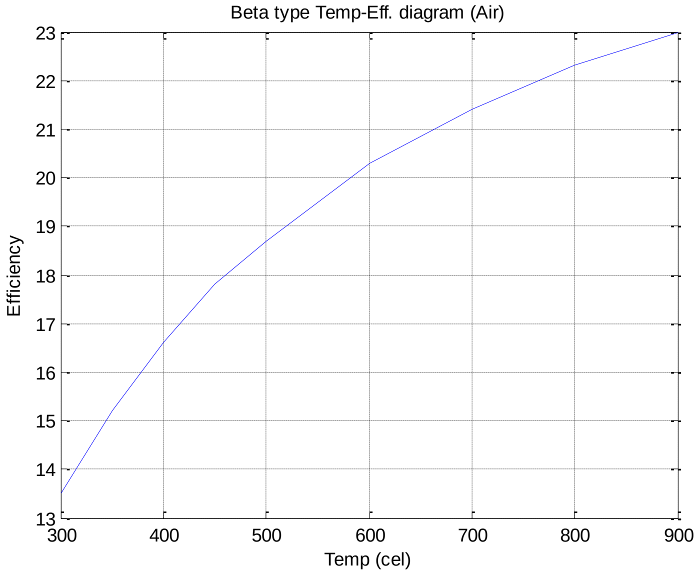
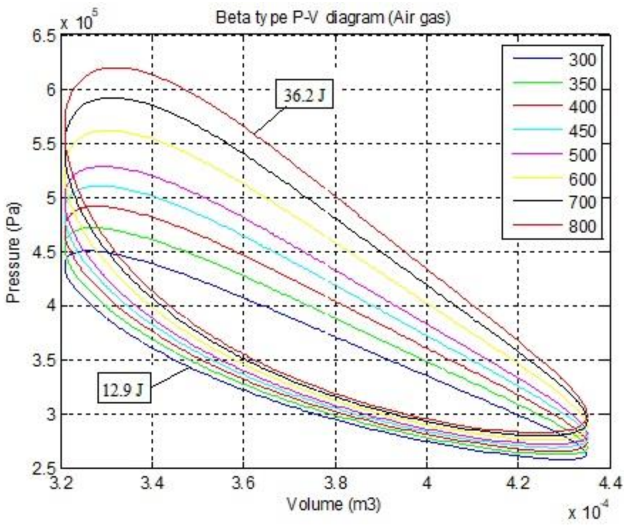
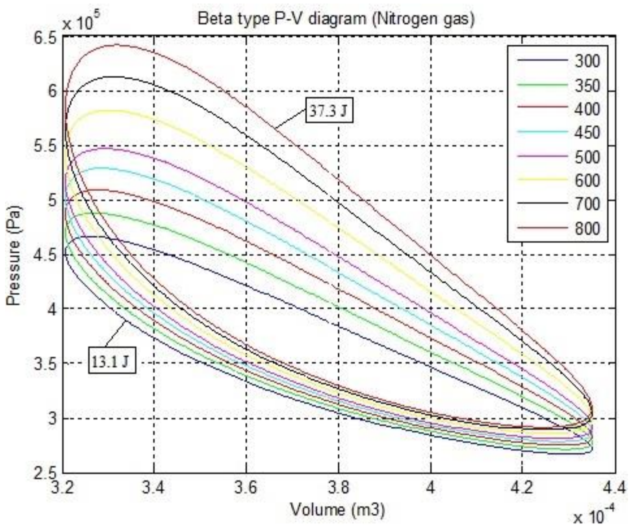


Figure 6 Efficiency versus applied temperature .

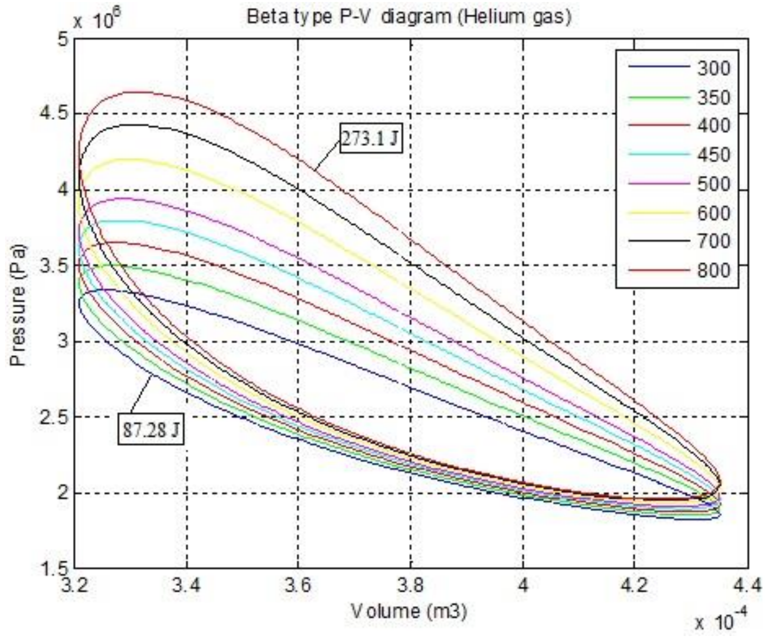
Fig. 7(a,b,c) show the *P-V* diagrams for three different working gases, air, nitrogen and helium with respect of changing temperature (T_h=300,400,500,600,700,800 K). While the cycling area for T_h=300 K is narrow as in figure, it has been enlarged as temperature is increased to T_h=800 K in the three cases of air, nitrogen and helium. Although, all diagrams have been obtained under 4 bars charge pressure, 700 rpm engine speed and a 30 °C cold end temperature, which has been kept constant by a cooled water system, the generated works differs as usual by working gases and temperature differences. For instance, when the applied temperature is 300 °C, work generation has been obtained as 12.9 J, 13.1 J and 87.28 J for air, nitrogen and helium respectively. Indeed, work generation increases upto 36.0 J, 37.3 J and 273 J, when temperature has been raised to 800 °C. The increase in the cyclic work generation has been found as three times for each gas, while the hot end temperature is ranging from 300 °C to 800 °C. One can conclude that using of helium instead of air or nitrogen gives more generating work than increasing the applied temperature.



(a)



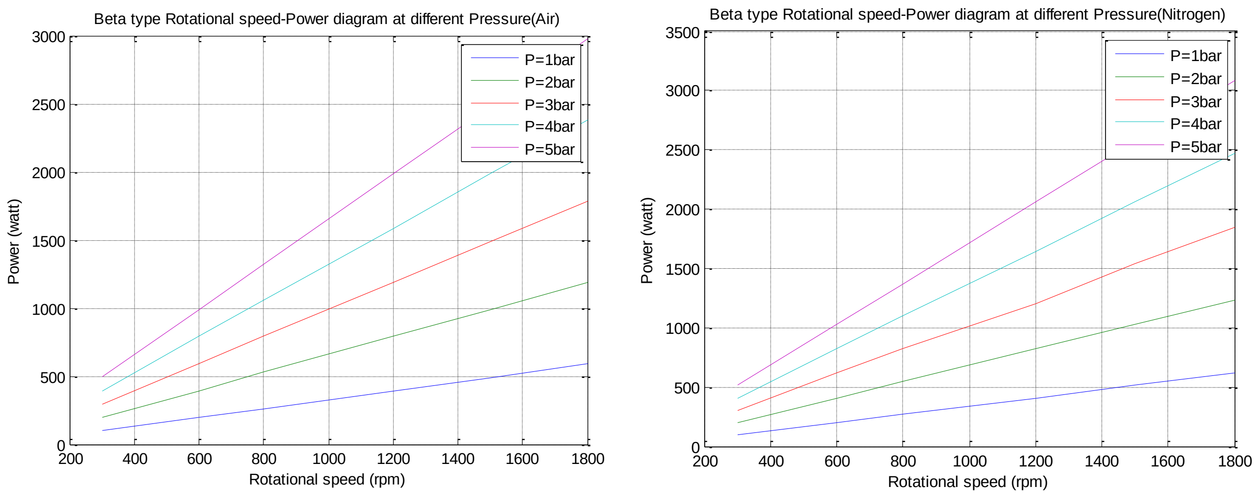
(b)



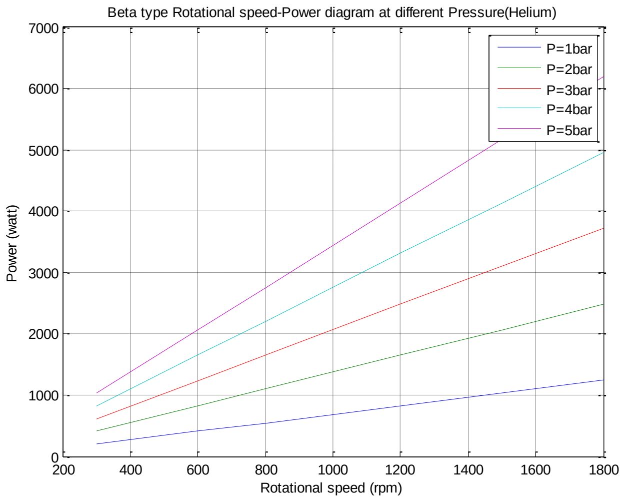
(c)

Figure. 7 Pressure vs Cylinder volume in the case of (a) Air, (b) Nitrogen, (c) Helium

Figs. 8 (a, b, c) proves that the increasing pressure also affects the power generation besides the type of working gas. For instance, power can be increased up to 6 kW from 1.3 kW by enhancing the pressure fivefold at 700 K. For the air and nitrogen gases power reaches 3 kW at 5 bars, while using of helium gas increasing the power to above 6 kW at the same pressure Thus, the pressure-dependence and working gas are an important finding for such engines.



(a) (b)



(c)

Figure 8 Power versus engine speed for various applied pressures 1,2,3,4,5 bars in the case of (a) Air, (b) Nitrogen, (c) Helium.

5. Conclusions

In this study, a new numerical tool has been introduced in order to estimate the parameters of a beta-type Stirling engine. The code works under Matlab/Simulink package. The findings are considered to be parallel to the literature in terms of speed, pressure, temperature and power using three types of working gases air, nitrogen and helium. One can easily apply different working gases and examine their effects to the machine output power values, besides the changing of applied pressure and temperature. The designed engine can produce 1.3 kW power at 1 bar and 700 K hot end temperature in the air medea. That power can be increased further 6 kW by using of helium as a working gas and increase pressure to 5 bars. According to literature, increasing pressure can decrease the thermal efficiency of the engine. Since the input heat to the expansion cylinder is increased with the increase of applied temperature. Then, the rotational speed and the output power are higher by using of helium or increasing the heat source temperature. However, the temperature T_h should be strictly limited by the thermal expansion of the materials of the heating head and that can produce a natural limit for the output power.

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